

**MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE**

**O. M. BEKETOV NATIONAL UNIVERSITY  
of URBAN ECONOMY in KHARKIV**

**METHODOLOGICAL GUIDELINES  
FOR “CALCULATION AND GRAPHICS  
TASKS OF HIGHER MATHEMATICS”**

*(for full-time students (bachelor) of the education level of the  
specialty 192 – Construction and civil engineering)*

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*Recommended by the department of Higher Mathematics  
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## PREFACE

Calculation and graphic task for full-time bachelors of the specialty 192 – Construction and Civil Engineering is designed to master the basic mathematical concepts and methods of solving tasks during the students' independent work to study the course “Higher Mathematics”. The presented in these guidelines tasks are professional orientation tasks and they clearly illustrate the practical application of the methods of linear algebra, analytical geometry, mathematical analysis (differential calculus) in solving applied tasks.

Educational and methodical complex of the discipline “Higher Mathematics” for students of the specialty 192 – Construction and Civil Engineering, which includes some lecture notes with the necessary theoretical material, reference books and tasks for practical classes and independent work for students, allows students to prepare for calculation and graphical task carefully

The calculation and graphic task must be drawn up on standard sheets of paper A4 (size 210 x 297 mm). Students should write their work only on the one side of the sheet. An example of the design of the title page is presented in the appendices. After your work is done, it must be bound and handed over to the teacher.

The term of performance of this work is defined by the teacher.

# 1 STATEMENT OF THE ISSUE FOR CALCULATION AND GRAPHIC TASK

Task 1. Construction of the object  $C(x_0, y_0)$  has started. The object is located away from the highway. For delivery of materials, equipment, workers it is necessary to build an auxiliary track under the following conditions:

In the points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  (on the plan of the area), that are directly connected by the existing highway, there is a warehouse of construction materials and parking of construction equipment, respectively. The object of the new building  $C(x_0, y_0)$  in the same system should be connected by the shortest way with the highway. Find the point of entry of such a road and its length on the highway, draw a figure if the location of objects  $A(x_1, y_1)$ ,  $B(x_2, y_2)$ ,  $C(x_0, y_0)$  are presented at the table 1.1.

Table 1.1 - Location of objects  $A(x_1, y_1)$ ,  $B(x_2, y_2)$ ,  $C(x_0, y_0)$

Variant	$A(x_1, y_1)$	$B(x_2, y_2)$	$C(x_0, y_0)$
<b>1</b>	(-20; 14)	(24; 4)	(-10; -12)
<b>2</b>	(-12; -6)	(-2; 18)	(8; 8)
<b>3</b>	(8; 12)	(-8; -2)	(-3; 3)
<b>4</b>	(10; -12)	(-17; 6)	(9; 7)
<b>5</b>	(-5; -11)	(8; 21)	(-7; 3)
<b>6</b>	(-6; 5)	(11; 4)	(9; 12)
<b>7</b>	(-10; -10)	(21; 12)	(10; -14)
<b>8</b>	(6; -5)	(-9; 10)	(3; 13)
<b>9</b>	(2; -10)	(15; 18)	(14; 3)
<b>10</b>	(-5; 17)	(6; -12)	(4; 11)

Task 2. At the preparatory stage of construction in the preparation of the estimate the task is to approve the norms of consumption of construction materials for the manufacture of construction mixtures. For the production of 1 ton of construction mixtures of types  $A_1$  and  $A_2$  (Table 1.2), we need the components  $B_1$  and  $B_2$ , for the manufacture of which, in turn, requires some building materials  $C_1$ ,  $C_2$  (their amount is given in Table 1.3). Determine the amount of productive materials  $C_1$ ,  $C_2$  for a production of 1 ton of products  $A_1$  and  $A_2$ .

Table 1.2 – Components of construction mixtures production

Products	Components		Building materials	$C_1$	$C_2$
$A_1$	$a$	$b$	$B_1$	$m$	$n$
$A_2$	$c$	$d$	$B_2$	$f$	$q$

Table 1.3 –Number of components for the production of construction mixtures

Variant	$a$	$b$	$c$	$d$	$m$	$n$	$f$	$q$
<b>1</b>	3	2	1	4	1	3	2	1
<b>2</b>	3	4	2	5	4	2	2	4
<b>3</b>	5	4	4	6	10	2	4	6
<b>4</b>	4	1	5	0	3	5	1	5
<b>5</b>	7	3	3	5	6	2	2	4
<b>6</b>	3	0	2	2	4	12	3	2
<b>7</b>	2	5	3	1	3	8	12	1
<b>8</b>	9	5	10	2	2	26	1	3
<b>9</b>	1	2	1	6	2	1	0	4
<b>10</b>	4	9	13	7	0	12	5	6

Task 3. Calculate the wages that are accrued to employees at each stage of construction, if we know (Table 1.4):

a) the cost of working time in hours at each workplace (matrix  $P$ );

б) production standards (matrix  $Q$ );

в) hourly payment (in USD.) at each workplace (matrix  $C$ ).

Table 1.4 – Labor costs, production rates and hourly payment

Variant	$P$	$Q$	$C$
1	$\begin{pmatrix} 4 & 3 & 21 & 7 \\ 9 & 2 & 85 & 3 \\ 4 & 6 & 18 & 5 \end{pmatrix}$	$\begin{pmatrix} 210 & 145 & 317 \\ 540 & 230 & 150 \\ 325 & 425 & 810 \end{pmatrix}$	$\begin{pmatrix} 11,6 \\ 15,3 \\ 13,5 \\ 12,8 \\ 17,3 \end{pmatrix}$
2	$\begin{pmatrix} 3 & 8 & 46 & 2 \\ 1 & 7 & 58 & 3 \\ 6 & 2 & 74 & 1 \end{pmatrix}$	$\begin{pmatrix} 150 & 295 & 356 \\ 427 & 750 & 245 \\ 265 & 250 & 310 \end{pmatrix}$	$\begin{pmatrix} 13,7 \\ 15,3 \\ 12,9 \\ 18,3 \\ 14,5 \end{pmatrix}$
3	$\begin{pmatrix} 6 & 4 & 59 & 2 \\ 5 & 8 & 13 & 6 \\ 6 & 2 & 51 & 7 \end{pmatrix}$	$\begin{pmatrix} 760 & 240 & 380 \\ 155 & 348 & 564 \\ 295 & 493 & 250 \end{pmatrix}$	$\begin{pmatrix} 16,2 \\ 18,5 \\ 14,7 \\ 15,9 \\ 14,3 \end{pmatrix}$
4	$\begin{pmatrix} 5 & 7 & 23 & 8 \\ 3 & 6 & 15 & 7 \\ 4 & 9 & 26 & 9 \end{pmatrix}$	$\begin{pmatrix} 150 & 230 & 645 \\ 284 & 165 & 454 \\ 357 & 159 & 310 \end{pmatrix}$	$\begin{pmatrix} 15,8 \\ 16,2 \\ 18,1 \\ 19,2 \\ 11,6 \end{pmatrix}$

