

SMOOTH ACCELERATION OF TRANSVERSE AXIS MOVEMENTS IN CNC THREADING MACHINING

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Abstract – Most of the complex details of modern technical products are manufactured on CNC equipment. The threaded surface is one of the most critical elements of such parts. Their reliability and durability largely depend on the accuracy of the machining process. Machining threaded surfaces requires precise synchronization of periodic tool movements with the continuous rotation of the spindle. Ensuring smooth movement along the transverse axis with a synchronized approach to the machined surface is one of the important tasks. A variant of solving this problem in a two-level CNC system is proposed. A mathematical model of the movement along the transverse axis with a smooth variation in acceleration has been developed. A technique is proposed using an S-shaped feed profile based on the \sin^2 function, which provides ease of calculation and high accuracy. The model is implemented in a two-level CNC. The results of experimental studies are shown that servo error decreases to a value of the order of $0...-1 \mu\text{m}$ before the end of the approach section, outside the workpiece.

Keywords: Threading, CNC, S-shaped Feedrate Profile, Smooth Acceleration, Axis Synchronization.

1. Introduction

Modern complex products include parts with helical and threaded surfaces in their design [1]. These are various parts of transmissions, worm and lead screws, parts of detachable connections and others. The production of such parts is carried out using CNC machining [1–3]. High demands are placed on the accuracy of threaded surfaces, which significantly affects both the functional characteristics of products and their reliability and durability, for example [4, 5]. The leaders of modern industrial production produce equipment that meets the requirements for the required accuracy. However, the task of ensuring accuracy does not lose its relevance, since these requirements are constantly increasing [6–8].

Threading is a dynamic process in which the movement of several machine parts is synchronized with the position of the rotating spindle [9]. The accuracy of movement of each organ is determined by the combination of characteristics of the individual elements involved in this process: motors [10], servos [11], software control algorithms [12–14].

The inertia of the movement of the machine tools determines the need to take into account the nature of the variation in the differential characteristics of the movement, such as speed, acceleration and jerk [15]. This problem is solved by developing models

for the S-shape feedrate. A large number of such models using different mathematical bases are proposed: with trapezoidal acceleration [16], cubic polynomial profile with a discontinuous jerk [17], polynomial profile with jerk continuous [18], based on trigonometric functions with continuous acceleration and discontinuous jerk [19], with continuous jerk [20]. However, the listed models are characterized by rather complex computational algorithms.

When threading, the axes of the transverse and longitudinal tool movement perform motions that are different in nature [9, 21]. The axis of transverse movement in a short section changes the position along the machining diameter and is largely subject to the formation of dynamic errors [22]. Movement of the transverse axis synchronized with the position of the rotating spindle during threading is considered in this work. Application of S-shaped feedrate profile with a smooth variation in the differential characteristics contributes to the reduction of these errors [14]. Models of S-shaped feedrate profile with smoothly-limited jerk, such as: seven-interval [23] and three-interval [24], which are applied to control of toolpath feedrate and three-interval model which is applied to control of longitudinal axis movements in threading process [14] have been proposed previously. In this paper a simple two-interval S-shaped feedrate profile model based on \sin^2 function that

provides continuity of the jerk and smoothness of acceleration is proposed. Model is applied to control the transverse axis movements in CNC threading machining.

2. Threading Control Organization in a Two-level CNC System

There is a wide variety of threaded surface designs. However, for the processing of these surfaces on CNC equipment, the same type of multi-pass threading cycles are used. Typical toolpath [14] used in threading cycles shown in the Fig. 1a. To machine the thread surface, it is necessary to carry out a synchronized movement of three axes at once: spindle rotation S , X-axis transverse movement and Z-axis longitudinal movement.

