MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

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Methodological recommendations
for practical classes and organizing independent work
on an academic discipline

"METROLOGY AND STANDARDIZATION"

(for the first (bachelor) level students of higher education all education forms
specialty 192 – Building and Civil Engineering,
of education program "Industrial and civil construction")

Kharkiv
O. M. Beketov NUUE
2024

Methodological recommendations for practical classes and organizing independent work on an academic discipline “Metrology and Standardization**” (**for the first (bachelor) level students of higher educational all education forms specialty 192 – Building and Civil Engineering, of education program “Industrial and civil construction”) / O. M. Beketov National University of Urban Economy in Kharkiv ; com. : E. S. Sedyshev, A. V. Naboka, D. G. Petrenko, Yu. M. Krul. – Kharkiv O. M. Beketov NUUE, 2024. – 37 p.

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Recommended by the department of building structures, record № 12 on March 2, 2024

CONTENT

INTRODUCTION

The course “Metrology and standardization” is one of the final among the disciplines dedicated to technologies, materials and structures in construction.

The main goal of the course is to give future specialists an idea of the place of the sciences “Metrology” and “Standardization” in the national economy and construction, as well as in interstate cooperation.

Methodical instructions are intended to acquaint students of specialty 192 – Construction and civil engineering with the subject of practical classes in the discipline “Metrology and standardization”, which is taught in 3–4 courses of full-time and extramural forms of education.

In the practical classes, students are offered problems that have practical significance and are found in technical fields, including statistical processing of test results of building materials and structures, for consideration and solution.

For practical classes, it is recommended to use various literary sources: textbooks, reference books, manuals, methodical instructions, normative literature. For the convenience of students, brief information on the theory of the question and formulas for calculations are provided for each section of the methodological instructions, and tables of reference coefficients are provided in the appendix.

This methodical guide is compiled in accordance with the program of the course “Metrology and standardization” for the preparation of bachelors of specialty 192 – Construction and civil engineering. Its content corresponds to the nature of the teaching of this discipline at the Department of Building Structures of O. M. Beketov National University of Urban Economy in Kharkiv based on 13 – 18 hours of practical classes.

1 BASIC MEASUREMENT METHODS

Measurement is the process of experimentally finding the values of a physical quantity using special measuring tools. To measure some physical quantity Q means equalize her with another size q, accepted per unit measurement and express the first in particles the latter in mathematical form:

$$Q = k \cdot q, \tag{1.1}$$

where k is any positive integer or fractional number, which shows how much Q is greater or less than q.

The value of a physical quantity can be obtained as a result of direct (*immediate) measurements* (measurement masses on scales, temperatures-thermometer, length-using linear measures etc.), or *indirect (mediocre)*, for which she is staying as a function of directly measured values(density by mass and geometric dimensions, strength of concrete by signal transit time in non-destructive methods measurements, definition of roll buildings according to angular and linear results measurements etc.).

The accuracy of the measurements can be assessed only in the presence of repeated measurements. In order to control and assess the accuracy, it is necessary to make at least two measurements of the same physical quantity.

Measurements are divided into necessary, which give only one result of the measured quantity, and repeated (additional), as a result of which several values of the measured quantity are obtained. Assessment of accuracy measurements may be done only if there are duplicate measurements. In order to control and evaluate accuracy, it is necessary to do at least two measurements, one and that same physical size.

The following main methods are used in technical industries and construction measurements:

– *direct assessment method,* according to which the value of the quantity is determined directly from the measuring device (pressure – by manometer, characteristics current-ammeter, voltmeter), this is probably the most common method measurements;

– *method comparison with measure,* for which measurable value is compared with size, reproduced measure (comparison of mass on scales with kettlebells, linear measurements with a tape measure, where is the length receive as a set of linear values);

– *method of coincidences,* for which difference between the measured size, and size, reproduced measure, measured by coincidentally evaluations scales; all linear are measured by this method values measuring devices with Vernier's (calipers, micrometers) and angle devices with Vernier's (theodolites). For example, a caliper (Fig. 1.1) measures the length using two rulers, the prices of divisions of which are in a certain ratio.

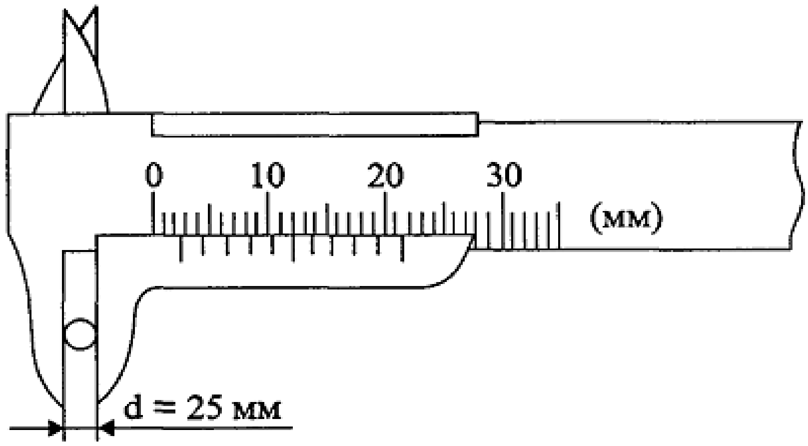


Figure 1.1 – To measure the diameter of the wire (2,5 mm) with a caliper by the coincidence method (Vernier's)

2 MEASUREMENT ERRORS AND THEIR CLASSIFICATION

The condition of any measurement is the existence of a valid value of the measured value. Due to the fact that external conditions can change during the test, multiple measurements of the same value do not turn out to be the same. The difference between the measurement result x and it's true value a is called the absolute measurement error Δ etc.

$$\Delta = x - a. \qquad (2.1)$$

Relative error measurements –

$$\frac{\Delta}{x} = \frac{x - a}{x} \quad (2.2)$$

Absolute error measurements, as a rule, consist of two components: *systematic* and *random*.

Systematic errors have certain sign and accumulate force taint functional by law as a result unilaterally active factors. They should be excluded from the results measurements by introduction corrections, which are calculated according to the functional law of error, or be compensated by the appropriate organization of the processing method measurements.

Random errors, what arise as a result of imperfect technology and methods measurements, changes in external conditions, due to rounding of results readings from devices etc., inevitable and fully exclude their, from the results measurements impossible

The impact of errors on test results depends significantly on the purpose of the test. If the tests are carried out in order to identify the nature of deformation and destruction of the structure, then the impact of errors will be less pronounced than when tests are carried out in order to obtain the numerical parameters of the investigated systems. In the latter case, more thorough preparation of the experiment is necessary.

Test errors increase with the complexity of measuring equipment and test methods. It is also worth remembering about the spontaneous change in instrument readings, that is, about the so-called “zero drift”. In the case of protractors, this is due to the gradual pulling out of the wire and weakening of the fastening; in pasted tensors – with hardening of the glue.

Statistical probabilistic methods are used when processing materials for tests of building materials and structures, because the strength and elasticity parameters of materials, load variations, and test errors are random, stochastic in nature.