Continuation of the table 1.4

Variant	$P$	$Q$	$C$
<b>5</b>	$\begin{pmatrix} 4 & 5 & 27 & 4 \\ 6 & 3 & 18 & 5 \\ 5 & 7 & 93 & 6 \end{pmatrix}$	$\begin{pmatrix} 654 & 255 & 317 \\ 250 & 654 & 854 \\ 340 & 645 & 480 \end{pmatrix}$	$\begin{pmatrix} 13,5 \\ 16,4 \\ 17,2 \\ 15,7 \\ 16,3 \end{pmatrix}$
<b>6</b>	$\begin{pmatrix} 7 & 4 & 23 & 1 \\ 4 & 8 & 72 & 7 \\ 5 & 9 & 32 & 1 \end{pmatrix}$	$\begin{pmatrix} 365 & 255 & 250 \\ 645 & 520 & 345 \\ 650 & 495 & 352 \end{pmatrix}$	$\begin{pmatrix} 15,2 \\ 11,6 \\ 14,3 \\ 13,5 \\ 12,1 \end{pmatrix}$
<b>7</b>	$\begin{pmatrix} 9 & 3 & 27 & 5 \\ 4 & 8 & 14 & 3 \\ 6 & 2 & 57 & 2 \end{pmatrix}$	$\begin{pmatrix} 210 & 431 & 258 \\ 395 & 542 & 670 \\ 450 & 290 & 510 \end{pmatrix}$	$\begin{pmatrix} 11,2 \\ 16,8 \\ 15,6 \\ 14,2 \\ 15,9 \end{pmatrix}$
<b>8</b>	$\begin{pmatrix} 5 & 7 & 18 & 3 \\ 8 & 1 & 43 & 6 \\ 2 & 5 & 76 & 9 \end{pmatrix}$	$\begin{pmatrix} 520 & 370 & 455 \\ 254 & 350 & 120 \\ 370 & 210 & 350 \end{pmatrix}$	$\begin{pmatrix} 18,3 \\ 17,2 \\ 11,5 \\ 16,4 \\ 15,5 \end{pmatrix}$
<b>9</b>	$\begin{pmatrix} 2 & 3 & 25 & 8 \\ 6 & 1 & 79 & 2 \\ 1 & 5 & 23 & 6 \end{pmatrix}$	$\begin{pmatrix} 255 & 360 & 465 \\ 369 & 250 & 421 \\ 241 & 540 & 610 \end{pmatrix}$	$\begin{pmatrix} 10,2 \\ 14,6 \\ 12,5 \\ 13,4 \\ 16,2 \end{pmatrix}$
<b>10</b>	$\begin{pmatrix} 5 & 1 & 26 & 4 \\ 1 & 6 & 32 & 9 \\ 7 & 5 & 87 & 4 \end{pmatrix}$	$\begin{pmatrix} 620 & 258 & 375 \\ 680 & 545 & 247 \\ 325 & 548 & 486 \end{pmatrix}$	$\begin{pmatrix} 14,3 \\ 11,1 \\ 12,6 \\ 13,5 \\ 17,8 \end{pmatrix}$

Task 4. To make a construction mixtures directly on the place, it is necessary to prepare metal cylindrical tanks of the



given volume  $V$ . What should be their size so that the least material is used for the production? Determine the size of the tank by the amount of material used (into  $m^2$ ) without taking into account the cost of cutting, provided that it is necessary to make tanks of volume  $V$  (Table 1.5).

Table 1.5 – Cylindrical tank volume,  $V$

Variant	$V$	Variant	$V$
<b>1</b>	650	<b>6</b>	765
<b>2</b>	545	<b>7</b>	370
<b>3</b>	250	<b>8</b>	490
<b>4</b>	380	<b>9</b>	570
<b>5</b>	945	<b>10</b>	585

Task 5. Part of the open containers on the construction place has the shape of a rectangular parallelepiped. Determine what should be the width and length  $h$  and volume  $V$  (Table 1.6) of the container for the manufacture of which will be spent the least amount of material. Calculate the consumed material amount (into  $m^2$ ).

Table 1.6 – Geometric measurements of the container

Variant	$h$	$V$	Variant	$h$	$V$
<b>1</b>	5	2	<b>6</b>	4	6
<b>2</b>	2	1	<b>7</b>	5	7
<b>3</b>	4	3	<b>8</b>	2	8
<b>4</b>	1	2	<b>9</b>	1	5
<b>5</b>	3	3	<b>10</b>	10	7

Task 6. **Assembly crane** is a one of the simplest mechanisms for lifting cargoes, it is made from wood or metal (Figure 1.1). Its stability is achieved by tensioning the steel cable. Vector diagram of the assembly crane is presented at the figure 1.2.



Figure 1.1 – Assembly crane

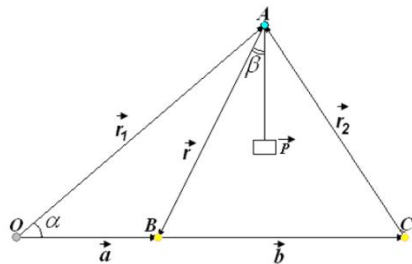


Figure1.2 – Vector diagram of the assembly crane

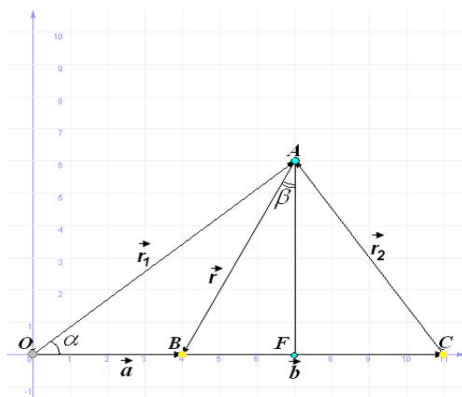


Figure 1.3 – Assembly crane at the given coordinates system

Write the condition of the closed system of forces acting on the mast in the unloaded state, in a scalar form. Select the coordinate system as it is shown at the figure 1.3 Find the lengths of the segments  $OA$  and  $AB$ , if  $|OB|, \alpha, \beta$  are given at the table 1.7.

Table 1.7 – Geometric measurements of the assembly crane

Variant	$ OB $	$\alpha$	$\beta$
<b>1</b>	4	$60^\circ$	$20^\circ$
<b>2</b>	3	$70^\circ$	$10^\circ$
<b>3</b>	5	$45^\circ$	$15^\circ$
<b>4</b>	6	$50^\circ$	$10^\circ$
<b>5</b>	2	$70^\circ$	$20^\circ$
<b>6</b>	7	$65^\circ$	$10^\circ$
<b>7</b>	4	$60^\circ$	$15^\circ$
<b>8</b>	5	$70^\circ$	$20^\circ$
<b>9</b>	3	$55^\circ$	$15^\circ$
<b>10</b>	6	$80^\circ$	$10^\circ$

Task 7. **Lattice (or rafter) truss** is a metal or wooden building structure, which consists of individual rods connected by welding, bolts, nails, and is used to cover factory buildings. The rods  $AE$  and  $EB$  are called stingrays, vertical rods are called racks, and inclined (sloping) are called braces (Figure 1.4). Racks and struts make up the rafters of the truss. In order to avoid harmful stresses that are possible due to temperature fluctuations, one of the supports of the truss remains loosely fixed (on rollers).