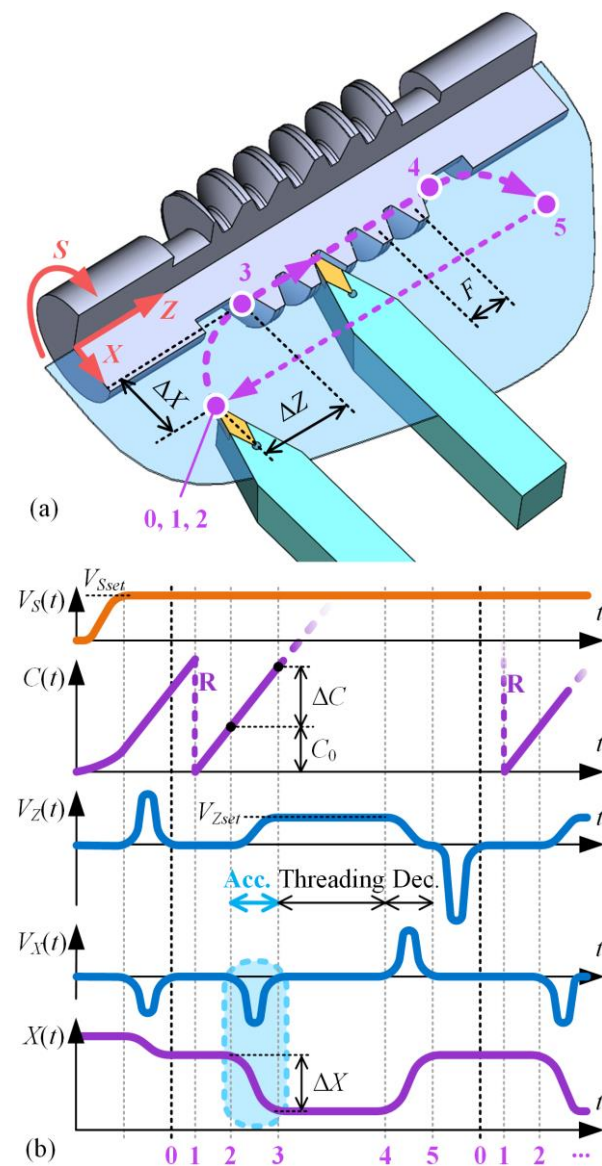


Figure 1: Threading: (a) – characteristic sections of tool paths; (b) – axis interaction diagram for threading; **R** – reset spindle angle value at zero label position

X-axis provides positioning on the diameter of the machined surface. Z-axis provides positioning along the pitch of the spiral line F . The dynamic characteristics of the spindle and tool movement axes differ significantly due to differences in inertia and axes design features. For this reason, the use of traditional interpolation of the axes movement for threading does not provide the necessary accuracy [14, 25–27]. The tool movement during the machining of threaded surfaces is controlled in the mode of synchronization of the X, Z-axes with the position of the rotating spindle S . In this work, synchronization with the axis of the transverse movement is considered.

One of the possible schemes for organizing control during threading shown in the Fig. 2. The initial information for machining in the G-code form is transferred from CAD/CAM to CNC. In used by the authors for experimental studies two-level CNC system [28, 29] the tasks of code interpretation and motion interpolation are distributed between two computers. “Interpreter” performs data conversion and “Machine code” generation in a format convenient for use in equipment control. These transformations do not require the use of real-time mode.

Modules “Interpolator” and “Regulator” operate in real time. To eliminate the “information hunger” when controlling the movement, the “Regulator” and “Interpolator” modules are clocked at a different pace. In the normal mode of movement along the trajectory, for each control cycle, the “Interpolator” module transmits to the “Regulator” module data on changes in the position and speed of movement.