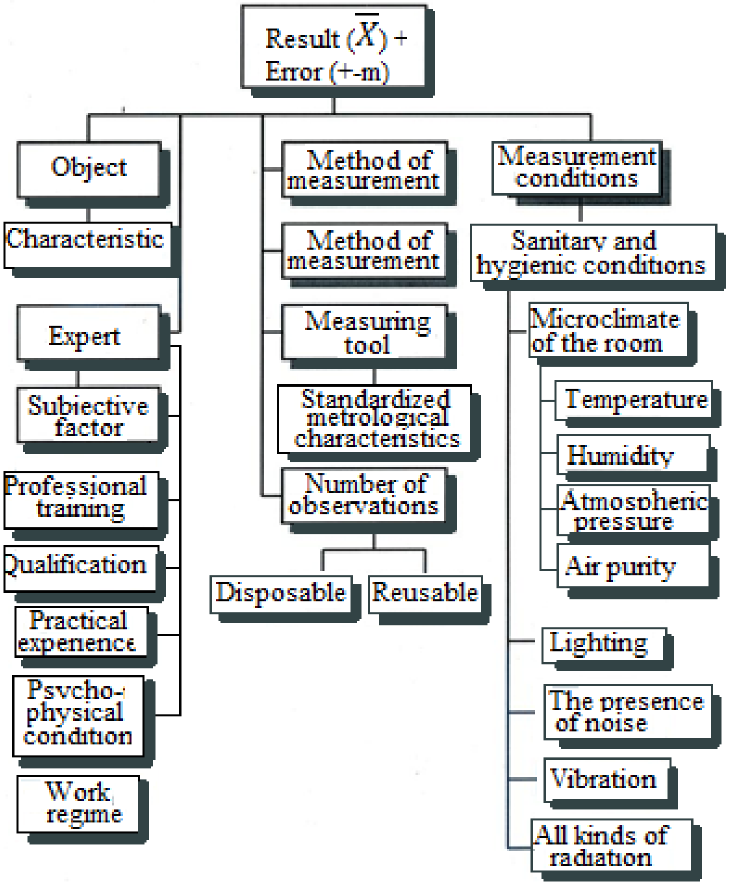


Figure 2.1 – Factors affecting measurement results

To increase accuracy *measurements*, during their implementation, it is worth follow following rules:

– if there is a systematic error in determining, i.e. her size significantly greater than the random error inherent in this data method, then it is enough to perform measurement only twice, because increasing their number will not increase the accuracy of the measurement result. Next, it is necessary to calculate and introduce a correction to the final result;

– if systematic errors are less *random*, then, by increasing the number measurements, you can get a result whose accuracy will be higher than that of one measurement. Having an array of measurement values, it is possible to carry out its mathematical processing.

3 SYSTEMATIC MEASUREMENT ERRORS AND METHODS OF THEIR ELIMINATION

3.1 General characteristics of systematic errors

Systematic errors are classified by the nature of the change and the reasons for its occurrence.

Depending on the nature of the change, systematic measurement errors are divided (Fig. 3.1) into permanent and variable (progressive, periodic and errors that change according to a complex law).

Depending on the causes, systematic measurement errors are: instrumental (equipment, instrument errors), measurement method errors, errors depending on changes in measurement conditions, and subjective.

Systematic errors distort the measurement result, so they must be excluded from the measurement result by introducing corrections or by adjusting the devices to minimize the systematic error.

The results of observations obtained in the presence of errors are called uncorrected.

Taking into account the influence of systematic errors can be done by:

– elimination of sources of errors before the start of measurement (according to the passport data of the measuring instrument, according to the results of verification of the measuring instrument, using measurement methods with minimal errors);

– determination of corrections and their introduction to the measurement result;

– estimation of the limits of non-excluded systematic errors.

To increase the accuracy of measurements with the exception of constant systematic errors the following methods are used: measurement with substitution, contrast, sign error compensation, randomization etc.

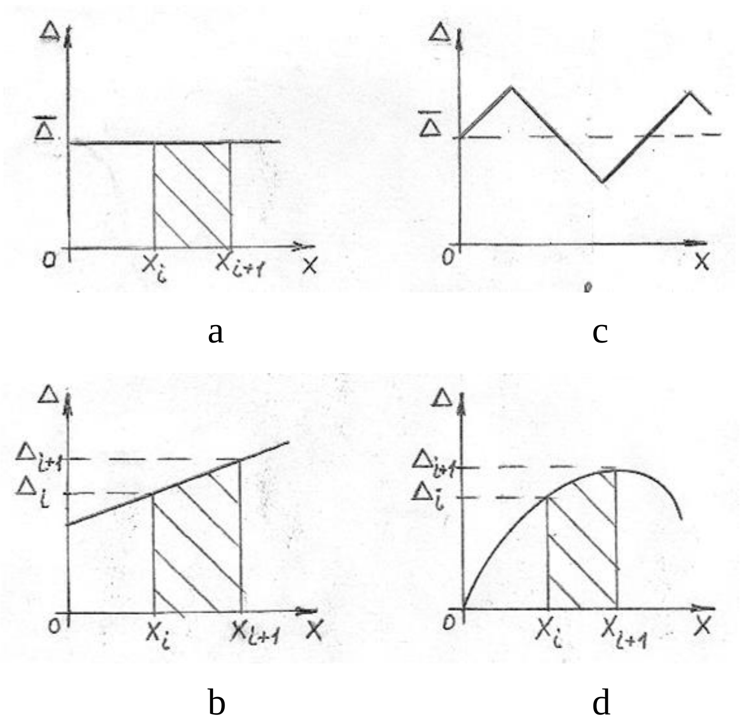


Figure 3.1 – Changes in systematic errors from measurement to measurement:
a – constant errors; b – progressive variables; c – periodic variables;
d – errors that change according to a complex law

3.2 Method of measurement with substitution

The method of measurement with substitution is a variant of the general method of comparison with a measure. The comparison is performed by replacing the measured value with a measure with a known value of the value so that no changes occur in the state of the measuring devices used.

Example 3.1. Weighing on spring scales that have constant systematic errors.

Weighing is performed in two steps (see Fig. 3.2). First, a weight is placed on the scale m_x and note the position of the pointer (on the note N). Then the weighed object is being replaced with weights of the same weight m_0, so that the pointer takes its position on the mark again N. Then $m_x = m_0$ and the systematic error of the scales will not be reflected in the weighing result.

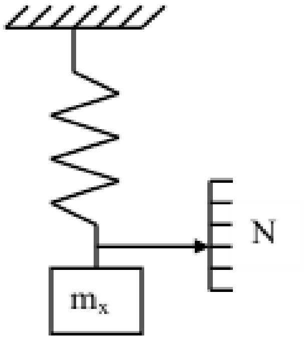


Figure 3.2 – Illustration of the measurement method with substitution (Borda's method)

3.3 The addition method

When applying the method of supplementing the value of the measured quantity, it is supplemented with a measure of the same quantity with the calculation that their sum, which is equal to the previously set value, acts on the comparison device.

Example 3.2. Weighing on spring scales, which have constant systematic errors (Fig. 3.2).

If m_x (the mass to be measured) add the mass of the measure m_0 so that their sum is equal to the position on the scale mark N, then $m_x = N - m_0$.

The measurement error decreases because the mass m_x we calculate from two given values.

3.4 Contrast method

(variety of the method of comparison with a measure)

The measurement is performed twice and is performed in such a way that in both cases the cause of the constant error indicates the result of observations that are different, but known according to the regularity of the action.

Example 3.3. Weighing on unequal scales (Fig. 3.3).

Condition of balance of weights: $m_x \cdot l_1 = m_0 \cdot l_2$,

where m_x – mass of the weighted cargo; m_0 – mass of weighs; l_1, l_2 - the length of the shoulders of the scales.

Let's rewrite the formula in the form $m_x = m_0 \frac{l_2}{l_1}$.

If the shoulders are the same, then $m_x = m_0$.

