**MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE**

**O. M. BEKETOV NATIONAL UNIVERSITY**
**of URBAN ECONOMY in KHARKIV**

Methodical recommendations
for prepare for lectures and practical classes,
self-dependent and test works
on the Subject

# «STRUCTURAL MECHANICS»

(for full-time foreigner students first (bachelor's) level of higher education specialty
192 – Construction and civil engineering)

**Kharkiv**
**O. M. Beketov NUUE**
**2023**

Methodical recommendations for prepare for lectures and practical classes, self-dependent and test works on the Subject «Strength of materials» (for full-time foreigner students first (bachelor's) level of higher education specialty 192 – Construction and civil engineering) / O. M. Beкetov National University of Urban Economy in Kharkiv; com. : A. A. Chuprynin, A. O. Garbuz, M. A. Zasiadko. – Kharkiv : O. M. Beketov NUUE, 2023. – 18 p.

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**CONTENTS**

# INTRODUCTION

Structural mechanics is one of the most important disciplines which makes the foundations of the future specialist of the civil engineering in the field of structural calculation and their elements for strength, stiffness and durability of machines and structures.

The course of Structural mechanics is studied by students after learning the courses of higher mathematics.

For the design of structures, it is necessary to learn the theoretical and practical methods of their calculation, which ensure the reliability of the structure, and its cost-effectiveness. In the conditions of exploitation of structures there is a constant problem of their calculation for high loads. Such calculations and knowledge can be obtained by studying the course Structural mechanics.

When acquiring a course on Structural mechanics, the most effective method is the students independent solving problems and control tasks. It contain theoretical statements and variants of problems for control work and an example of its application.

Each student is given out a problem to perform the calculation and graphical work, and in order to eliminate possible questions when performing home control work, practical classes are held to analyses the basic positions of homework.

After receiving a note on the supervisory work, the student must correct the mistakes indicated by the teacher, make the necessary corrections, even if the work has been approved.

If the work is not approved, correct the same or in a separate drawing and resubmit the work for reconsideration. Independence in the execution of calculated and graphical work is of paramount importance for mastering program material. Detailed instructions for completing and design of the problem are given below.

# 1 FORMATION OF CALCULATION AND GRAPHICAL WORK

1. Work is executed on sheets of standard A4 format.

2. The cover is made of dense paper for drawing. On the title page there should be the name and number of the calculation and graphic problem, name of the discipline, last name, first name of the student, his variant, the name of the faculty, the group, the surname and initials of the teacher.

3. The solution of each problem should begin with the indication of its number, names, writing down complete problem task, numerical output data and draw calculation scheme.

4. The solution to the problem should be accompanied by short explanations, drawings and sketches.

5. Drawings and graphs are executed necessarily on a certain scale. In the drawings one must indicate the letter designation and numerical values of all values used in the calculations.

6. When solving the problem, you must first obtain the result in algebraic form, and then substitute the corresponding numerical values. The results obtained in numerical form should be indicated and units of measurement must be specified.

## 2 OUTPUT DATA AND PROBLEM TO WORK

The initial data for the problem should be taken from tables 1–2 and Figure 1–3. Specific numerical tasks for each control work the student chooses himself from the tables in accordance with his personal cipher on the last three digits of the number of the gradebook. For example, record book № 81135 educational cipher, 135: 1 – being first, 3 – second, 5 – third digit of code.

To perform task 1, you must use the rod scheme from figure 1 and the numerical data of table 1. In the problem it is necessary to calculate longitudinal forces in a given panel by rational methods.

To perform task 2 it is necessary to consider the system in figure 2–3 with the numerical data of table 2. For given three-hinged arch, it is necessary to for a point (at d=$\alpha l$/4 distance from the left support) to calculate of bending moments, shear and longitudinal forces.