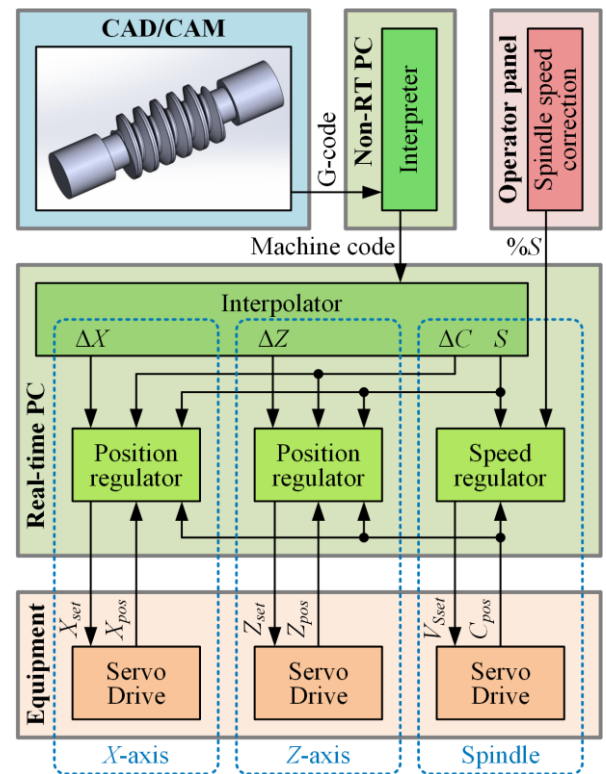


Figure 2: Threading control organization

Modules “Regulator” for different axes can be located both on one computing device and on different ones. To unify communication, data for the “Regulator” is transmitted in packets through a virtual network. Such an organization implies a significant limitation on the number and type of transmitted parameters. These restrictions on data transfer to the “Regulator” module are also valid for the synchronization mode with the position of a rotating spindle. This circumstance determines the need to use a synchronization model that does not require a large amount of data to initialize it and execute the movement.

A typical trajectory and a diagram of changes in the movement parameters of synchronized axes during threading is shown in Fig. 1. The threading cycle is performed at a constant specified spindle speed S , while the specified (calculated) rate of change in the position of the spindle is determined as

$$V_{Sset} = 6 \cdot S. \quad (1)$$

The actual position of the rotating spindle is contained in parameter $C(t) = C_{pos}$. Before the cycle start, the tool is positioned at the starting point (time moment 0). At zero label position (time moment 1) $C(t)$ value is reset, modules “Regulator” of X and Z axes go into spindle tracking mode. After reaching the spindle position C_0 (time moment 2), the acceleration stage for the Z -axis begins and the X -axis approaches the machining diameter. At this stage, the longitudinal axis should accelerate to V_{Zset} in the section of the specified length ΔZ (time moment 3), and the X -axis should change its position by the value ΔX . Change of position of the transverse axis should be completed at the machining size with a minimum servo error and without oscillation. Spindle speed S and spiral line pitch F determine the required speed of movement of the longitudinal axis V_{Zset} and the magnitude of the coordinated movements of the spindle ΔC of the transverse axis ΔX and longitudinal axis ΔZ . The threading stage is executed at the speed of the synchronized movement V_{Zset} to the end of the threaded section, and the transverse axis maintains the positioning size (time moment 4). The synchronized motion mode is completed by retracting the tool from the part surface along the X -axis and decelerating the Z -axis to zero speed (time moment 5). The cycle is repeated after returning to the starting point in the traditional interpolation mode.

For the proposed model, the parameters S , C_0 , ΔC , ΔZ are used as initial data. This set of data, in accordance with the communication scheme of the modules (Fig. 2), is sufficient to initialize the synchronized movement of the axes.

In this paper mathematical models of movement of the transverse axis synchronized with the spindle based on movement with smooth acceleration are considered. An experimental study of the accuracy of the transverse axis movements was carried out using the proposed models.

3. Mathematical Model of Motion with Smooth Acceleration in the Area of Approach of the Transverse Axis

An important characteristic for algorithms used in real-time control units is the minimization of computational burden. Previously proposed seven model of S -shaped feedrate profile with smoothly-limited jerk [23] and tree-interval model [24] provide the necessary nature of the variation in differential characteristics, but require some calculations and logical operations to initialize the profile parameters. In contrast to the previously considered models, this paper proposes a two-interval S -shaped smooth acceleration feedrate profile, which provides the necessary character of the change in differential characteristics and does not require complex calculations to initialize the profile parameters.