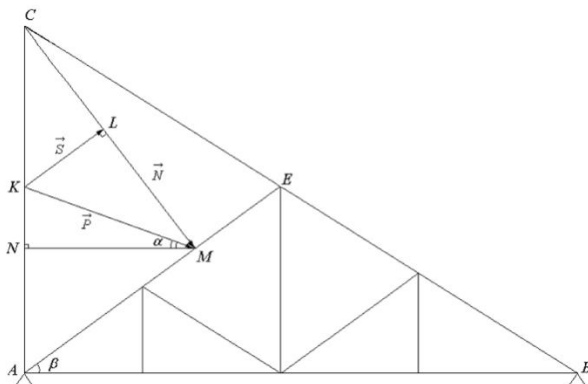


Figure. 1.4 – Scheme of a lattice truss

It is known that the right support of the lattice truss is rigidly fixed, and the left support is on the rollers (Figure 1.5). Determine the value of the wind pressure on the left slope of the truss, if the angle of inclination of the slope  $\beta$ , if the angle between the force vector  $\vec{P}$ , which characterizes the wind pressure on the left slope of the truss, and the horizon line  $\alpha$ , and the magnitude of the vector  $\vec{P}$ , are given at the table 1.8.

Table 1.8 – Geometric dimensions of a lattice truss

Variant	$\alpha$	$\beta$	$ \vec{P} $
<b>1</b>	15°	40°	60
<b>2</b>	10°	30°	50
<b>3</b>	12°	35°	40
<b>4</b>	9°	30°	55
<b>5</b>	13°	45°	65
<b>6</b>	15°	40°	50

Continuation of the table 1.8

Variant	$\alpha$	$\beta$	$ \vec{P} $
<b>7</b>	20°	30°	60
<b>8</b>	17°	45°	45
<b>9</b>	11°	50°	35
<b>10</b>	18°	40°	65

Task 8. To move the load from the point  $M$  to the point  $N$  we use the constant force  $\vec{F}$ . Determine the work of the cargo replacing and the angle between the vectors  $\vec{F}$  and  $\overrightarrow{MN}$ , if the coordinates of points  $M$  and  $N$ , and the force vector  $\vec{F}$  are given at the table 1.9.

Table 1.9 – Coordinates of the points  $M$  and  $N$ , and force vector  $\vec{F}$ 

Variant	$\vec{F}$	$M$	$N$
<b>1</b>	(6; 15; 19)	(-6; 5; -9)	(3; 8; -1)
<b>2</b>	(3; 10; -5)	(7; -3; -8)	(5; -6; -4)
<b>3</b>	(5; -10; 12)	(4; 7; -3)	(6; -5; -2)
<b>4</b>	(-11; 6; -5)	(-2; -4; -3)	(-5; 8; 4)
<b>5</b>	(7; -1; -8)	(6; -8; 5)	(-7; -2; -1)
<b>6</b>	(-6; -7; 5)	(-3; 7; 5)	(2; -8; -2)
<b>7</b>	(8; 12; 7)	(5; -6; -2)	(-2; 7; 5)
<b>8</b>	(9; -5; 10)	(8; -4; 9)	(5; -6; -5)
<b>9</b>	(2; 11; -3)	(-7; -5; -2)	(-4; 3; 1)
<b>10</b>	(-5; 12; 10)	(-1; -7; 5)	(-2; 2; -3)

Task 9. During the designing of a channel for laying pipes, it is necessary to take into account the angle of a natural slope of the soil.

**The angle of the natural soil slope** is the largest acute angle between the slope and the horizon, at which the soil particles will be in a stable position (Figure 1.5).

Distance  $L$  from the foundation to the middle of the ditch in which the pipe is laid is determined by the formula

$$L = h \cdot \operatorname{ctg} \alpha + 0,5b + 0,4, \quad (1.1)$$

where  $h$  is a depth of laying the bottom of the pipe from the base of the foundation,  $m$ ;  $b$  is a ditch width,  $m$ ;  $\alpha$  is an angle of the natural soil slope.

Draw the relation graph of the distance  $L$  from the ditch width  $b$  ( $1 \leq b \leq 10$ ), if the depth of laying the pipe  $h$  and an angle of the natural soil slope  $\alpha$  are given at the table 1.10.

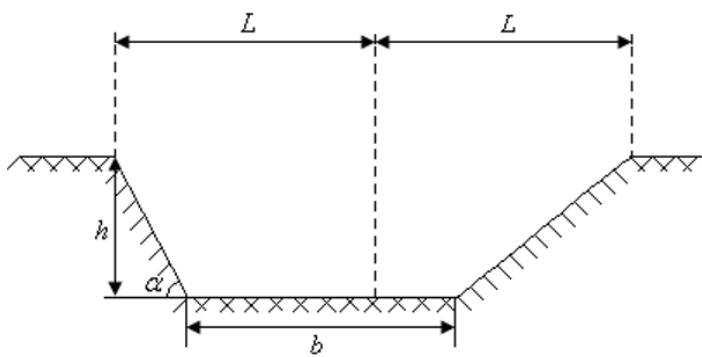


Figure 1.5 – Scheme of a marking of the ditch for lying of pipes

Table 1.10 – Original data of the ditch marking for laying pipes

Variant	$h$	soil type	$\alpha$
<b>1</b>	2,0	clay saturated with water	20
<b>2</b>	1,5	clay moisture	45
<b>3</b>	1,8	gravel round	30
<b>4</b>	2,1	chernozem is dry	40
<b>5</b>	1,7	clay saturated with water	20
<b>6</b>	1,4	clay moisture	45
<b>7</b>	2,3	gravel round	30
<b>8</b>	1,9	chernozem is dry	40
<b>9</b>	1,6	clay saturated with water	20
<b>10</b>	2,2	clay moisture	45

Task 10. A suspension bridge is crossing the ditch. The ropes of the suspension bridge approximately take the form of a parabola (Figure 1.6). Compose the equation of this parabola if the deflection of the rope and the length of the span are given at the table 1.11.

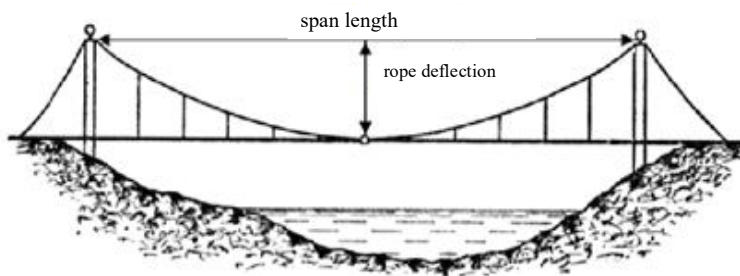


Figure 1.6 – Scheme of the suspension bridge

Table 1.11 – Geometric measurements of the suspension bridge

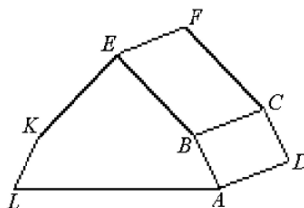
№ variant	The rope deflection	The span length	№ variant	The rope deflection	The span length
<b>1</b>	4	22	<b>6</b>	5	24
<b>2</b>	3	20	<b>7</b>	4	26
<b>3</b>	5	26	<b>8</b>	3	16
<b>4</b>	2	18	<b>9</b>	5	30
<b>5</b>	4	28	<b>10</b>	2	20

Task 11. It is necessary to calculate the angle between the attic roof slopes (Figure 1.7), if we know completely or partially the coordinates of the points (Figure 1.8) in Cartesian frame (Table 1.12).

