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MODIFIED INSTRUMENTAL VARIABLES METHOD WITH SLIDING WINDOW

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The task of constructing a mathematical model of the object under study is not only of interest in itself, but is also part of the optimization problem, the quality of the solution of which depends significantly on the complexity of the model used. Therefore, in practice, it is often justified to simplify the mathematical model of an object and present it in the form of regression equations. In this case, the problem of estimating model parameters of the form

$$Y_n = X_n C_n^* + \Xi_n \tag{1}$$

where $Y_n = \left(y_1, y_2, ..., y_n\right)^T$ is the vector of output signals $n \times 1$; $X_n = \left(x_1^T, x_2^T, ..., x_n^T\right)$ – matrices of input variables $n \times N$; $C_n^* = \left(c_{1n}^*, c_{2n}^*, ..., c_{Nn}^*\right)^T$ – vector of estimated parameters $N \times 1$; $\Xi_n = \left(\xi_1, \xi_2, ..., \xi_n\right)^T$ – interference vector; n is discrete time and, subject to the usual assumptions of classical regression analysis, can be successfully solved using the least squares method (LSM). In this case, the essential assumptions are the absence of correlation between useful signals and interference, the absence of interference in the observed visible signals and the constancy of the estimated parameters c^*.

If, in the presence of correlated noise, the use of a generalized OLS is quite effective, then violation of other assumptions sharply reduces the effectiveness of the OLS.

Thus, if variables are measured with noise and there is a correlation between them, OLS estimates will be biased. In this case, it is advisable to use methods based on the use of specifically selected instrumental variables [1]. The instrumental variable method (IVM) estimate has the form

$$C_n = \left(W_n^T X_n\right)^{-T} W_n^T Y_n \ , \tag{2}$$

where W_n is the $n \times N$ matrix of instrumental variables.

In [2], a modification of the MIP was proposed, constructed by analogy with the generalized LSM and effective with corrected interference Ξ_n.

If the assumption of stationarity of parameters is violated, then the assessment, firstly, must be recurrent and, secondly, contain some mechanism for assessing the value of the information used. Such a mechanism can be either exponential weighting of information, which gives greater weight to newly received information, or some kind of sliding window, which also gives equal weight to a certain (usually fixed) amount of information taken into account [3].

An exponentially weighted recurrent MIP can be obtained by analogy with [2]. The goal of this work is to obtain a rcurrent form of MIP (RMIP) with a sliding window.

Let us denote the estimate obtained at the n-step over L previous steps, by analogy with [4] as follows:

$$C_{n/L} = \left(W_{n/L}^T X_{n/L}\right)^{-T} W_{n/L}^T Y_{n/L} \ , \tag{3}$$

where $X_{n/L} = \left(x_{n-L}^T, x_{n-L+1}^T, ..., x_n^T\right)^{-1}$, $W_{n/L} = \left(W_{n-L}^T, W_{n-L+1}^T, ..., W_n^T\right)^{-1}$ are $N \times (n-L)$ matrices; $Y_{n/L} = \left(y_{n-L}^T, y_{n-L+1}^T, ..., y_n^T\right)^{-1}$ – vector $n-L$; $L > n$ is the size of the sliding window (memory of the algorithm).

For the purpose of recurring forms, the modification of MMP is necessary obtained $jn(n+1)$ – step by L measuring,

$$C_{(n+1)/L} = \left(W_{(n+1)/L}^T X_{(n+1)/L}\right)^{-T} W_{(n+1)/L}^T Y_{(n+1)/L} \tag{4}$$

If you move to (4) the matrix and the vector are the same size as those in (3). If the ocean is half-way $(n+1)$-thous, it will take a larger number of measurements No, you can read the following statement:

$$C_{(n+1)/L} = \left(W_{n/L}^T X_{n/L} + w_{n+1} x_{n+1}^T - w_{n-L} x_{n-L}^T\right)^{-1} \left(W_{n/L}^T Y_{n/L} + w_{n+1} x_{n+1} - w_{n-L} y_{n-L}^T\right). \tag{5}$$

As can be seen from (5) and as noted in [4], a feature of algorithms with a fixed window is that the observation matrix $X_{n/L}$ (and, therefore, the IP $W_{n/L}$ matrix) and the vector $Y_{n/L}$ at each step are formed either by including newly received information $x_{n+1}(w_{n+1})$ and y_{n+1} or by initially eliminating outdated information and then introducing new information.