In Fig. 3 shown a diagram of variation the differential characteristics of the movement of the axis of the transverse movement in the synchronization mode with the position of the rotating spindle.

To describe the movement of the transverse axis, a model is used as a function of the parameter p , defined as the ratio of the spindle position ΔC_{pos} to the given speed of its rotation V_{Sset} or S , taking into account the Eq. (1):

$$p = \Delta C_{pos} / V_{Sset} = \Delta C_{pos} / (6 \cdot S). \quad (2)$$

The approach section of the transverse axis corresponds to its movement in the interval of parameter change from $p_0 = 0$ to p_{acc} :

$$p_{acc} = \Delta C / V_{Sset} = \Delta C / (6 \cdot S). \quad (3)$$

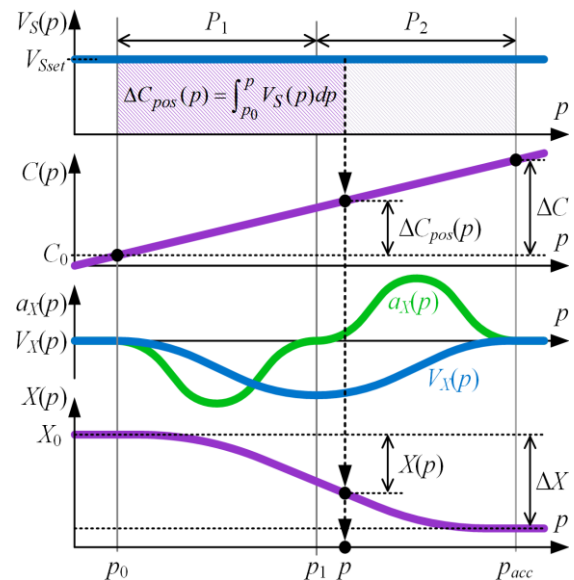


Figure 3: Diagram of variation the differential characteristics of the movement of the X-axis transverse movement synchronized with the rotation of the spindle S

Let's write an expression for a two-interval model of a smooth change in acceleration:

$$a_X(p) = \begin{cases} a_p \cdot \sin^2(\omega \cdot \tau(p)), & p \in P_1; \\ -a_p \cdot \sin^2(\omega \cdot \tau(p)), & p \in P_2. \end{cases} \quad (4)$$

Variation the position of the transverse axis starts at the time corresponding to the value of the parameter p_0 and finishes at the time corresponding p_{acc} upon reaching a variation in the position of the X -axis by the value ΔX . At each i interval parameter from it start is defined as

$$\tau(p) = (p - p_{i-1}), \quad p \in P_i. \quad (5)$$

The position change function is defined according to the S -shape feedrate profile (Fig. 3), which consists of two intervals:

- P_1 corresponds to reaching the maximum axis travel speed $V_X(p)$. On this interval, an acceleration changes from $a_X(p_0) = 0$ to $a_X(p_1) = 0$ and reaches its maximum value in the middle of the interval $a_X(p_1/2) = a_{XP}$. The value of parameter change is $\tau(p_1) = \Delta p_1$;

- P_2 corresponds to a decrease in the movement speed from the maximum value $V_X(p)$ to a complete stop $V_X(p_2) = 0$. At this interval an acceleration changes from $a_X(p_1) = 0$ to $a_X(p_{acc}) = 0$ and reaches its maximum value in the middle of the interval $a_X(p_2/2) = -a_{XP}$. The value of parameter change is $\tau(p_{acc}) = \Delta p_2 = \Delta p_1$.