**Attic roof** is a sloping roof, which is characterized by a structure consisting of two parts: upper, sloping; and the bottom – steep. The attic formed thanks to such design is called an attic.



Figure 1.7 – Building with an attic roof



1.8-Schematic image of an attic roof



Table 1.12 – Coordinates the points where the slopes of the mansard cover should lie

Variant	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>K</i>
<b>1</b>	(100; 0; 0)	(70; 0; 100)	(70; 300; 100)	(100; 300; 0)	(20; 0; 150)	(20; 300; 150)	(-30; 0; 100)
<b>2</b>	(130; 0; -50)	(100; 0; 70)	(100; 300; 70)	(130; 300; -50)	(0; 0; 120)	(0; 300; 120)	(-100; 0; 70)
<b>3</b>	(20; 50; 100)	(20; 0; 170)	(100; 0; 170)	(100; 50; 100)	(20; -30; 200)	(100; -30; 200)	(20; -60; 170)
<b>4</b>	(20; 130; 20)	(40; 90; 50)	(140; 140; 50)	(120; 180; 20)	(60; 50; 70)	(160; 100; 70)	(80; 10; 50)
<b>5</b>	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)
<b>6</b>	(100; 20; 0)	(70; 20; 100)	(70; 300; 100)	(100; 300; 0)	(0; 20; 120)	(0; 300; 120)	(-70; 20; 100)
<b>7</b>	(20; 150; 50)	(20; 130; 80)	(100; 130; 80)	(100; 150; 50)	(20; 70; 100)	(100; 70; 100)	(20; 10; 80)
<b>8</b>	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)
<b>9</b>	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)	(100; 0; 0)
<b>10</b>	(20; 130; 0)	(20; 90; 30)	(100; 90; 30)	(100; 130; 0)	(20; 50; 40)	(100; 50; 40)	(20; 10; 30)

Task 12. Calculate the approximate increase in the volume of a cylindrical column with the height  $H$  and a radius  $R$ , on which the plaster layer  $\Delta R$  was applied (Table 1.13).

Table 1.13 – Geometric shape of a cylindrical colony

Variant	$H$ , м	$R$ , см	$\Delta R$ , см
<b>1</b>	3	30	0,5
<b>2</b>	5	35	0,6
<b>3</b>	4	25	0,4
<b>4</b>	2	20	0,3
<b>5</b>	6	30	0,5
<b>6</b>	3	25	0,4
<b>7</b>	4	20	0,6
<b>8</b>	5	30	0,5
<b>9</b>	6	25	0,3
<b>10</b>	3	20	0,6

Task 13. Energy savings in a production should not lead to a decline in the quality of work and a deteriorating of working conditions. The design of lighting systems on a construction place is reduced to the following task.

At what distance  $h$  from the horizontal plane is it necessary to put the light bulb so that at a given point  $A$  on the plane the illumination was the largest ( $OA = a$ ) (Figure 1.9)? Calculate the height of the column  $h$  (with accuracy  $\alpha = 0,01$ ) with a lighting device, if a distance  $a$  (in meters) is given at the table 1.14.

Table 1.14 – Distance  $a$  from the column with a lighting device

Variant	$a$ , m	Variant	$a$ , m
<b>1</b>	5	<b>6</b>	9
<b>2</b>	2	<b>7</b>	11
<b>3</b>	4	<b>8</b>	8
<b>4</b>	3	<b>9</b>	6
<b>5</b>	7	<b>10</b>	10

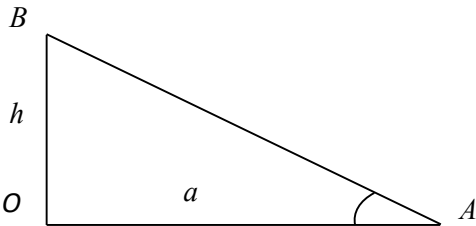


Figure 1.9 – Location of the lighting device relative to the horizontal surface

Task 14. Suspension bridge ropes (Figure 1.6) take the form of a chain line.

**Chain line** is a curve on the plane, which is given in the Cartesian coordinate system by the equation

$$y = \frac{a\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)}{2}, \quad (1.2)$$

where  $a$  is some positive number.

Compose an equation and draw the curvature of the bridge, if the span length is given at the table 1.15.

Table 1.15 – Span length  $2a$  of the suspension bridge

Variant	$2a$ , м	Variant	$2a$ , м
<b>1</b>	20	<b>6</b>	18
<b>2</b>	16	<b>7</b>	24
<b>3</b>	14	<b>8</b>	10
<b>4</b>	22	<b>9</b>	26
<b>5</b>	12	<b>10</b>	30

Task 15. The construction of the research center is provided with a continuous cash flow with a rate  $I(t) = -a_2t^2 + a_1t + a_0$  (UAH million / year), where  $t$  is a time, during  $N$  years with an annual interest rate  $p$ , (%). Determine the discounted value of this flow according to dates at the table 1.16.

Table 1.16 – Data for calculating the discounted value of cash flow

Variant	$a_2$	$a_1$	$a_0$	$N$	$p$
<b>1</b>	-2	10	5	20	4
<b>2</b>	1	20	3	15	5
<b>3</b>	-3	5	7	20	2

Continuation of the table 1.16

Variant	$a_2$	$a_1$	$a_0$	$N$	$p$
4	-4	8	-4	25	10
5	2	25	8	15	8
6	3	30	9	10	20
7	-2	7	-5	20	5
8	2	30	35	25	4
9	-4	20	3	20	8
10	1	15	4	15	10

16. Exhibition pavilion, the cross section of which has the shape of a parabolic segment (Figures 1.10, 1.11). Calculate the construction volume and floor area of the structure, if the geometric dimensions of the pavilion are given at the table 1.17.



Figure 1.10 – Exhibition pavilion

**Building volume** is the volume of the room, which is limited by the building structure.

The building volume for the exhibition pavilion (Figure 1.10) is equal to the cross-sectional area multiplied by the height of this segment.