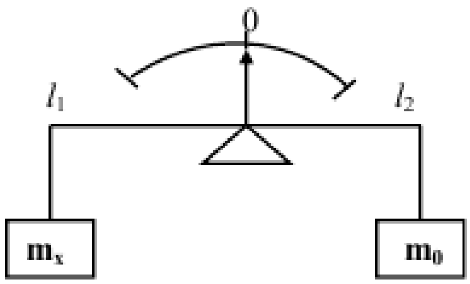


Figure 3.3 – Illustration of the contrast measurement method

If $l_1 \neq l_2$, then a systematic error occurs every time when weighing $\Delta_c = m_0\left(\frac{l_2}{l_1} - 1\right)$. To eliminate this error, weighing is performed in two stages. First the cargo m_x, is balanced by weights m_{01}. Then $m_x \cdot l_1 = m_{01} \cdot l_2$. Then the cargo m_x move to the cup of scales where the weights were and re-balance with the weights m_{02} and $m_{02} \cdot l_1 = m_x \cdot l_2$. Eliminated from the equation $\frac{l_2}{l_1}$ we will get $m_x = \sqrt{m_{01} \cdot m_{02}}$.

3.5 Differential method

The differential method is characterized by measuring the difference between the measured value and the known value, which is reproduced by the measure. The method makes it possible to obtain a result of relatively high accuracy when using relatively rough measuring tools.

Example 3.4 Measuring the length of the sample X, if the length of the measure is known $l < X, X = l + a,$ *where a – is a measurement value.*

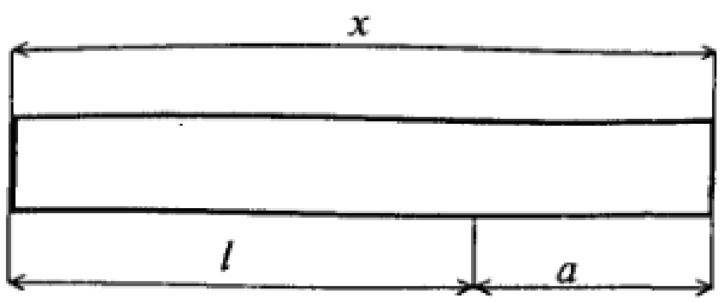


Figure 3.4 – Differential method of length measurement (to example 3.4)

3.6 Zero method

The zero method is similar to the differential method, but the difference between the measured value and the measure is reduced to “0”. This should be controlled by the device (zero indicator).

The method has advantages in that the measure can be many times smaller than the value being measured.

Example 3.3 *Unequal scales (Fig. 3.3).*

Equilibrium equation when the arrow points to zero $m_x \cdot l_1 = m_0 \cdot l_2$.

Then, for example, the measured value can be calculated as $m_x = m_0 \frac{l_2}{l_1}$.

3.7 Method of randomization

The essence of the randomization method is that the same value is measured by different random methods (devices). Systematic errors of each of them for the entire population are different random variables. Therefore, when the number of methods (devices) used increases, systematic errors are mutually compensated.

3.8 The sign error compensation method

The method involves a measurement with two observations, which are performed so that a constant systematic error enters the result of each of them with different signs. It is excluded when calculating the average value.

For example, to compensate for the error caused by the Earth’s magnetic field during geophysical measurements, the first measurement is carried out at any position of the device, and for the second measurement, the device is rotated in the horizontal plane by 180^{o}. If in the first case the magnetic field was added to the field of the device and caused a positive error, then when the device is turned, the Earth's magnetic field affects the reading of the device with a negative error equal to the previous one.

With sign error compensation, verification is performed at the “zero point” before the work of geodetic levels and theodolites.

3.9 Methods of eliminating variable errors

Analysis of signs of uncorrected errors OK. If the signs of uncorrected errors alternate with any regularity, then a variable systematic error is observed. If the sequence of “+” signs in the error is replaced by a sequence of “– ” signs, or vice versa, then a monotonically variable systematic error is present. If the groups of “+” and “–” signs in the error alternate, then there is a periodic systematic error.

Graphical method. One of the simplest ways to detect variable systematic error in observation results. It consists in a graphic presentation of a sequence of uncorrected values of the results of observations. On the graph, a smooth curve is drawn through the plotted points, which expresses the trend in the change of the measurement result, if it exists. If the trend is not observed, then the variable systematic error can be considered practically absent.

For example, a special case of an error that changes with some regularity is an error progressive according to a linear law, for example, proportional to time (according to the type of Fig. 3.1, b). In this case, the error can be estimated and eliminated as follows. If it is known that when measuring a constant value X_0 the systematic error varies linearly in time, $X_{зм} = X_0 + C \cdot t$ (where C=const), then it is enough to make two observations to exclude it X_1 and X_2 with time fixation t_1 and t_2. Then the desired value of the quantity will be

$$X_0 = \frac{X_1 \cdot t_2 - X_2 \cdot t_1}{t_2 - t_1}.$$

4 ERRORS AND ACCURACY CLASSES OF MEASURING INSTRUMENTS

4.1 Types of errors of measuring devices depending on the nature of the change in the physical quantity

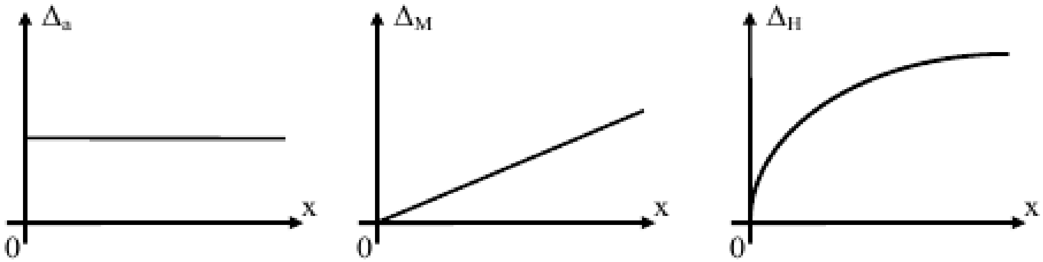
Static error of the measuring device – an error that occurs when measuring a value that is assumed to be constant (measurement of the length, diameter of the rod etc.).

Dynamic error of the measuring instrument – an error that occurs when measuring a variable (in the process of measurement) of a value, for example, measuring the temperature on a furnace using a thermo steam.

There are such concepts as basic and additional error of measurement tools. The main error appears under normal (passport) operating conditions of the measuring instrument. An additional error occurs if the device works in conditions other than normal. The main errors of measurement tools include:

1. Additive (obtained by addition) – additive errors are also called "zero" errors, that is, the device shows constant errors at all values of the quantity being measured. If the additive error is systematic, then it is eliminated by correcting the zero value of the output signal. The occurrence of a random additive error is due to the internal shortcomings of the measuring device (friction, "zero" drift, signal fluctuations).

2. Multiplicative (obtained by multiplication) – multiplicative errors are also called sensitivity errors of measuring devices. The multiplicative error varies linearly with the change in the measured value. The reasons for its occurrence are changes in the conversion factor of individual elements and nodes of measurement systems.



a b c

Figure 4.1 – Graphs of changes in errors: a – additive; b – multiplicative; c – non-linear

3. Hysteresis errors (or reverse course) is an error that is difficult to eliminate. It can arise from backlash, friction on contacts and springs, elastic effects). The hysteresis error is estimated by the variation of the readings of the measuring device (the difference between the readings of the device on forward and reverse strokes):

$$w = x_{nx} - x_{ox} .$$

4.2 Accuracy classes of measuring instruments

The accuracy class is a characteristic of the measuring instrument, which is expressed by the limit of permissible values of its basic and additional errors, as well as other characteristics that affect accuracy. That is, the accuracy class makes it possible to judge the limits of the error of a measuring instrument of this type.