In accordance with the symmetry of the change in acceleration in the proposed model

$$\begin{aligned} p_{acc} &= \Delta p_{acc} = \Delta p_1 + \Delta p_2 = 2 \cdot \Delta p_1; \\ \Delta p_1 &= 0.5 \cdot \Delta p_{acc}. \end{aligned} \quad (6)$$

The function of the speed and position changing of the transverse axis will be obtained as a result of integration

$$V_X(p) = \int_{p_0}^{p_{acc}} a(p) dp; \quad X(p) = \int_{p_0}^{p_{acc}} V_X(p) dp. \quad (7)$$

Additionally the following boundary conditions should be observed to achieve the smoothness of motion during acceleration: $V_X(p_0) = V_X(p_{acc}) = 0$; $X(p_0) = X_0$; $X(p_{acc}) = X_0 + \Delta X$. The value of the distance traveled ΔX for the approach section of the transverse axis is specified in the machine code.

The acceleration variation law is based on the \sin^2 function with period defined by angular frequency ω (s^{-1}) [23]. Value of ω is calculated from the condition $\sin^2(\omega \cdot \Delta p_1) = 0$:

$$\omega = \pi / \Delta p_1.$$

In accordance with the proposed model, we write equation to determine the velocity

$$V_X(p) = \begin{cases} a_p \cdot FV(\tau(p)), & p \in P_1, \\ V_X(\tau(p_1)) - a_p \times \\ \times FV(\tau(p)), & p \in P_2, \end{cases} \quad (8)$$

where $FV(p) = 0.5 \cdot \tau(p) - (\sin(2 \cdot \omega \cdot \tau(p)) / (4 \cdot \omega))$.

The position of the axis of longitudinal movement is defined as

$$X(p) = X_0 + \Delta X(p). \quad (9)$$

Equation to determine the distance traveled during acceleration:

$$\Delta X(p) = \begin{cases} a_p \cdot FX(\tau(p)) + X_0, & p \in P_1, \\ V_X(p_1) \cdot \tau(p) - a_p \times \\ \times FX(\tau(p)) + X(p_1), & p \in P_2, \end{cases} \quad (10)$$

where $FX(p) = 0.25 \cdot \tau(p)^2 - (\sin(\omega \cdot \tau(p)) / (2 \cdot \omega))^2$.

In accordance with the proposed model, the maximum acceleration amplitude will be reached in the middle of the intervals P_1, P_2 . It is not difficult to verify that in accordance with (4):

$$a_X(0.5 \cdot \Delta p_1) = a_p; \quad a_X(0.5 \cdot \Delta p_2) = -a_p.$$

From Eq. (8), the speed achieved at the end of the first interval is determined:

$$V_X(p_1) = 0.5 \cdot a_p \cdot \Delta p_1. \quad (11)$$

From Eq. (10), taking into account (6), we determine the distance over which the axis of lateral movement will move at the end of the interval P_2 when the parameter value p_{acc} is reached:

$$X(p_{acc}) = X_0 + 0.5 \cdot a_p \cdot (\Delta p_1)^2. \quad (12)$$

From Eq. (12), taking into account (3), obtained expression for determining the value of a_p required to move by ΔX when the parameter p changes from p_0 to p_{acc}

$$a_p = 288 \cdot \Delta X \cdot S^2 / \Delta C^2. \quad (13)$$

In accordance with the proposed mathematical model, the calculation of the movement parameters of the axis of transverse movement during threading

in the area of approach to the machining diameter is performed in this order:

- determination of the intervals for variation the parameters p , p_{acc} by Eq. (3) and Δp_1 by Eq. (6);
- calculation of the required jerk a_p by Eq. (13);
- in the process of threading, depending on the position of the spindle ΔC_{pos} , the parameter p is calculated according to Eq. (2) and the movement and positioning parameters according to the model (8), (10).