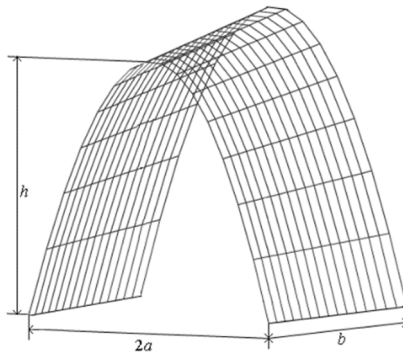


Figure 1.11 – Schematic representation of the exhibition pavilion

Table 1.17 – Geometric measurements of the exhibition pavilion

Variant	$a$	$b$	$h$
<b>1</b>	5	20	6
<b>2</b>	6	15	5
<b>3</b>	8	12	7
<b>4</b>	5	21	4
<b>5</b>	4	17	3
<b>6</b>	7	14	5
<b>7</b>	8	19	9
<b>8</b>	6	23	7
<b>9</b>	5	18	6
<b>10</b>	4	22	5

## 2 EXAMPLE OF TYPICAL VARIANT TASK SOLUTION

Task 1. Let the coordinates of the points  $A(1,1)$ ,  $B(6,6)$  and  $C(2,5)$  be known (Figure 2.1). Let the highway be a straight line that passes through points  $A$  and  $B$ . Using the equation of a line passing through two points

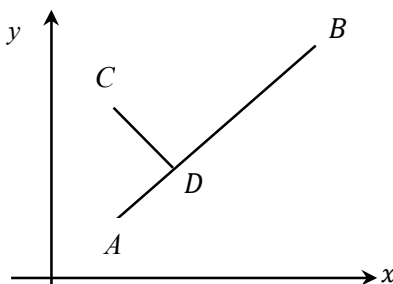


Figure 2.1 – Scheme of a highways

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} \quad (2.1)$$

we get the highway equation

$$\frac{x-1}{6-1} = \frac{y-1}{6-1}.$$

After some identical transformations we get it:  $y = x$ .

According to the condition of the task, we should connect an object located at a point  $C$  in the same coordinate system by the shortest path to the highway. However, we know that the shortest distance is perpendicular to the line  $AB$ . According to the condition of perpendicularity we have:

$$k_{CD} = -\frac{1}{k_{AB}} = -1.$$

Make the equation of a line with a given angular coefficient passing through a given point

$$y - y_0 = k(x - x_0). \quad (2.2)$$

We get the equation of a line  $CD$ :

$$y - 5 = -(x - 2), \quad y = -x + 7.$$

Find the point of intersection of the lines  $AB$  and  $CD$ , solve the system of lines equations:

$$\begin{cases} y = -x + 7; \\ y = x; \end{cases} \begin{cases} x = \frac{7}{2}; \\ y = \frac{7}{2}. \end{cases}$$

Finally, we have the coordinates of the desired point  $D\left(\frac{7}{2}; \frac{7}{2}\right)$ .

**Answer :**  $D\left(\frac{7}{2}; \frac{7}{2}\right)$ .

Products	Components		Building materials	$C_1$	$C_2$
$A_1$	$a$	$b$	$B_1$	$m$	$n$
$A_2$	$c$	$d$	$B_2$	$f$	$q$

Task 2. We will make the equation for finding components  $B_1, B_2$  to define quantity of necessary materials, proceeding from the table 1.2. Since the building materials  $C_1, C_2$  are part of each mixture  $B_1, B_2$  and  $C_1, C_2$ , consisting of elements,  $m$  and  $f$ ;  $n$  and  $q$ ; respectively, we obtain the following equations:

$$mC_1 + nC_2 = B_1;$$

$$fC_1 + qC_2 = B_2;$$

we have it in a matrix form:

$$\begin{pmatrix} m & n \\ f & q \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}. \quad (2.3)$$

Speculating similarly, we make the matrix equation for the finding building mixes  $A_1, A_2$ :

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} aB_1 + bB_2 \\ cB_1 + dB_2 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}. \quad (2.4)$$

From the equation (2.3) express unknowns  $C_1, C_2$ :

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} m & n \\ f & q \end{pmatrix}^{-1} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix},$$

and from the equation (2.4) express unknowns  $B_1, B_2$ :

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}.$$

Thus,

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} m & n \\ f & q \end{pmatrix}^{-1} \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix},$$

where

$$\begin{pmatrix} m & n \\ f & q \end{pmatrix}^{-1} = \frac{1}{\begin{vmatrix} m & n \\ f & q \end{vmatrix}} \cdot \begin{pmatrix} q & -n \\ -f & m \end{pmatrix};$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

So,



$$\begin{aligned}
\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} &= \frac{1}{\begin{vmatrix} m & n \\ f & q \end{vmatrix}} \cdot \begin{pmatrix} q & -n \\ -f & m \end{pmatrix} \cdot \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \\
&= \frac{1}{mq - fn} \cdot \frac{1}{ad - bc} \begin{pmatrix} qd + nc & -qb - na \\ -fd - mc & fb + ma \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \\
&= \frac{1}{(mq - fn) \cdot (ad - bc)} \cdot \begin{pmatrix} (qd + nc)A_1 + (-qb - na)A_2 \\ (-fd - mc)A_1 + (fb + ma)A_2 \end{pmatrix}.
\end{aligned}$$

Therefore,

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{(mq - fn) \cdot (ad - bc)} \cdot \begin{pmatrix} (qd + nc)A_1 + (-qb - na)A_2 \\ (-fd - mc)A_1 + (fb + ma)A_2 \end{pmatrix} \quad (2.5)$$

Substitute at the formula (2.5) the values of variables  $a, b, c, d, m, f, n, q$  from the table 1.3 according to the variant number, and substitute units instead of  $A_1, A_2$ . Suppose that according to the condition of the task we have  $a = 2, b = 3, c = 2, d = 1, m = 1, f = 1, n = 1, q = 0$ , then:

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \frac{1}{(-1) \cdot (2 - 6)} \begin{pmatrix} (0 + 2) + (0 - 2) \\ (-1 - 2) + (3 + 2) \end{pmatrix} = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}.$$

So, the production of one ton of products  $A_1$  and  $A_2$  requires building materials  $C_1 = 0, C_2 = 1/2$ .

**Answer:**  $C_1 = 0, C_2 = 1/2$ .

Task 3. To determine the amount of salary accrued to employees at each stage of construction, according to the table 1.4: the cost of working time in hours at each workplace (matrix  $P$ ); production standards (matrix  $Q$ ); payment (in USA) at each workplace (matrix  $C$ ).

According to production standards and labor costs at each workplace is the volume of output as a product of matrices:  $S = Q \cdot P$ ; according to the tariff of hourly payment at each workplace is determined by the amount of payments to employees by the formula

$$X = S \cdot C = Q \cdot P \cdot C. \quad (2.6)$$

Let us know the cost of working time in hours at each workplace (matrix  $P$ ); production standards (matrix  $Q$ ); hourly payment at each workplace (matrix  $C$ ):

$$P = \begin{pmatrix} 2 & 14 & 5 & 0 \\ 5 & 32 & 0 & 1 \\ 11 & 01 & 4 & 3 \end{pmatrix}; Q = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 3 & 0 & 1 \end{pmatrix}; C = \begin{pmatrix} 1,5 \\ 1,4 \\ 1,25 \\ 1,3 \end{pmatrix}.$$