Table 1 – Initial data for task 1

| Initial number cipher's | ℓ, m | h, m | Number panelm (leftward) | Second number cipher's | $F_1$, kN | $F_2$, kN | $F_3$, kN | $F_4$, kN | Third number cipher's | Number scheme |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 21 | 4 | 2 | 1 | 20 | 11 | 25 | 10 | 1 | 2 |
| 2 | 18 | 3 | 3 | 2 | 15 | 12 | 20 | 15 | 2 | 3 |
| 3 | 30 | 6 | 4 | 3 | 35 | 13 | 30 | 20 | 3 | 4 |
| 4 | 15 | 4 | 5 | 4 | 40 | 14 | 10 | 25 | 4 | 2 |
| 5 | 21 | 5 | 2 | 5 | 25 | 15 | 25 | 30 | 5 | 3 |
| 6 | 24 | 6 | 3 | 6 | 15 | 16 | 20 | 10 | 6 | 4 |
| 7 | 20 | 3 | 4 | 7 | 10 | 17 | 70 | 15 | 7 | 2 |
| 8 | 22 | 4 | 5 | 8 | 40 | 18 | 25 | 20 | 8 | 3 |
| 9 | 26 | 5 | 2 | 9 | 25 | 19 | 65 | 25 | 9 | 4 |
| 0 | 28 | 6 | 3 | 0 | 15 | 20 | 25 | 30 | 0 | 2 |