Obviously, the calculations necessary to initialize the motion parameters of the transverse axis in accordance with the proposed mathematical model are quite simple. Performing these calculations is not a problem both when used in the CNC system, and when using hardware calculations in the servo drive. The model provides the ability to control movement in the area of approach to the size of processing on a limited set of initial data. The set of necessary data consists of the following values: change in the position of the axis of the transverse movement ΔX , change in the position of the spindle ΔC and the value of the specified spindle speed S . Due to errors in the drive settings, the actual spindle speed may differ from the specified one. In addition, during processing, the spindle speed can be adjusted (Fig. 2). However, Eq. (10) is a function of the parameter p , and, taking into account (2), it depends only on the change in the actual position of the spindle (C_{pos}). Thus, the proposed model of the movement of the transverse axis provides tracking of the actual position of the spindle, regardless of the actual speed of its rotation. The resulting model is not sensitive to deviations in the value of the spindle speed.

4. Experimental Investigations of the Transverse Axis Movement

Experimental studies of the movement of the transverse axis when approaching the machining size were carried out on the equipment with two-level CNC system. Movement of the machine transverse axis with AC servomotor Estun EMG-10APA22 was investigated. Registration and processing of experimental data was carried out using a CNC-based software and hardware complex [14, 28, 29]. The movement of the tool was studied in the process of machining left hand worm screw (Fig. 1a). The experiment investigated the behavior of servo error $\delta(t)$ during movement of the transverse movement axis X in the tool supply area to the processing size. The sensitivity of the proposed model to a change of the spindle speed was experimentally tested. The comparison of servo error $\delta(t)$ obtained during the processing of the helical surface at different spindle speeds is performed. Variation the spindle speed was carried out using spindle speed correction (Fig. 2).

In Fig. 4 shown diagrams of differential characteristics and axis servo error $\delta(t)$ for three modes of spindle rotation.

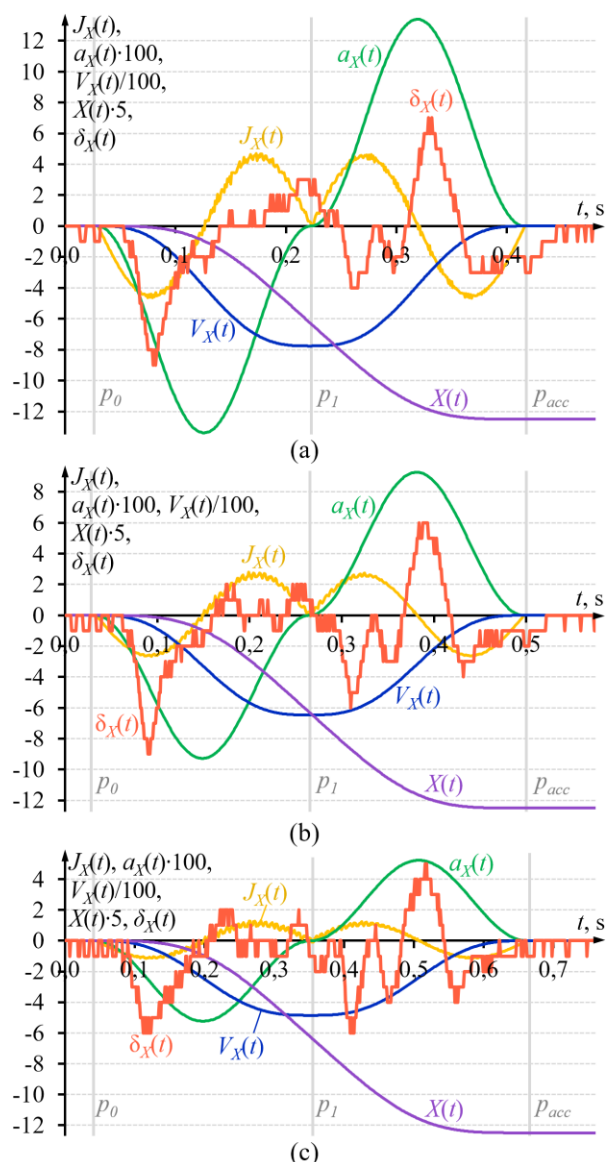


Figure 4: Transverse axis servo error $\delta(t)$:
(a) $\%S = 100$, (b) $\%S = 80$, (c) $\%S = 60$