Calculate the product of matrices:

$$\begin{aligned} S = Q \cdot P &= \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 3 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 14 & 5 & 0 \\ 5 & 32 & 0 & 1 \\ 11 & 01 & 4 & 3 \end{pmatrix} = \\ &= \begin{pmatrix} 2+10+0 & 1+6+0 & 4+4+0 & 5+0+0 & 0+2+0 \\ 4+20+55 & 2+12+0 & 8+8+5 & 10+0+20 & 0+4+15 \\ 6+0+11 & 3+0+0 & 12+0+1 & 15+0+4 & 0+0+3 \end{pmatrix} = \\ &= \begin{pmatrix} 12 & 7 & 8 & 5 & 2 \\ 79 & 14 & 21 & 30 & 19 \\ 17 & 3 & 13 & 19 & 3 \end{pmatrix}; \\ X = S \cdot C &= \begin{pmatrix} 12 & 7 & 8 & 5 & 2 \\ 79 & 14 & 21 & 30 & 19 \\ 17 & 3 & 13 & 19 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1,5 \\ 1,4 \\ 1,25 \\ 1,25 \\ 1,3 \end{pmatrix} = \end{aligned}$$

$$= \begin{pmatrix} 18 + 6,8 + 10 + 6,25 + 2,6 \\ 118,5 + 19,6 + 26,25 + 37,5 + 24,7 \\ 25,5 + 4,2 + 16,25 + 23,75 + 3,9 \end{pmatrix} = \begin{pmatrix} 43,65 \\ 226,55 \\ 73,60 \end{pmatrix}.$$

$$\textbf{Answer} : X = \begin{pmatrix} 43,65 \\ 226,55 \\ 73,60 \end{pmatrix}.$$

Task 4. In order to find the optimal proportions of the tank (Figure 2.1), it is necessary to solve the task of the extreme. The volume of the cylinder is determined by the formula

$$V = \pi r^2 h, \quad (2.7)$$

then,

$$h = \frac{V}{\pi r^2}. \quad (2.8)$$

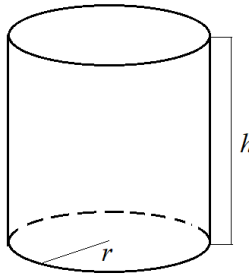


Figure 2.1 – Cylindrical tank

Construction requires the least amount of material, so the surface area should be minimal at the specified volume. The total surface area of the cylinder is calculated by the following formula

$$S_{n.n.} = 2\pi r(r + h) = 2\pi r^2 + 2\pi rh. \quad (2.9)$$

But in the specified statement of the task the metal cylinder has only one basis (bottom), then rewrite (2.8), taking into account the expression found for  $h$ , and we have:

$$S_{n.n.} = \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r \frac{V}{\pi r^2} = \pi r^2 + \frac{2V}{r}, \quad (r \neq 0).$$

That is, the total surface area is a function of the radius of the base of the cylinder:  $S_{n.n.} = S(r)$ . Find the derivative of this function  $S_{n.n.}$ .

$$S'_{n.n.} = 2\pi r - \frac{2V}{r^2}.$$

Determine in which value of  $r$  the function derivative will be equal to zero:  $S'_{n.n.} = 0$ .

$$2\pi r - \frac{2V}{r^2} = 0; \frac{\pi r^3 - V}{r^2} = 0, \quad \pi r^3 - V = 0, \quad \pi r^3 = V,$$

$$r = \sqrt[3]{\frac{V}{\pi}}.$$

It is necessary to find the second order function derivative, so, we get it:  $S''_{n.n.} = 2\pi + \frac{4V}{r^3}$ .

Accordingly to condition  $S''_{n.n.} \left( \sqrt[3]{\frac{V}{\pi}} \right) > 0$ , we make conclusion, that the function  $S_{n.n.}$  reaches its least value at the point  $t = \sqrt[3]{\frac{V}{\pi}}$ .

$$h = \frac{V}{\pi r^2} = \frac{V}{\pi \sqrt[3]{\left(\frac{V}{\pi}\right)^2}} = \sqrt[3]{\frac{V}{\pi}}.$$

Therefore, the lowest material costs can be achieved when

$$h = \sqrt[3]{\frac{V}{\pi}}, \quad r = \sqrt[3]{\frac{V}{\pi}}. \quad (2.10)$$

Calculate the height and radius of the cylinder, provided that the desired volume  $V = 150 \text{ l}$  with accuracy  $\alpha = 0,01$  (suppose that  $\pi = 3,14$ ):

$$h = \sqrt[3]{\frac{150}{3,14}} \approx 3,63 \text{ (m)}; \quad r = \sqrt[3]{\frac{150}{3,14}} \approx 3,63 \text{ (m)}.$$

**Answer:**  $h = r = 3,63 \text{ (m)}$ .

Task 5. According to the condition of the task, the container has the shape of a rectangular parallelepiped without an upper face (Figure 2.2). Introduce the denotation:  $x$  is a width,  $y$  is a length,  $z$  is a height. According to the introduced symbols, the volume of the container and the surface area are calculated by to the following formulas

$$V = xyz, \quad (2.11)$$

$$S = xy + 2xz + 2yz. \quad (2.12)$$

Suppose that according to the condition of the task it is

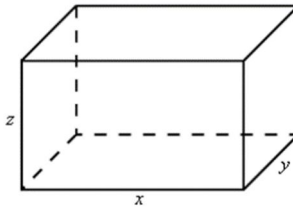


Figure 2.2 – Container in the form of a rectangular parallelepiped

necessary to make a container of a volume  $6m^3$  and a height  $2m$ .

After substitution at the equation (2.11) a volume value of a parallelepiped, we get:  $6 = xyz$ ; express the variable  $z$  from a volume expression using  $x$  and  $y$ , and substitute it at the equation (2.12):

$$z = \frac{6}{xy}; \quad S = xy + 2xz + 2yz = xy + \frac{12}{y} + \frac{12}{x}.$$

Thus, the function of the surface of a parallelepiped is a function of two variables, so to find the minimum cost of material, it is necessary to solve the problem of the extreme of the function of two variables:

1) find the partial derivative of the first order

$$\frac{\partial S}{\partial x} = y - \frac{12}{x^2}; \quad \frac{\partial S}{\partial y} = x - \frac{12}{y^2};$$

2) find the stationary points from the condition

$$\begin{cases} \frac{\partial S}{\partial x} = 0; \\ \frac{\partial S}{\partial y} = 0; \end{cases} \begin{cases} y - \frac{12}{x^2} = 0; \\ x - \frac{12}{y^2} = 0; \end{cases} \begin{cases} y = \frac{12}{x^2}; \\ x = \frac{12}{\left(\frac{12}{x^2}\right)^2} = 0; \end{cases}$$

$$12x - x^4 = 0; \quad x(12 - x^3) = 0; \quad \begin{cases} x_1 = 0; \\ y_1 = 0; \end{cases} \begin{cases} x_2 = \sqrt[3]{12} \\ y_2 = \sqrt[3]{12} \end{cases}$$

According to the task we have  $x > 0$ ,  $y > 0$ , therefore, we are satisfied the value  $x = \sqrt[3]{12}$ ,  $y = \sqrt[3]{12}$ .