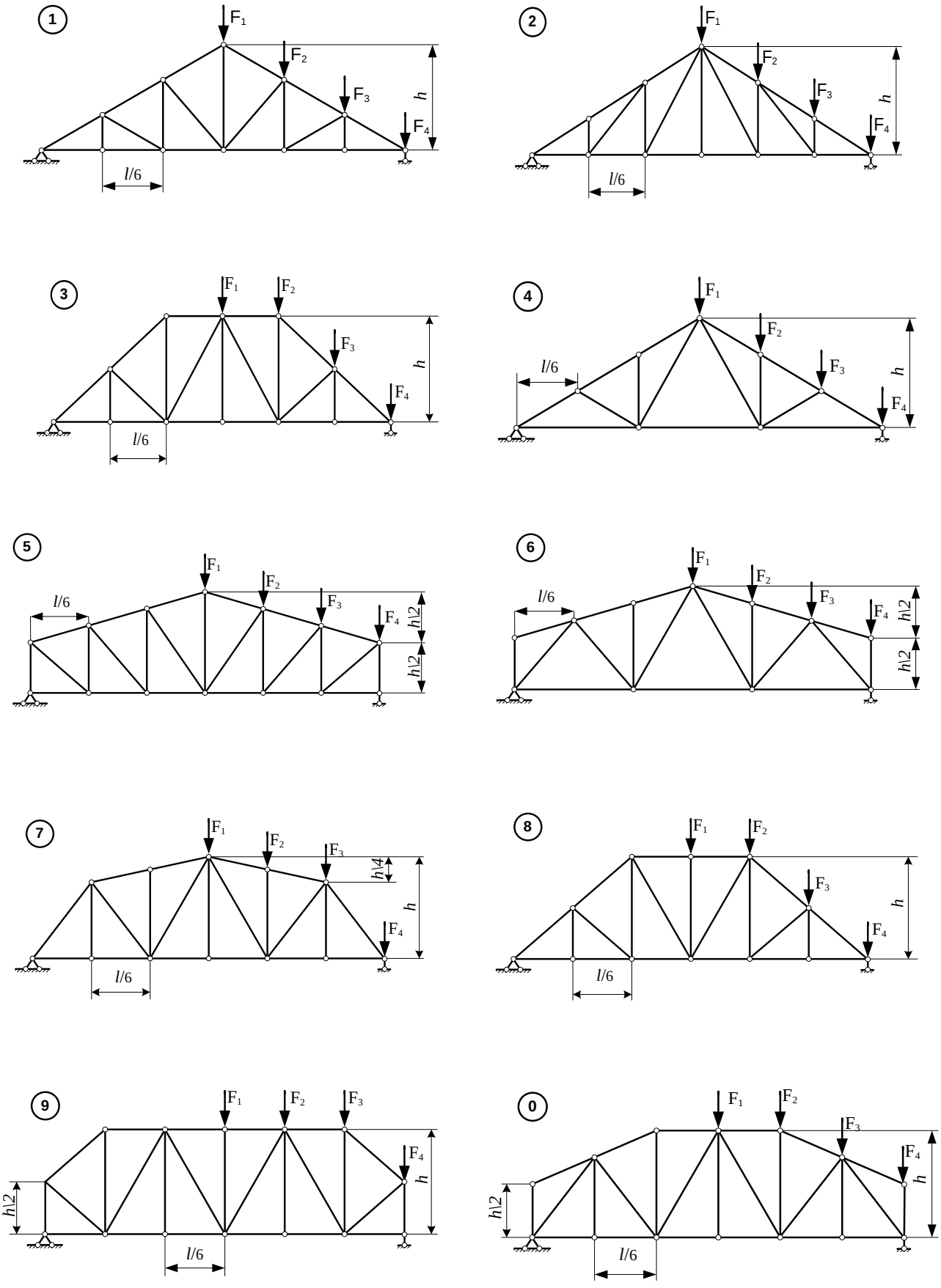


Figure 1 – Scheme for task 1

Table 2 – Initial data for task 2

| Initial number cipher's | $q_1$ kN/$m$ | $q_2$ kN/$m$ | F, kN | Second number cipher's | $l$,$m$ | α | f/$l$ | Third number cipher's | Outline of the arch |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 0 | 60 | 1 | 26 | 0,2 | 0,31 | 1 | Parabolic |
| 2 | 0 | 15 | 40 | 2 | 34 | 0,25 | 0,32 | 2 | Circular |
| 3 | 20 | 0 | 70 | 3 | 18 | 0,3 | 0,33 | 3 | Semi ellipse |
| 4 | 0 | 25 | 80 | 4 | 28 | 0,35 | 0,34 | 4 | Sinusoidal |
| 5 | 30 | 0 | 60 | 5 | 20 | 0,4 | 0,35 | 5 | Parabolic |
| 6 | 0 | 35 | 40 | 6 | 32 | 0,45 | 0,36 | 6 | Circular |
| 7 | 40 | 0 | 70 | 7 | 22 | 0,5 | 0,37 | 7 | Semi ellipse |
| 8 | 0 | 45 | 80 | 8 | 16 | 0,55 | 0,38 | 8 | Sinusoidal |
| 9 | 50 | 0 | 90 | 9 | 14 | 0,6 | 0,39 | 9 | Parabolic |
| 0 | 0 | 55 | 40 | 0 | 24 | 0,65 | 0,4 | 0 | Circular |