All modes are characterized by the occurrence of axis servo error $\delta(t)$ at the beginning of the movement. This servo error is generated outside of the workpiece and varies within $\pm 8 \mu\text{m}$. By the end of the tool approach to the machining size, when approaching point 3 (Fig. 1a), which corresponds to the achievement of the value of the p_{acc} servo error parameter it decreases to a value of the order of $0 \dots -1 \mu\text{m}$ and remains so throughout the entire time of machining a regular thread section. This allows to set the thread start point (Fig. 1a, pos. 3) directly on the detail and facilitates the execution of technological operations for processing details with a limited length of the approach section.

Checking the sensitivity of the model to changes in the actual spindle speed was carried out using “Spindle speed correction” and without changing the “Machine code” (Fig. 2). With the studied variation of the spindle speed, the model provided an automatic variation of the required values of the differential characteristics and the exact achievement of the set position synchronously with the position of the spindle.

Based on the results of the research, the proposed mathematical model S-shape feedrate profile with smooth acceleration for synchronization of threading movements was introduced into the production process. On Fig. 5a shows the process of developing the manufacturing technology of the left hand worm screw, and Fig. 5b example of manufactured parts.

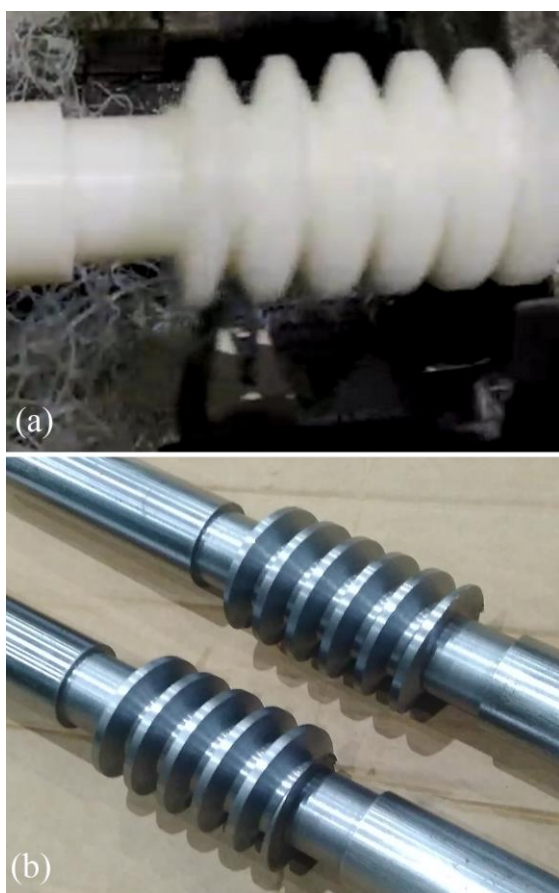


Figure 5: Example of left hand worm screw manufacturing: (a) – process adjustment; (b) – manufactured parts example

5. Conclusions

The paper considers the actual problem of ensuring the accuracy of technological equipment when synchronizing tool movements with the position of a rotating spindle in a two-level CNC system.

Movement of the transverse axis when the tool is approached by the machining size synchronized with the spindle have been considered. A set of param-

eters necessary to specify the movement in the tool approach section is determined, taking into account the limitations of the amount of data transmitted to the controller module in the CNC system. A mathematical model of the movement of the transverse axis with a smooth change in acceleration has been developed. S-shaped feedrate profile with using the \sin^2 function is applied. The proposed model provides extremely simple calculations when initializing the parameters of the model of the movement of the transverse axis of a rotating spindle synchronized with the movement. The model provides high accuracy of reaching the end position regardless of the actual spindle speed. Experimental studies have shown that servo error decreases to a value of the order of $0...-1 \mu\text{m}$ before the end of the approach section, outside the workpiece. The proposed model is implemented in the two-level CNC and used in the manufacturing process.

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