Methods of assigning accuracy classes of devices (Fig. 4.2):

– 1-st method (used for measurements). The serial number of the accuracy class is indicated (1-st, 2-nd etc.). For example, the dynamometer is an exemplary 3-rd accuracy class. The procedure for calculating errors is determined by the documentation (passport) that is attached to the measure;

– the 2-nd method specifies the accuracy class for devices with a uniform scale (Fig. 4.3) and additive errors. The class is given in the form of an R number, which is expressed as a interest of the scale range. The main error of the device should not exceed the number R = (**1,0**; **1,5**; **2,0**; **2,5**; **4,0**; **5,0**; **6,0**);

– the 3-rd method specifies the accuracy class for devices with an uneven scale (Fig. 4.4) and additive errors. At the same time, the basic relative error in interest of the value of the reading is normalized. The number characterizing the accuracy class is given in a circle®;

– the 4-th method is used for devices with comparable additive and multiplicative errors. Accuracy is given by two numbers $^{c}/_{d}$, where the number *c* is responsible for additive, and *d* for the multiplicative component;

– the 5-th method (for devices with an uneven scale). The accuracy class is specified by the number *R* above the sign ∨, which normalizes the main reduced error in interest of the length of the main range of the scale.

It is possible to use a double classification system, i.e., to assign to a single measure or measuring instrument both its category and class of accuracy. It has already been pointed out that the category characterizes the degree to which a measure of measuring instrument approaches its maximum attainable accuracy with the existing technique for measuring the given physical value. The class of accuracy indicates the tolerated and actually attainable limiting measurement error.

The formula for the limits of permissible errors	Examples of limits of permissible fundamental error	Class designation accuracy		Note
		in the documentation	on measuring instruments	
$\Delta = \pm a$	-	Accuracy class M	M	-
$\Delta = \pm(a + bx)$	-	Accuracy class C	C	-
$\gamma = \dfrac{\Delta}{X_N} = \pm p$	$\gamma = \pm 1{,}5$	Accuracy class 1,5	1,5	If X_N is expressed in units of the measured quantity
	$\gamma = \pm 1{,}5$	Accuracy class 1,5	$\underset{\vee}{1{,}5}$	If X_N is expressed by the length of the scale
$\delta = \dfrac{\Delta}{x} = \pm q$	$\delta = \pm 0{,}5$	Accuracy class 0,5	0,5	-
$\delta = \pm\left[c + d\left(\left\|\dfrac{X_K}{x}\right\| - 1\right)\right]$	$\delta = \pm\left[0{,}02 + 0{,}01\left(\left\|\dfrac{X_K}{x}\right\| - 1\right)\right]$	Accuracy class 0,02/0,01	0,02/0,01	-

Figure 4.2 – Accuracy classes of measuring equipment

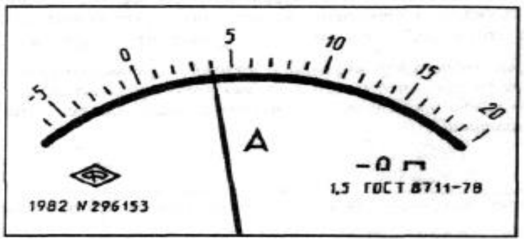


Figure 4.3 – Front panel of an ammeter of accuracy class 1,5

with a uniform scale

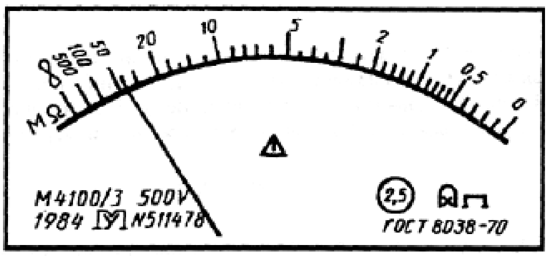


Figure 4.4 – Front panel of a megohmmeter of accuracy class 2,5

with an uneven scale

Example 4.1.

An ammeter of accuracy class 1,0 with measurement limits of – 10 A...+25 A shows 5A. Determine the limit of the permissible absolute error.

Solution.

The accuracy class of the ammeter is set by the limit of the permissible reduced relative error $\delta = \pm\frac{\Delta}{X_N} \cdot 100\% = 1\%$.

Normative value for the range of measurements

$$X_N = |-10| + |+25| = 35 \text{ A.}$$

Limit of permissible absolute error

$$\Delta = \pm\frac{\delta \cdot X_N}{100\%} = \pm\frac{1{,}0 \cdot 35}{100} = \pm 0{,}35 \text{ A.}$$

Example 4.2.

Accuracy class electric energy meter 0,5 shows consumption of 500 kWh. Determine the limit of the permissible absolute error of the counter.

Solution.

Relative error of the counter

$$\delta = \pm\frac{\Delta}{X} \cdot 100\% \leq \pm 0{,}5\%.$$

Limit of permissible absolute error

$$\Delta = \pm\frac{\delta \cdot X}{100\%} = \pm\frac{0{,}5 \cdot 500}{100} = \pm 2{,}5 \text{ kWh}$$

Example 4.3.

An ammeter of the accuracy class 0,2/0,1 with a uniform scale and a range of 0...50 A shows 10 A. Determine the limit of the permissible absolute error.

Solution.

Limit of permissible relative error

$$\delta = \pm\left[c + d \cdot \left(\left|\frac{X_N}{X}\right| - 1\right)\right]\% = \pm\left[0{,}2 + 0{,}1 \cdot \left(\left|\frac{50}{10}\right| - 1\right)\right] = \pm 0{,}6\%.$$

Limit of permissible absolute error

$$\Delta = \pm\frac{\delta \cdot X}{100\%} = \pm\frac{0{,}6 \cdot 10}{100} = \pm 0{,}06\text{A.}$$

Example 4.4.

An ammeter with a scale range of 0...50 A shows a reading of 25 A. Ignoring other types of measurement errors, estimate the limits of the permissible absolute error of this reading when using different devices with accuracy classes: **0,5**/**0,2**; 0,5 and **0,5**.

Let`s define the limit of permissible absolute error:

– for a device of accuracy class **0,5**/**0,2** we have:

$$\delta = \pm\left[c + d \cdot \left(\left|\frac{X_N}{X}\right| - 1\right)\right]\% = \pm\left[0{,}5 + 0{,}2 \cdot \left(\left|\frac{50}{25}\right| - 1\right)\right] = \pm 0{,}7\%,$$

$$\Delta = \pm\frac{\delta \cdot X}{100\%} = \pm\frac{0{,}7 \cdot 25}{100} = \pm 0{,}175 \text{ A};$$

– for a device with an accuracy class 0,5:

$$\Delta = \pm\frac{\delta \cdot X}{100\%} = \pm\frac{0{,}5 \cdot 25}{100} = \pm 0{,}125 \text{ A};$$

– for a device with an accuracy class of **0,5**:

$$\Delta = \pm\frac{\delta \cdot X_N}{100\%} = \pm\frac{0{,}5 \cdot 50}{100} = \pm 0{,}25 \text{ A}.$$

5 RANDOM MEASUREMENT ERRORS

5.1 General characteristics of random errors and point estimates of the true value of the quantities being measured

Random errors (including gross errors and misses) vary randomly during repeated measurements of the same quantity. Random error cannot be excluded from the measurement results, but its influence can be reduced due to repeated measurements of the same value and mathematical processing of experimental data.

Gross errors and omissions appear due to mistakes or incorrect actions of the performer, as well as during short-term various changes (temperature, vibration, shock of the device) during the measurement.

The result of the measured value always contains systematic and random errors, so the errors of the measurement results can generally be considered as a random value.

Then the systematic error is the mathematical expectation of this quantity, and the random error is the centered random variable.