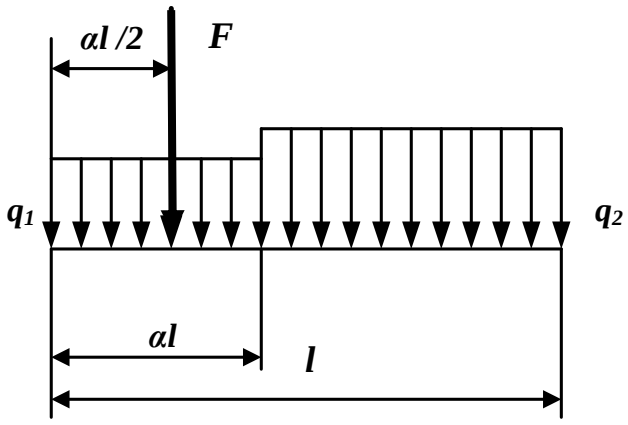


Figure 2 – Load for task 1

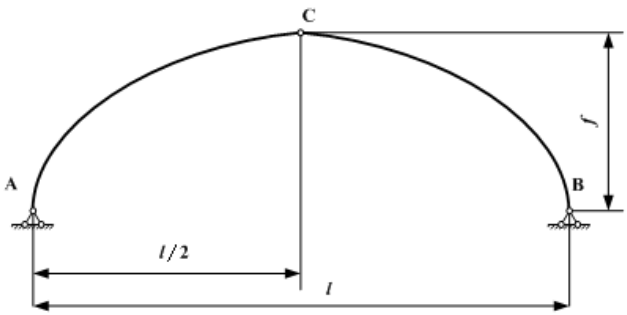


Figure 3 – Scheme for task 2

The main calculated formulas for determining the geometric and trigonometric characteristics of arches different shapes.

*Parabolic arch:*

$$y = \frac{4f}{l^2} x(l - x);\ tg\varphi = \frac{4f}{l^2}(l - 2x) \Rightarrow \sin\varphi;\ \cos\varphi.$$

*Circular Arch:*

$$r = \frac{f}{2} + \frac{l^2}{8f};\ y = \sqrt{r^2 - \left(\frac{l}{2} - x\right)^2} - r + f;\ \sin\varphi = \frac{l/2 - x}{r}; \Rightarrow \cos\varphi.$$

*Ellipse Arch:*

$$y = \frac{2f}{l}\sqrt{x(l - x)};\ tg\varphi = \frac{f}{l} \cdot \frac{l - 2x}{\sqrt{x(l - x)}} \Rightarrow \sin\varphi;\ \cos\varphi.$$

*Sinusoidal Arch:*

$$y = f \cdot \sin\frac{\pi x}{l};\ tg\varphi = \pi\frac{f}{l}\cos\frac{\pi x}{l} \Rightarrow \sin\varphi;\ \cos\varphi.$$

## 3 EXAMPLES OF CALCULATIONS

### Problem № 1

*Output data.*

Make the calculation of the truss shown in Figure 4 at constant load $q = 12\ кН/м$, $F_1 = F_4 = 6$ kN, $F_2 = F_3 = 12$ kN, choose panel № 5.

*Solution*:

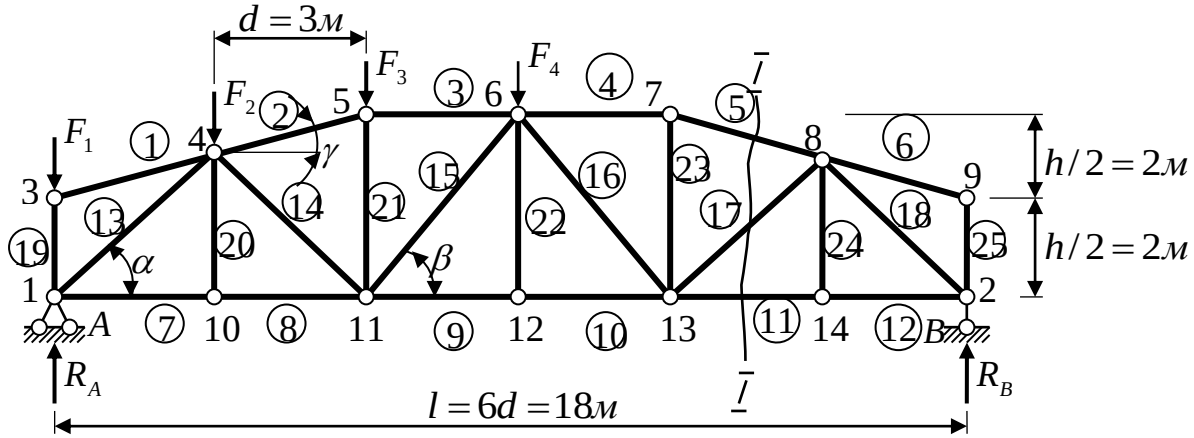


Figure 4 – Task 1

*Support's reactions*:

$$\sum M_A = -F_2 d - F_3 \cdot 2d - F_4 \cdot 3d + R_B \cdot 6d = 0;$$

$$R_B = \frac{F_2 d + F_3 \cdot 2d + F_4 \cdot 3d}{6d} = \frac{12 \cdot 3 + 12 \cdot 2 \cdot 3 + 6 \cdot 3 \cdot 3}{6 \cdot 3} = 9;$$

$$\sum M_B = F_1 \cdot 6d + F_2 \cdot 5d + F_3 \cdot 4d + F_4 \cdot 3d - R_A \cdot 6d = 0;$$

$$R_A = \frac{F_1 \cdot 6 \cdot d + F_2 \cdot 5d + F_3 \cdot 4d + F_4 \cdot 3d}{6d} =$$

$$= \frac{6 \cdot 6 \cdot 3 + 12 \cdot 5 \cdot 3 + 12 \cdot 4 \cdot 3 + 6 \cdot 3 \cdot 3}{6 \cdot 3} = 27;$$

**Check:** $\sum y = R_A + R_B - F_1 - F_2 - F_3 - F_4 = 27 + 9 - 6 - 12 - 12 - 6 = 0.$

We number the joints and truss rods. The first is to appoint a pinned joint support, the second – a sliding joint. The other joints are numbered in an arbitrary –

preferably logical – sequence. The truss rods are numbered in any order. The numbers of the rods in Figure 4 are given in circles.

*We calculate the trigonometric functions of the angles of the rods inclination:* from triangle 1-4-10:

$$\alpha = 45^0;\ \sin\alpha = \cos\alpha = 0{,}707;$$

from triangle 6-11-12:

$$\sin\beta = \frac{l_{6-22}}{l_{6-11}} = \frac{4}{\sqrt{4^2+3^2}} = 0{,}8;\ \cos\alpha = \frac{l_{11-12}}{l_{6-11}} = \frac{3}{\sqrt{4^2+3^2}} = 0{,}6;$$

from the triangle constrained by the rod 2 and its projections on the horizontal and vertical axes we obtain:

$$\sin\gamma = \frac{1}{\sqrt{1^2+3^2}} = 0{,}316;\ \cos\gamma = \frac{3}{\sqrt{1^2+3^2}} = 0{,}949.$$

*Calculate the efforts in the given panel* № 5.

Draw a section 1-1 (Fig. 4) and consider the right side of the truss (Fig. 5).

Efforts in the rods 12-13 rationally determine the method of moments, since the other two cut rods 5-6 and 6-12 intersect at point 6. This point is moment point for the rods 12-13, therefore, we have:

$$\sum M_6 = R_B \cdot d - N_{12-13} \cdot h_{12-13} = 0,$$

where the arm of rod $N_{12-13}$ is $h_{12-13} = \frac{h + h/2}{2} = \frac{4+2}{2} = 3m.$

Now you can calculate the search force:

$$N_{12-13} = \frac{R_B \cdot d}{h_{12-13}} = \frac{9 \cdot 3}{3} = 9.$$

To determine the force in the rod 17 we find the point of intersection of the other two cut rods (5 and 11). This is point *C*. So, again, we use the method of moments

$$\sum M_C = -R_B \cdot c + N_{17} \cdot h_{17} = 0.$$

The distance to the moment point is determined from the similarity of triangles 7-13-*C* and 2-9-*C:*

$$\frac{c+2d}{h}=\frac{c}{h/2};\ (c+2d)\cdot\frac{h}{2}=c\cdot h;\ c=\frac{d\cdot h}{h/2}=2d=2\cdot 3=6м.$$

The arm effort is determined from triangle 13-C-D:

$$h_{17}=(c+2d)\cdot\sin\alpha=(6+2\cdot 3)\cdot 0{,}707=8{,}48м.$$

Now the effort is search: $N_{6-12}=\frac{R_B\cdot c}{h_{17}}=\frac{9\cdot 6}{8{,}48}=6{,}37.$