Let's check whether the function  $S(x, y)$  reaches a minimum at the point  $(\sqrt[3]{12}; \sqrt[3]{12})$ . To do this, we find the partial derivatives of the second order:

$$\frac{\partial^2 S}{\partial x^2} = \left(y - \frac{12}{x^2}\right)'_x = \frac{24}{x^3} \cdot \frac{\partial^2 S}{\partial x^2} \Big|_{(\sqrt[3]{12}; \sqrt[3]{12})} = \frac{24}{12} = 2;$$

$$\frac{\partial^2 S}{\partial y^2} = \left(x - \frac{12}{y^2}\right)'_y = \frac{24}{y^3} \cdot \frac{\partial^2 S}{\partial y^2} \Big|_{(\sqrt[3]{12}; \sqrt[3]{12})} = \frac{24}{12} = 2;$$

$$\frac{\partial^2 S}{\partial x \partial y} = \left(y - \frac{12}{x^2}\right)'_y = 1; \frac{\partial^2 S}{\partial x \partial y} \Big|_{(\sqrt[3]{12}; \sqrt[3]{12})} = 1.$$

Calculate the square shape at the point  $(\sqrt[3]{12}; \sqrt[3]{12})$ :

$$\Delta = 2 \cdot 2 - 1^2 = 3 > 0.$$

According to the condition  $\Delta > 0$ ,  $\frac{\partial^2 S}{\partial x^2} < 0$  we can make the conclusion, that a function  $S$  reaches its minimum value at the found point.

Find the height of the container

$$z = \frac{6}{xy} = \frac{6}{\sqrt[3]{12} \cdot \sqrt[3]{12}} = \frac{1}{2} \sqrt[3]{12}.$$

$$\textbf{Answer:} x = \sqrt[3]{12}; y = \sqrt[3]{12}; z = \frac{1}{2} \sqrt[3]{12}.$$

Task 6. Look at the figure 1.2 and the figure 1.3, there are the segment  $AB$  is a crane mast, and the segment  $OA$  is a cable. Accordingly to the task, we can write down the vectors

$$\overrightarrow{AB} = \vec{r}, \overrightarrow{OA} = \vec{r}_1, \overrightarrow{CA} = \vec{r}_2, \overrightarrow{OB} = \vec{a}, \overrightarrow{BC} = \vec{b}.$$

The acute angle formed by a segment  $OA$  with abscissa axis is denoted as  $\alpha$ , the acute angle formed by a segment  $AB$  with ordinate axis is denoted as  $\beta$ .

According to Newton's third law, the forces of interaction of two material points are equal in magnitude and have opposite directions, and act along the line connecting these points. By this law in any mechanical system in the absence of external forces, the geometric sum of all internal forces is zero.

The vector scheme of the crane is a combination of two schemes with two contours  $OBAO$  and  $BCAB$ . Therefore, the closed condition for a complete system can be hung as the sum of the closed conditions for each of the parts separately. In vector form, these conditions take the form:

$$(\vec{a} - \vec{r} - \vec{r}_1) + (\vec{b} + \vec{r}_2 + \vec{r}) = 0. \quad (2.13)$$

Using the addition rule of vectors, we get

$$\vec{b} + \vec{r}_2 + \vec{r} = 0.$$

Hence the condition of a closed system takes the form

$$\vec{a} - \vec{r} - \vec{r}_1 = 0. \quad (2.14)$$

Find the projection of the condition (2.14) on the abscissa axis:

$$proj_{Ox} \vec{a} = |\vec{a}| \cos 0 = |\vec{a}|;$$

$$proj_{Ox} \vec{r} = -|\vec{r}| \cos(90 - \beta) = -|\vec{r}| \sin \beta;$$

$$proj_{Ox} \vec{r}_1 = |\vec{r}_1| \cos \alpha.$$



Therefore, the projection of the condition of the crane closure on the abscissa axis takes the form:

$$|\vec{a}| + |\vec{r}| \sin \beta - |\vec{r}_1| \cos \alpha = 0. \quad (2.15)$$

Find similarly the projection of the condition (2.14) on the ordinate axis:

$$proj_{Oy} \vec{a} = |\vec{a}| \cos 90 = 0;$$

$$proj_{Oy} \vec{r} = -|\vec{r}| \cos \beta;$$

$$proj_{Ox} \vec{r}_1 = |\vec{r}_1| \cos(90 - \alpha) = |\vec{r}_1| \sin \alpha.$$

Hence the projection of the condition of the crane closure on the ordinate axis takes the form:

$$|\vec{r}_1| \sin \alpha - |\vec{r}| \cos \beta = 0. \quad (2.16)$$

Calculate the length of segments  $OA$ ,  $AB$  taking into account that  $|\vec{OB}| = 6$ ,  $\alpha = 40^\circ$ ,  $\beta = 15^\circ$ . For this, considering the conditions (2.15), (2.16), make the equations system:

$$\begin{cases} |\vec{r}| \sin 15^\circ - |\vec{r}_1| \cos 40^\circ = -6; \\ |\vec{r}_1| \sin 40^\circ - |\vec{r}| \cos 15^\circ = 0. \end{cases}$$

Let's solve the obtained system, having preliminary approximately calculated on the calculator the values of trigonometric functions with accuracy to 0,001.

$$\begin{cases} 0,259|\vec{r}| - 0,766|\vec{r}_1| = -6; \\ 0,966|\vec{r}| - 0,643|\vec{r}_1| = 0. \end{cases}$$

We find the solution of this system, for example, using the Cramer's rules:

$$\Delta = \begin{vmatrix} 0,259 & -0,766 \\ 0,966 & -0,643 \end{vmatrix} \approx -0,167 + 0,740 = 0,573;$$

$$\Delta_1 = \begin{vmatrix} -6 & -0,766 \\ 0 & -0,643 \end{vmatrix} = 3,858;$$

$$\Delta_2 = \begin{vmatrix} 0,259 & -6 \\ 0,966 & 0 \end{vmatrix} = 5,796.$$

Here from, we get

$$AB = |\vec{r}| = \frac{3,858}{0,573} \approx 6,733m; \quad OA = |\vec{r}_1| = \frac{5,796}{0,573} \approx 10,115m.$$

**Answer:**  $AB \approx 6,733m; OA \approx 10,115m.$

Task 7. According to the Figure 1.4 enter the denotation  $\vec{P}$  is a vector of a wind pressure force on the left slope of the truss,  $\vec{N}$  is a force vector directed along the normal to the roof,  $\vec{S}$  is a force vector, directed parallel to the roof,  $\alpha$  is an angel between a vector  $\vec{P}$  and a horizon line, points  $A$  and  $B$  aretruss supports,  $\beta$  is a skew slope angel, that is, the angle between the segments  $AE$  and  $EB$ .

According to the vectors addition rules, we can present a vector  $\vec{P}$  in a sum form as:

$$\vec{P} = \vec{N} + \vec{S}.$$

Only force  $\vec{N}$  will act on the truss, because the wind will slide on the roof under the action of a force  $\vec{S}$ .

Consider a right triangle  $ABC$  ( $\angle CAB = 90^\circ$ ) (Figure 1.4). There is a point  $E$  is a midpoint of the segment  $BC$ ,  $\angle EAB = \beta$ . Point  $M$  is an arbitrary point belonging to the segment  $AB$ , a point  $N$  is at he base of the perpendicular drawn from the point  $M$  to the line  $AC$ , a point  $K$  belongs to the

segment  $AC$ , herewith  $\angle KMN = \alpha$ . Let the length of the perpendicular  $KL$  be equal to 30 ( $kN$ ), and angels  $\alpha$  and  $\beta$  acquire values accordingly:  $\alpha = 20^\circ, \beta = 25^\circ$ .

Consider right triangles

$ACM$  ( $\angle AMC = 90^\circ$ ),  $KCL$  ( $\angle CLK = 90^\circ$ ),  $KNM$  ( $\angle KNM = 90^\circ$ ).