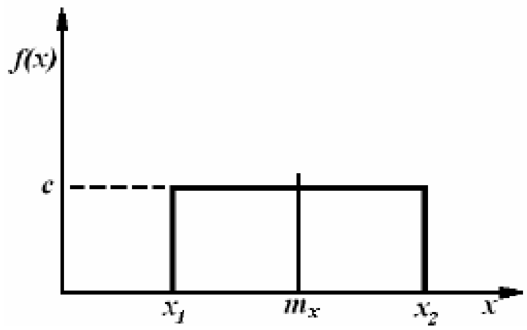
It's a distribution law, which determines the nature of the behavior of various results of individual measurements and serves as a complete description of the quantity, and therefore of the error.

If the continuous random variable x is uniformly distributed in the interval with the boundaries of x1 and x2 (Fig. 5.1, a), then the distribution density in this interval is constant, and outside its boundaries is equal to zero

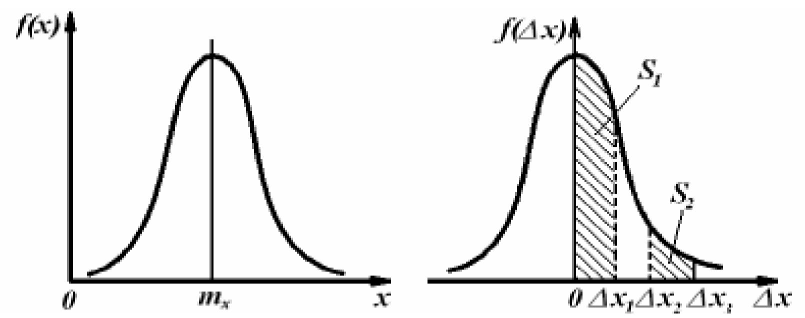
$$c = \frac{1}{x_1 - x_2} \qquad (5.1)$$

Statistical processing of the results of tests of building materials and structures shows that most of the analyzed random variables change according to the normal law of the Gaussian distribution (Fig. 5.1, b) with the density of the distribution according to the formula

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(x_i - \bar{x})^2}{2\sigma^2}}. \qquad (5.2)$$



a



b

Figure 5.1 – Graphs of the distribution of the random variable X (probability density functions):

a – uniform distribution; b – normal distribution (according to the Gauss law)

The distribution law can be characterized by numerical characteristics that are used to quantify the error. The main numerical characteristics of distribution laws are mathematical expectation and variance or root mean square deviation σ_x, which is more often used instead of variance.

Scores expressed by one number are called point estimates of these parameters. Point assessment of mathematical expectation of measurement results $M[X]$ is the average arithmetic value of the measured value $\overline{x} = \sum_1^n x_i/n$, (5.3)

where n – number of unit measurements (sample);

x_i – the result of the i unit measurement.

Point estimation of variance $D[X]$ – there is a statistical dispersion that characterizes the spread (dispersion) of the values

$$S_x = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}. \quad (5.4)$$

Point assessment -the mean square deviation can be determined by the formula

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}} \quad (5.5)$$

Point estimates can be considered as random variables, the values of which depend on the sample size n. The larger the sample and the more accurately defined the distribution function of the values of the measured physical quantity, the more accurately the true (valid) value of the physical quantity is estimated using the arithmetic mean, and the spread of the measurement results using the root mean square deviation.

The coefficient of variation can be used for preliminary estimation of the distribution law of the parameter

$$\nu_x = \frac{\sigma_x}{\overline{x}}\,. \quad (5.6)$$

At $\nu_x \leq 0{,}33\ldots0{,}35$ it can be assumed that the distribution of a random variable obeys a normal law.

5.2 Interval evaluations of measurement results

In tasks where the reliability of measurement results needs to be assessed, knowledge of point estimates is not enough.

When the error distribution is theoretically infinite (with a normal distribution law), then in this case we can only talk about an interval beyond which an error will not spread with a certain probability. This interval is called a confidence interval, characterizing its probability as a confidence probability, and the limits of this interval are error values. The confidence interval and confidence probability are chosen depending on the specific conditions of measurement.

Confidence interval with bounds in the form of an absolute error value Δ can be written as

$$\bar{x} - \Delta < x < \bar{x} + \Delta. \qquad (5.7)$$

In the theory of probability it is proved that within $\bar{x} \pm \sigma_x$ 68,3 % of all measured values are within the limits $\bar{x} \pm 2\sigma_x$ are up to 95,4 %, and within $\bar{x} \pm 3\sigma_x$ – 99,7 %. Therefore, standard confidence probabilities P = 0,68; 0,95; 0,99; 0,999 are often used in the statistical processing of results. For tests of building materials and structures, most often P = 0,95. For soil characteristics during engineering and geological tests, P = 0,85 and 0,95 are taken.

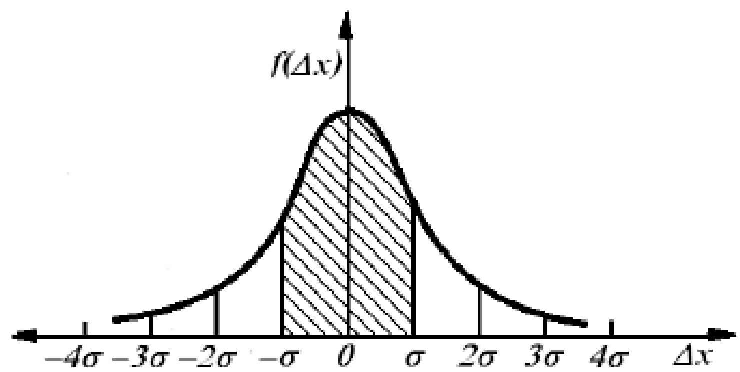


Figure 5.2 – To the concept of confidence interval

However, in order for the mean square deviation to approach the standard, it is necessary to perform many measurements, which is not always possible when testing materials and structures. To find an estimate of the confidence interval with a small number of measurements (samples), we use the Student's distribution in the form of the Student`s coefficient, which depends on the number of measurements n and the probability P:

$$t_p = \frac{\Delta \cdot \sqrt{n}}{\sigma}. \tag{5.8}$$

The Student's coefficient is determined according to special tables depending on the number of experiments and the probability of the value falling into the given interval.

The confidence interval of the investigated value x with a given probability can be written by the expression:

$$\bar{x} - t_p \cdot \frac{\sigma}{\sqrt{n}} < x < \bar{x} + t_p \cdot \frac{\sigma}{\sqrt{n}}. \tag{5.9}$$

It can be seen from formula (5.9) that the values of the confidence intervals increase with an increase in the confidence probability and decrease with an increase in the number of measurements.

Example 5.1.

When measuring the dimensions (height) of a standard cube for concrete tests, a number of results were obtained (*Xi*), cm: 15,3; 15,0; 15,1; 15,2; 15,2; 15,4. It is necessary to determine the confidence interval of the sample with a confidence probability of P = 0,90.

Decision.

The arithmetic mean of the sample $\overline{X} = \frac{91{,}2}{6} = 15{,}2$ cm.

Mean square deviation

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}} = \sqrt{\frac{2 \cdot 0{,}1^2 + 2 \cdot 0{,}2^2}{6-1}} = 0{,}141.$$

Student coefficient according to table A.1 $t_p = 2{,}02.$

Boundary of the confidence interval $\Delta = \frac{t_p \cdot \sigma_x}{\sqrt{n}} = \frac{2{,}02 \cdot 0{,}141}{\sqrt{6}} = 0{,}12$ cm

Confidence interval $\overline{X} \pm \Delta = 15{,}2 \pm 0{,}12$ cm.