Similarly, using moment point 13, we determine the force $N_5$ :

$$\sum M_{13}=R_B\cdot 2d+N_5\cdot h_5=0;$$

$$h_5=h\cdot\cos\gamma=4\cdot 0{,}949=3{,}80м;$$

$$N_5=-\frac{R_B\cdot 2d}{h_5}=-\frac{9\cdot 2\cdot 3}{3{,}8}=-14{,}21.$$

The forces in vertical rods 23 and 24 are determined from the equilibrium of joints 7 and 14, respectively. This was done when calculating the effort of t*he method isolation of joints*.

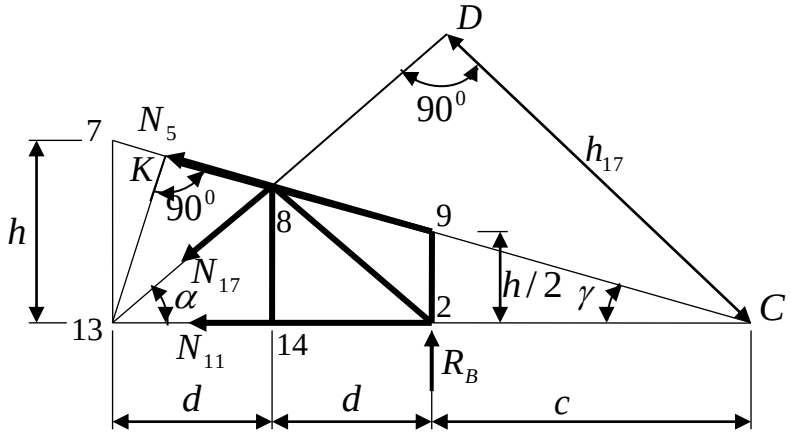


Figure 5 – Section 1-1

## Problem № 2

*Output data.*

For the arch, in Figure 6 to calculate of bending moments, shear and longitudinal forces from the given load at the points K (distance from the left support 2 m).

*Solution.*

*Determine the reactions of supports.*

Vertical reactions:

$$\sum M_A = 0; -q \cdot 7 \cdot 3{,}5 - F \cdot 10 + V_B \cdot 14 = 0,$$

$$V_B = \frac{4 \cdot 7 \cdot 3{,}5 + 6 \cdot 10}{14} = 11{,}29 kH;$$

$$\sum M_B = q \cdot 7 \cdot 10{,}5 + F \cdot 4 - V_A \cdot 14 = 0,$$

$$V_A = \frac{4 \cdot 7 \cdot 10{,}5 + 6 \cdot 4}{14} = 22{,}71 kH.$$

Check vertical reactions:

$$\sum y = V_A + V_B - q \cdot 7 - F = 22{,}71 + 11{,}29 - 4 \cdot 7 - 6 \equiv 0.$$

To determine the thrust we use the equation of equilibrium bending moment about the hinge C:

$$\sum_{лів} M_C = -q \cdot 7 \cdot 3{,}5 + V_A \cdot 7 - H \cdot 4 = 0,$$

$$H = \frac{-4 \cdot 7 \cdot 3{,}5 + 22{,}71 \cdot 7}{4} = 15{,}24 kH.$$

Check the calculated value of the thrust:

$$\sum_{пр} M_C = -F \cdot 3 - H \cdot 4 + V_B \cdot 7 = -6 \cdot 3 - 15{,}24 \cdot 4 + 11{,}29 \cdot 7 = 0{,}07 \approx 0.$$

Determine the radius of the arc of the circle on which the delineated arch axis:

$$r = \frac{f}{2} + \frac{l^2}{8f} = \frac{4}{2} + \frac{14^2}{8 \cdot 4} = 8{,}125 м.$$

Take cross sections to determine internal forces. Required are sections at the points K (Fig. 6, *a*).