The corresponding algorithms implement the "accumulation-reset" and "reset-accumulation" rules.

In this case, the arrival of a new observation at the $(n+1)$th step allows us to obtain a new estimate for the $(L+1)$th observation

$$C_{(n+1)/(L+1)} = \left(W^T_{(n+1)/(L+1)} X_{(n+1)/(L+1)}\right)^{-1} W^T_{(n+1)/(L+1)} Y_{(n+1)/(L+1)} \qquad (6)$$

where $X_{(n+1)/(L+1)} = \left(X_{n/L} \vdots x^T_{n+1}\right)$, $W_{(n+1)/(L+1)} = \left(X_{n/L} \vdots w^T_{n+L}\right)$ are $N \times (L+1)$ matrices.

Let's denote

$$P_{n/L} = \left(W^T_{n/L} X_{n/L}\right)^{-1}. \qquad (7)$$

Then (6) will take the form

$$C_{(n+1)/(L+1)} = P_{(n+1)/(L+1)} W_{(n+1)/(L+1)} Y_{(n+1)/(L+1)} \qquad (8)$$

Where

$$P_{(n+1)/(L+1)} = P_{n/L} + w_{n+1} x^T_{n+1} . \qquad (9)$$

$$c_{(n+1)/L} = c_{(n+1)/(L+1)} - \frac{P_{(n+1)/(L+1)} w_{n-L}}{1 - x^T_{n-L} P_{(n+1)/(L+1)} w_{n-L}} \left(y_{n-L} - c^T_{(n+1)/(L+1)} x_{n-L} \right).$$

Applying the matrix inversion lemma to (9), we obtain

$$P_{(n+1)/(L+1)} = P_{n/L} - \frac{P_{n/L} w_{n+1} x^T_{n+1} P_{n/L}}{1 + x^T_{n+1} P_{n/L} w_{n+1}},$$

and substituting (10) into (8) and taking into account the block representation and, after simple transformations we have the following expression for the estimate corresponding to the accumulation of information

$$c_{(n+1)/(L+1)} = c_{n/L} + \frac{P_{n/L} w_{n+1}}{1 + x^T_{n+1} P_{n/L} w_{n+1}} \left(y_{n+1} - c^T_{n/L} x_{n+1} \right).$$

If outdated information is reset, the dimension of the matrices and vectors used in constructing the assessment decreases. Wherein

$$P_{(n+1)/L} = P_{(n+1)/(L+1)} - w_{n-L} x^T_{n-L}$$

or after applying the matrix inversion lemma

$$P_{(n+1)/L} = P_{(n+1)/(L+1)} + \frac{P_{(n+1)/(L+1)} w_{n-L} x^T_{n-L} P_{(n+1)/(L+1)}}{1 - x^T_{n-L} P_{(n+1)/(L+1)} w_{n-L}}.$$

The corresponding expression for the estimate after simple transformations will have the form

$$c_{(n+1)/L} = c_{(n+1)/(L+1)} - \frac{P_{(n+1)/(L+1)} w_{n-L}}{1 - x_{n-L}^T P_{(n+1)/(L+1)} w_{n-L}} \left(y_{n-L} - c_{(n+1)/(L+1)}^T x_{n-L} \right).$$

Thus, the recurrent MIP algorithm with a sliding window includes two procedures, the first of which, described by expressions (10) and (11), corresponds to the accumulation of new information (calculation of an auxiliary estimate when new information arrives), and the second, represented by relations (12) and (13), – resetting the outdated one.

The following must be said about the choice of initial values of the matrix (namely, for this algorithm it is involved in the calculations of matrices and). Since this algorithm starts working only after the number of observations becomes equal to the number of unknown parameters, i.e. at the initial stage coincides with the usual RMIP, then the choice occurs as in the usual RMIP.

By analogy with the above, it is not difficult to obtain relations for the RMIP algorithm with a sliding window, operating according to the "dump-accumulate" rule.

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