This means that the true value of the measured size with a probability of 90 % is within 15,08...15,32 mm for a given number of measurements.

5.3 Exclusion of gross errors when processing measurement results

Chauvin's criterion

If the number of unit measurements $n \leq 10$ the Chauviné criterion is often used. The result holds to a rough error under the condition (5.10).

$$\left|X_i - \overline{X}\right| > \begin{matrix} 1{,}6\sigma_x \text{ at } n = 3; \\ 1{,}7\sigma_x \text{ at } n = 6; \\ 1{,}9\sigma_x \text{ at } n = 8; \\ 2{,}0\sigma_x \text{ at } n = 10. \end{matrix} \tag{5.10}$$

Example 5.2.

When measuring the diameter of the rod Ø $20^{(+0{,}33)}$ the following results were obtained: 20,32; 20,18; 20,38 mm. It is necessary to check whether or not the last result contained a gross error.

Solution.

Sample $n = 3$, so let's apply the Chauviné criterion.

The arithmetic mean of the sample

$$\overline{X} = \frac{20{,}32+20{,}18+20{,}38}{3} = 20{,}29 \text{ mm.}$$

Mean square deviation

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}} = \sqrt{\frac{0{,}03^2+0{,}11^2+0{,}09^2}{3-1}} = 0{,}098.$$

$$\left|X_i - \overline{X}\right| = |20{,}18 - 20{,}29| = 0{,}11 < 1{,}6\sigma_x = 1{,}6 \cdot 0{,}098 = 0{,}158.$$

The result does not contain a gross error, and parts can be sent for rejection only according to the requirements of the product's accuracy class.

Example 5.3.

According to the of example 5.1, check from which sizes the misses start.

Solution.

Sample $n = 6$, so let's apply the Chauviné criterion.

The arithmetic mean of the sample $\overline{X} = 15{,}2$ cm.

Mean square deviation $\sigma_x = 0{,}141$.

$$\left|X_i - \overline{X}\right| = |15{,}4 - 15{,}2| = 0{,}2 < 1{,}7\sigma_x = 1{,}7 \cdot 0{,}141 = 0{,}24 \text{ cm}.$$

The sample has no rough results. Sizes smaller than 14,96 cm and larger than 15,44 cm can be considered misses.

Romanovsky criterion

If the number of unit measurements $n \leq 20...25$, then gross errors (misses) have a significant impact on the results of statistical processing. To exclude misses, the Romanovsky criterion is defined

$$\frac{\left|X_i - \overline{X}\right|}{\sigma_x} < t', \quad (5.11)$$

Where t' – is taken according to table A.2, depending on the specified confidence probability.

If the inequality is not observed, then such a result (X_i) from the sample is rejected.

Example 5.4.

According to the data of example 5.1, we will establish gross errors (misses) according to the Romanovsky criterion with a confidence probability of P = 0,95

The coefficient of the Romanovsky criterion at $n = 6$ according to table A.2 $t' = 2{,}78$.

For X_i = 15,4 cm – $\frac{\left|X_i - \overline{X}\right|}{\sigma_x} = \frac{|15{,}4 - 15{,}2|}{0{,}141} = 1{,}42 < t' = 2{,}78$.

The result is not rough and it should not be excluded from the sample.

The rule of three sigma

If the number of unit measurements $20 < n \leq 50$, then the criterion can be applied$3\sigma_x$. The essence of the rule of three sigma is that if the random variables are normally distributed, then the absolute values of their deviation from the mathematical expectation do not exceed three times the mean square deviation. A questionable result should be excluded from a series of single measurements if $\left|X_i - \overline{X}\right| > 3\sigma_x$. (5.12)

6 EXAMPLES OF STATISTICAL PROCESSING OF MEASUREMENT RESULTS

Example 6.1.

It is necessary to determine the guaranteed class of concrete based on the results of the compression test of 6 cubes.

The compressive strength of each cube is given in table 6.1 – $X_i = f_{ci}$.

In tabular form, we will calculate the arithmetic mean of the sample, the deviation from the arithmetic mean, and the squares of the deviation values.

Table 6.1 – To the statistical processing of cube test results

Cube number	Cube strength, MPa $X_i = f_{ci}$	Deviation from the average, MPa $X_i - \overline{X}$	$(X_i - \overline{X})^2$
1	25,4	– 1,83	3,35
2	26,2	– 1,07	1,14
3	27,7	0,43	0,18
4	28,0	0,73	0,53
5	28,1	0,83	0,69
6	28,2	0,93	0,86
	$\overline{X} = 27{,}27$	$\sum \approx 0$	$\sum \approx 6{,}75$

Mean square deviation –

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}} = \sqrt{\frac{6{,}75}{6-1}} = 1{,}16 \text{ MPa.}$$

Coefficient of variation –

$$\nu = \frac{\sigma_x}{\overline{X}} = \frac{1{,}16}{27{,}27} = 0{,}0425 \text{ (4,25 \%).}$$

For the strength of building materials, the norms recommend taking a confidence probability $P = 0{,}95$, that is, when testing materials or structures, the established measurement error should not exceed 5 % of cases.

At $P = 0{,}95$ – Student's coefficient according to table A.1 $t_P = 2{,}57$ and the coefficient of the Romanovsky criterion according to table A.2 $t' = 2{,}78$.

The maximum deviation from the average in the 1st result. Let’s check it for roughness

$$\frac{|X_i - \overline{X}|}{\sigma_x} = \frac{|-1{,}83|}{1{,}16} = 1{,}58 < t' = 2{,}78.$$

This result is not a mistake and it should not be excluded from the processing results.

Measurement error –

$$\Delta = \frac{t_P \cdot \sigma_x}{\sqrt{n}} = \frac{2{,}57 \cdot 1{,}16}{\sqrt{6}} = 1{,}22 \text{ MPa}.$$

Relative measurement error –

$$\delta = \frac{\Delta}{\overline{X}} = \frac{1{,}22}{27{,}27} = 0{,}044 \text{ (4,47 \%)} < 5\%.$$

Confidence interval of the sample: at $P = 0{,}95$ – $\Delta = \pm 2\sigma_X$

$$(\overline{X} - 2\sigma_x) < X < (\overline{X} + 2\sigma_x),$$

$$27{,}27 - 2 \cdot 1{,}16 = 24{,}95 < X < 27{,}27 + 2 \cdot 1{,}16 = 29{,}59 \text{ MPa}.$$

At the lower limit of the interval, the tested concrete can be classified as C20/25.

In practice, when calculating the calculated characteristics of materials for building structures, correcting coefficients are used, which take into account the sample size and the guaranteed security of the test results.

A formula is used to ***determine the characteristic value of concrete strength***

$$f_{ck} = \overline{f_c}(1 - \beta \cdot \nu), \tag{6.1}$$

where $\overline{f_c}$ – the arithmetic mean of the sample (based on the results of testing individual samples);

$\nu = \frac{\sigma_x}{\overline{f_c}}$ – coefficient of variation, which is calculated during statistical processing of the sample;

β – correction coefficient for security (trust probability) $P = 0{,}95$ according to table 6.2.

Table 6.2 – Correction factor to formula 6.1

Number samples n	5	7	9	12	15	20	50	∞
β	3,34	2,8	2,58	2,39	2,28	2,16	1,94	1,64

Example 6.2.

According to the results of problem 6.1, we will determine the class of concrete using the correction coefficient of the sample.

At $P = 0{,}95$ and $n = 6$ (according to table 6.2) – $\beta = \frac{3{,}34+2{,}80}{2} = 3{,}07$.

From problem 6.1 – $\nu = 0{,}0432, \overline{f_c} = 23{,}27$ MPa.