Further calculations are performed write the coordinates *x* of the assigned points, for example, for point 1 we have $x_2 = 2$ m.

Calculated $\sin\varphi = \frac{\frac{l}{2} - x}{r}$, for point K we get $\sin\varphi = \frac{7-2}{8{,}125} = 0{,}615$. Value $\cos\varphi$ calculated any known way, for example, for point K we get $\arcsin\varphi = 37{,}98^0$ and, following $\cos 37{,}98^0 = 0{,}788$. C calculated ordinate points of the arch axis by the formula

$$y = \sqrt{r^2 - \left(\frac{l}{2} - x\right)^2} - r + f\ .$$

So, for point K we get $y = \sqrt{8{,}125^2 - (7-2)^2} - 8{,}125 + 4 = 2{,}28$ *m.*

Beams efforts $M^0$ and $Q^0$ determine from beam calculation (Fig. 6, b). So , for section K we get:

$$M^0 = V_A \cdot 2 - q \cdot 2 \cdot 1 = 22{,}71 \cdot 2 - 4 \cdot 2 \cdot 1 = 37{,}42\,kNm;$$

$$Q^0 = V_A - q \cdot 2 = 22{,}71 - 4 \cdot 2 = 14{,}71 kN.$$

In the next columns, the values of internal forces in the arch are calculated.

Calculated value $H \cdot y$, for section K we get $15{,}24 \cdot 2{,}28 = 34{,}75\,кНм.$ Has the resulting moments $M = M^0 - H \cdot y$, for section K is $M = 37{,}42 - 34{,}75 = 2{,}67\,kNm.$

Calculated product $Q^0 \cdot \cos\varphi$. For point K is $14{,}71 \cdot 0{,}788 = 11{,}59\,kN.$ *C*ontains $H \sin\varphi$. The value for point K is $15{,}24 \cdot 0{,}615 = 9{,}37\,kN.$ The values of the shear forces in the arch sections are calculated in column 13 – $Q = Q^0 \cos\varphi - H \sin\varphi$. For section K we get $Q = 11{,}59 - 9{,}37 = 2{,}27\,кН.$