Strength of concrete

$$f_{ck} = f_c(1 - \beta \cdot \nu) = 27{,}27(1 - 3{,}07 \cdot 0{,}0425) = 23{,}71 \text{ MPa}.$$

According to the calculated characteristic value of strength, concrete can be classified as C20/25.

Example 6.3.

During non-destructive tests to determine the strength of concrete using the Schmidt sclerometer, the results shown in table 6.3 were obtained.

The concrete test hammer or sclerometer from PCE-Instruments is used for the non-destructive compression strength measurement of concrete. In such a way, concrete is tested for its load-bearing capacity and strength. The measuring principle is such that the concrete test hammer hits the concrete. Depending on the concrete hardness, the body rebounds, more or less. The rebound is measured by the device and can be converted to a compressive strength. Since a conventional measurement cannot be carried out in the laboratory when the concrete is built-in, a concrete test hammer is used on site. The measurement is carried out directly on the surface of the concrete and is determined with the help of a conversion table for compressive strength classes. The compressive strength is determined by means of a one-axis, short pressure load without damaging the concrete.

Table 6.3 – To the statistical processing of the results of the Schmidt sclerometer test

Result number	Indication of the device H	$\left(H_i - \overline{H}\right)$	$\left(H_i - \overline{H}\right)^2$
1	33	1,8	3,24
2	30	- 1,2	1,44
3	32	0,8	0,64
4	30	-1,2	1,44
5	29	- 2,2	4,84
6	26	- 5,2	27,04
7	38	6,8	46,24
8	40	8,8	77,44
9	25	- 6,2	38,44
10	29	- 2,2	4,84
	$\overline{H} = 31{,}2$	$\sum = 0$	$\sum = 205{,}6$

The mean square deviation of the sample of test results –

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}} = \sqrt{\frac{205{,}6}{10-1}} = 4{,}779.$$

Coefficient of variation –

$$\nu = \frac{\sigma_x}{\overline{\overline{H}}} = \frac{4{,}7796}{31{,}2} = 0{,}1532 \text{ (15,32 \%).}$$

Let's check the interval of rough results according to the Chauvin test. At $n = 10$ $\left|H_i - \overline{H}\right| < 2\sigma_x = 2 \cdot 4{,}779 = 9{,}6$, that is, there are no rough results in the sample.

You can check result No. 8 for roughness according to the Romanovsky criterion. With confidence probability $P = 0{,}95$ and $n = 10$ according to table A.2 coefficient $t' = 2{,}37$.

$$\frac{\left|H_i - \overline{H}\right|}{\sigma_x} = \frac{8{,}8}{4{,}7796} = 1{,}84 < t' = 2{,}37.$$

Result № 8 is not rough.

The strength of concrete according to the results of non-destructive tests will be taken according to the schedule or calibration tables developed for the sclerometer. Example: $\overline{f_c} = 0{,}85\overline{H} = 0{,}85 \cdot 31{,}2 = 26{,}52$MPa.

Characteristic value of cubic strength of concrete

$$f_{ck} = f_c(1 - \beta \cdot \nu) = 26{,}52 \cdot (1 - 2{,}52 \cdot 0{,}1532) = 16{,}28 \text{ MPa},$$

where by interpolation is the coefficient $\beta = 2{,}58 - \frac{2{,}58-2{,}39}{3} = 2{,}52$.

According to the results of non-destructive tests, concrete can be classified as C12/15.

To increase the accuracy of measurements, if there is a random error, you need to increase the sample (the number of hits with the sclerometer). For example, when choosing $n = 25$ adjustment factor $\beta = 2{,}10$ and at the average strength and coefficient of variation as for the previous calculation characteristic value of the cubic strength of concrete

$f_{ck} = f_c(1 - \beta \cdot \nu) = 26{,}52 \cdot (1 - 2{,}10 \cdot 0{,}1532) = 17{,}98$ MPa, which can increase the class of concrete to C16/20.

Yield strength, or temporary resistance of steel according to the results of sample tests, they are calculated according to the formula

$$f_{yk} = \overline{f_y} \cdot \left(1 - \alpha_y \cdot \sigma_x\right), \qquad (6.2)$$

where $\overline{f_y}$ – the arithmetic mean of the sample (based on the results of testing individual samples);

σ_x – standard deviation of the sample;

α_y – correction coefficient for security (trust probability) $P = 0{,}95$ (according to table 6.4).

Table 6.4 – Correction factor to formula 6.2

Number samples n	10	15	20	25	30	≥ 40
α_y	2,911	2,569	2,396	2,298	2,220	2,125

7 APPROXIMATION OF EXPERIMENTAL DATA BY FUNCTIONAL DEPENDENCIES. REGRESSION EQUATION

If we connect the experimental points on the graph with straight lines, we will get a broken line, the shape of which will not be restored upon repeated measurements.

The distance of the line from the point in each direction horizontally and vertically indicates the value of the error, respectively, along the abscissa and ordinate axis. If the root mean square deviation of the obtained function from the experimental points is minimal, then the equation for the parameters of the function can be obtained $y = \phi(x)$. The method of least squares is based on this.

If as a result of the experiment we get several values of the function y_i at given points x_i, then it can be approximated by some analytical function $\phi(x)$ which includes n-number of constants $a_к$.

A necessary condition for the best rms approximation will be the minimization of the sum of the rms deviations

$$\sum_{i=1}^{n}[y_i - \phi(x_i)]^2 \to 0. \tag{7.1}$$

The correlation between two values is established by the correlation coefficient (for approximation, it should be at least 0,5).

$$r_{xy} = \frac{1/n \sum_{i=1}^{n}(x_i - \overline{x}) \cdot (y_i - \overline{y})}{\sigma_x \cdot \sigma_y}, \ |r_{xy}| \leq 1, \tag{7.2}$$

where are the mean square deviations along the coordinates

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}, \ \sigma_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \overline{y})^2}{n-1}}. \tag{7.3}$$

Example 7.1. *Building a graded dependency based on test results.*

Build a graduated dependence for determining the strength of concrete with Kashkarov’s hammer (by the impression on the surface).

Tests were performed on cubes of the design class of concrete C25/30 on 6 series of control samples (3 cubes each). At least 5 prints were made on each cube. Then, each batch of cubes was tested on a press to failure with the determination of the average strength in the series.

The aggregate of cubes gave concrete strength within limits $f_{ci} = 27\ldots41$ MPa (see table 7.1).

The correlation coefficient at

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(Z_i - \overline{Z})^2}{n-1}} = \sqrt{\frac{117{,}7 \cdot 10^{-4}}{5}} = 4{,}85 \cdot 10^{-2},$$

$$\sigma_y = \sqrt{\frac{\sum_{i=1}^{n}(f_{ci} - \overline{f_c})^2}{n-1}} = \sqrt{\frac{138{,}84}{5}} = 5{,}27.$$

$$r_{xy} = \frac{{}^1\!/_n \sum_{i=1}^{n}(Z_i - \overline{Z}) \cdot (f_{ci} - \overline{f_c})}{\sigma_x \cdot \sigma_y} = \frac{126{,}9 \cdot 10^{-2}}{6 \cdot 4{,}85 \cdot 10^{-2} \cdot 5{,}27} = 0{,}827 > 0{,}5.$$

A regression equation can be constructed.

In the first approximation, we will use a hyperbola in the form as a regression equation

$$f_c = a_0 + \frac{a_1}{H}, \text{ or } f_c = a_0 + a_1 \cdot Z \text{ , where } Z = {}^1\!/_H.$$

The coefficients in the equation (intermediate calculations were carried out in table 7.1):

$$a_1 = \frac{\sum_1^n (Z_i - \overline{Z}) \cdot (f_{ci} - \overline{f_c})}{\sum_1^n (Z_i - \overline{Z})^2} = \frac{126{,}9 \cdot 10^{-2}}{117{,}7 \cdot 10^{-4}} = 107{,}8,$$

$$a_0 = \overline{f_c} - a_1 \cdot Z = 33{,}8 - 107{,}8 \cdot 0{,}72 = -43{,}8.$$

Then graduated dependence $f_c = \frac{107{,}8}{H} - 43{,}8$ (MPa).

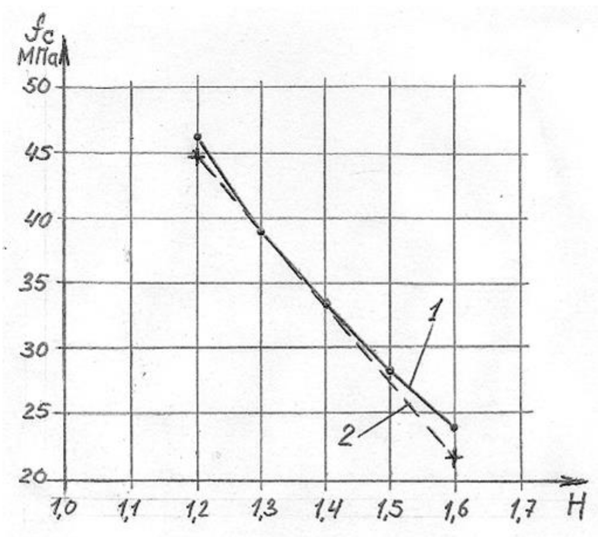


Figure 7.1 – Graphs of regression equations for problem 7.1:

1 – hyperbolic regression; 2 – linear regression

Table 7.1 – Estimated data before construction graduated hyperbola dependence

Number series	H_i	f_{ci}, MPa	$Z_i = \frac{1}{H_i}$	$f_{ci} - \overline{f}_c$	$(f_{ci} - \overline{f_c})^2$	$Z_i - \overline{Z}$ x 10^{-2}	$(Z_i - \overline{Z})^2$ x 10^{-4}	$(f_{ci} - \overline{f}_c) \cdot (Z_i - \overline{Z})$ x 10^{-2}
1	1,50	27	0,658	- 6,8	46,24	- 6,2	38,4	42,2
2	1,48	29	0,676	- 4,8	23,04	- 4,4	19,4	21,1
3	1,38	34	0,725	- 0,2	0,04	0,5	0,25	- 0,1
4	1,40	34	0,714	- 0,2	0,04	- 0,6	0,36	0,1
5	1,30	38	0,767	4,2	17,46	4,7	22,1	19,7
6	1,28	41	0,781	7,2	51,24	6,1	37,.2	43,9
	$\overline{H} = 1{,}39$	$\overline{f_c} = 33{,}8$	$\overline{Z} = 0{,}72$	$\sum = 0$	$\sum = 138{,}84$	$\sum = 0$	$\sum = 117{,}7$ x 10^{-4}	$\sum = 126{,}9$ x 10^{-2}

On the graph (Fig. 7.1), the obtained graduated dependence of "1" in the given interval is practically linear, therefore it is advisable to use a simpler linear equation of the type $f_c = a_0 + a_1 \cdot H$.

Based on the data in table 7.1, we will calculate the coefficients of the linear equation in table 7.2.

Table 7.2 – Calculated data for the construction of the linear graduated dependencies

H_i	f_{ci}, MPa	$H_i – H$	$f_{ci} - \overline{f}_c$	$(H_i – H)^2$ x 10^{-3}	$(H_i – H) \cdot (f_{ci} - f_c)$
1,50	27	+ 0,11	- 6,8	12,1	- 0,748
1,48	29	+ 0,09	- 4,8	8,1	- 0,432
1,38	34	+ 0,01	- 0,2	0,1	- 0,002
1,40	34	- 0,01	- 0,2	0,1	0,002
1,30	38	- 0,09	4,2	8,1	- 0,378
1,28	41	- 0,11	7,2	12,1	- 0,792
$\overline{H} = 1{,}39$	$\overline{f_c} = 33{,}8$	$\sum = 0$	$\sum = 0$	$\sum = 40{,}6$ x 10^{-3}	$\sum = -2{,}35$

The coefficients in Eq

$$a_1 = \frac{\sum_1^n (H_i - \overline{H}) \cdot (f_{ci} - \overline{f_c})}{\sum_1^n (H_i - \overline{H})^2} = \frac{-2{,}35}{40{,}6 \cdot 10^{-3}} = -57{,}9,$$

$$a_0 = \overline{f_c} - a_1 \cdot \overline{H} = 33{,}8 + 57{,}9 \cdot 1{,}39 = 114{,}3.$$

Then graduated dependence $f_c = 114{,}3 - 57{,}9 \cdot H$ (MPa).

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APPENDIX A

Table A.1 – Values Student's coefficient t_p depending on sample numbers n

n	Confidence probability *R*			*n*	Confidence probability *R*		
	0,9	0.95	0.99		0.9	0,95	0,99
2	6,31	12,71	63,70	18	1,74	2,11	2,90
3	2,92	4,30	9,92	19	1,73	2,10	2,88
4	2,35	3,18	5,84	20	1,72	2,09	2,86
5	2,13	2,78	4,60	25	1,71	2,06	2,80
6	2,02	2,57	4,03	30	1,70	2,05	2,76
7	1,94	2,45	3,71	35	1,69	2,03	2,73
8	1,89	2,37	3,50	40	1,68	2,02	2,71
9	1,86	2,31	3,36	45	1,68	2,02	2,69
10	1,83	2,26	3,25	50	1,68	2,01	2,67
11	1,81	2,23	3,17	60	1,67	2,00	2,66
12	1,79	2,20	3,11	70	1,67	1,99	2,65
13	1,78	2,18	3,06	80	1,66	1,99	2,64
14	1,77	2,16	3,01	90	1,66	1,99	2,63
15	1,76	2,15	2,98	10	1,66	1,98	2,63
16	1,75	2,13	2,95	120	1,66	1,98	2,62
17	1,74	2,12	2,92	∞	1,65	1,96	2,58

Table A.2 – Values coefficient of the Romanovsky criterion t', depending on the number of samples n

n	Confidence probability *R*		*n*	Confidence probability *R*	
	0,95	0,99		0,95	0,99
2	15,56	77,96	10	2,37	3,41
3	4,97	11,46	12	2,29	3,23
4	3,56	6,53	14	2,24	3,12
5	3,04	5,04	16	2,20	3,04
6	2,78	4,36	18	2,17	3,00
7	2,62	3,96	20	2,15	2,93
8	2,51	3,71	30	2,08	2,80
9	2,43	3,54	∞	1,96	2,58

Електронне навчальне видання

Методичні рекомендації
до проведення практичних занять
із навчальної дисципліни

«МЕТРОЛОГІЯ І СТАНДАРТИЗАЦІЯ»

(для здобувачів першого (бакалаврського) рівня вищої освіти всіх форм навчання зі спеціальності 192 – Будівництво та цивільна інженерія, освітньо-професійна програма «Промислове та цивільне будівництво»)

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За авторською редакцією
Комп'ютерне верстання *Є. С. Сєдишев, А. В. Набока*

План 2024, поз. 398М

Підп. до друку 11.04. 2024. Формат 60 × 84/16.
Ум. друк. арк. 2,2.

Видавець і виготовлювач:
Харківський національний університет
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Свідоцтво суб'єкта видавничої справи:
ДК № 5328 від 11.04.2017.