For longitudinal forces. For section K is

$$Q^0 \sin\varphi = 14{,}71 \cdot 0{,}615 = 9{,}05;$$

$$H \cos\varphi = 15{,}24 \cdot 0{,}788 = 12{,}01$$

and

$$N = -(Q^0 \sin\varphi + H \cos\varphi) = -(9{,}05 + 12{,}01) = -21{,}00\,kN.$$

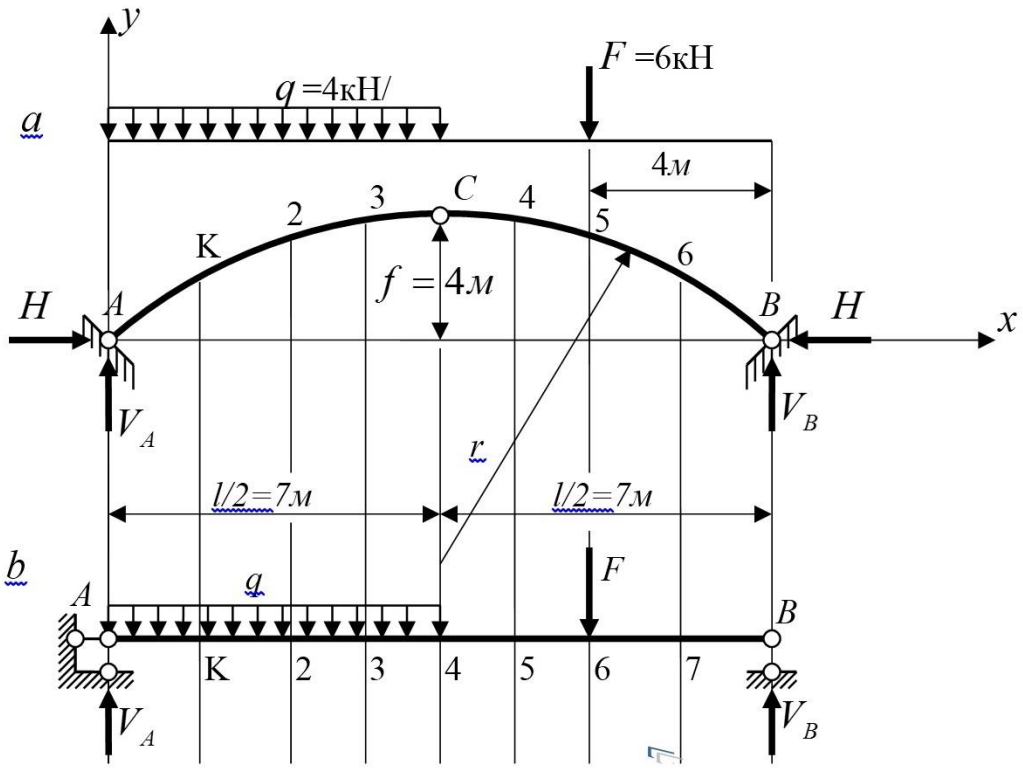


Figure 6 – Task 2

# 4 CRITERIA FOR THE EVALUATION OF CALCULATION WORK

According to the Calculation and Graphic Work (CGW) a student gets maximum mark, if completed within the time limit (3 weeks from the moment of giving a problem), using computer technology, is executed carefully, contains an analysis of the given results.

In the case of executing CGW without the use of a computer or a delay for 2 weeks (using a computer) student gets 90 % from the maximum mark. When executing CGW with a delay of more than for 2 weeks, the student gets 80 % of the maximum mark, with a delay of more than month – 60 % of the maximum mark.

## REFERENCES

1. Methodological Guidelines for practical classes, independent and calculator-graphical works on the Subject «Mechanics of Materials» (Tension – Compression) (for the second year full-time Bachelor degree students of the specialty 192 – Construction and civil engineering) [Electronic resource] / O. M. Beкetov National University of Urban Economy in Kharkiv ; comp. : N. V. Sereda, A. O. Garbuz, T. A. Suprun. – Electronic text data. – Kharkiv : O. M. Beketov NUUE, 2018. – 24 p. – Regime of access: https://eprints.kname.edu.ua/50048/, free (date of the application: 07.09.2023). – Header from the screen.

2. Methodological Guidelines for practical classes, independent and calculator-graphical works on the Subject «Mechanics of Materials» (Bending calculation. Drawing the diagrams) (for the second year full-time Bachelor degree students of the specialty 192 – Construction and civil engineering) [Electronic resource] / O. M. Beketov National University of Urban Economy in Kharkiv ; comp. : N. V. Sereda, A. O. Garbuz, T. A. Suprun. – Electronic text data. – Kharkiv : O. M. Beketov NUUE, 2019. – 30 p. – Regime of access: https://eprints.kname.edu.ua/52671/, free (date of the application: 07.09.2023). – Header from the screen.

3. Methodological Guidelines for practical classes, self-dependent and calculator-graphical works on the Subject «Strength of Materials» (Calculation of statically indeterminate frame by force method) (for the second year full-time students of the specialty 192 – Construction and civil engineering ) [Electronic resource] / O. M. Beкetov National University of Urban Economy in Kharkiv ; comp. : N. V. Sereda, A. O. Garbuz, O. O. Chuprynin, T. A. Suprun. – Electronic text data. – Kharkiv : O. M. Beketov NUUE, 2020. – 24 p. – Regime of access: https://eprints.kname.edu.ua/56460/, free (date of the application: 07.09.2023). – Header from the screen.

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для підготовки до лекцій і практичних занять, виконання контрольних завдань і самостійної роботи з навчальної дисципліни

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(Англ. мовою)

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