From the triangle  $ACM$ , we get  $\angle CAM = 90^\circ - \beta$ , then  $\angle ACM = \beta$ .

From the triangle  $KCL$ , we get  $\angle CKL = 90^\circ - \beta$ .

From the triangle  $KNM$ , we get  $\angle NKM = 90^\circ - \alpha$ .

It follows from the condition  $\angle CKL + \angle NKM + \angle LKM = 180^\circ$

$$\begin{aligned}\angle LKM &= 180^\circ - (\angle CKL + \angle NKM) = \\ &= 180^\circ - (90^\circ - \beta + 90^\circ - \alpha) = \alpha + \beta = 20^\circ + 25^\circ = 45^\circ.\end{aligned}$$

Calculate the magnitude of the vector  $\vec{N}$ :

$$|\vec{N}| = |\vec{P}| \cdot \sin \angle LKM = 30 \cdot \sin 45^\circ = 30 \cdot \frac{\sqrt{2}}{2} = 15\sqrt{2}.$$

**Answer:** The value of the wind pressure on the left slope of the truss is equal to  $15\sqrt{2}kN$ .

Task 8. Let the initial position of the cargo be determined by a point  $M(2; -5; 3)$ , and the final position of the cargo is determined by the point  $N(4; 6; -1)$ . Therefore, the movement of the cargo under the action of a force  $\vec{F} = 3\vec{i} + 5\vec{j} + 2\vec{k}$  will be carried out in the direction of the vector  $\vec{S} = \overrightarrow{MN}$ . The work of a moving the cargo under the action of a constant force is calculated by the formula

$$A = \vec{F} \cdot \vec{S}. \quad (2.17)$$

That is, to calculate the work it is necessary to find the scalar product of vectors

$$A = F_x S_x + F_y S_y + F_z S_z.$$

Find the coordinates of the movement vector:

$$\overrightarrow{MN} = \vec{S} = \overrightarrow{(4 - 2; 6 - (-5); -1 - 3)} = \overrightarrow{(2; 11; -4)},$$

and calculate the force vector value

$$A = 3 \cdot 2 + 5 \cdot 11 + 2 \cdot (-4) = 6 + 55 - 8 = 54.$$

**Answer:**  $A = 54$ .

Task 9. In the general case, a function  $L$  is a function of two variables;  $b, h, \alpha$ . But according to the task condition, specific values  $h, \alpha$ , are given, then a function  $L$  will be a function of the one variable  $b$ . Distance  $L$  from the foundation to the middle of the ditch in which the pipe is laid, is calculated by the formula (1.10):

$$L = h \cdot \operatorname{ctg} \alpha + 0,5b + 0,4,$$

when  $h = 1,5 \text{ m}$ ,  $\alpha = 30^\circ$ .

We need to draw the relation graph of the distance between the foundation and the middle of the ditch for the soil category is round gravel. We find an approximate value  $\operatorname{ctg} 30^\circ$  using tables of trigonometric functions or ordinary calculator that has this option:

$$\operatorname{ctg} 30^\circ = 1,73.$$

Substitute the obtained value into the formula (1.10) and get the following relation:

$$L = 1,5 \cdot 1,73 + 0,5b + 0,4.$$

This relation is a linear relation. In this case the graph of this relation is a straight line. We know that we need only two points to draw the straight line graph. Find it.

Let's draw a graph of the relation  $L(b)$  (Figure 2.3).

$b$	0	1
$L$	2,995	3,495

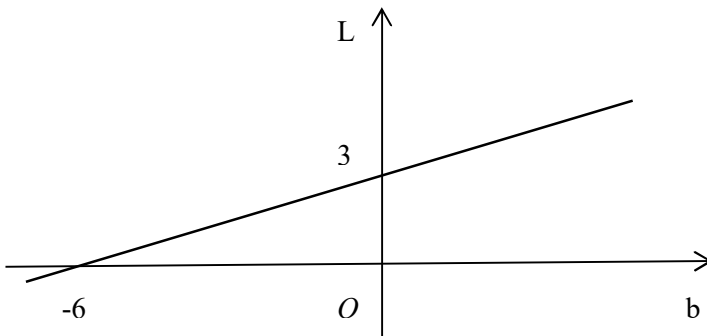


Figure 2.3 – Graph of the relation  $L(b)$

**Answer:** a relation graph of the distance between the foundation and the middle of the ditch from the width of the ditch for the category of soil is gravel round is presented at the figure 2.3.

Task 10. To make the equation of the parabola for the suspension bridge ropes, choose the coordinate system as it's shown at the figure 2.4.

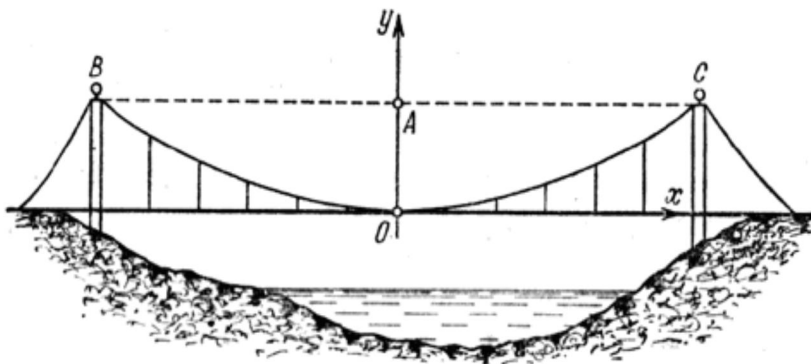


Figure 2.4 – Suspension bridge ropes coordinate system

Then the vertex of the parabola point  $O$  will coincide with the coordinate system origin point. Points  $B$  and  $C$  bound the parabola, and a point  $A$  is the point of intersection of the segment  $BC$  and the perpendicular, which is drawn from a point  $O$  to this segment.

Let the rope deflection be equal to  $OA = 6$  m, and the span length  $BC = 24$  m. Points  $O$ ,  $B$  and  $C$  belong to the parabola, therefore, in the coordinate system (Figure 2.4) the coordinates of the points will be as follows:  $O(0; 0)$ ,  $C(12; 6)$ ,  $B(-12; 6)$ .

Let's make the canonical equation of a parabola with a vertex  $O(0; 0)$ , the parabola branches are directed upwards, the parabola is symmetrical to the axis  $Oy$  and passes through the points  $C(12; 6)$  and  $B(-12; 6)$ .

Use the formula

$$x^2 = 2py, \quad (2.18)$$

where  $p \in R$ .

Find the parameter value  $p$ . To do this, substitute the coordinates of the points  $B$  or  $C$  at the formula (2.18).

Take the coordinates of a point  $C(12; 6)$ , then

$$12^2 = 2p \cdot 6; \quad p = 12.$$

Thus, the parabola canonic equation looks like this

$$x^2 = 24y.$$

**Answer:** the ropes of the suspension bridge approximately take the form of a parabola, which is given by the equation  $x^2 = 24y$  in the Cartesian coordinates system is shown at the figure 2.4.

Task 11. To find the angle between two slopes of the roof, we apply the formula for finding the cosine of the angle between two planes, which are given by their general equations:

$A_1x + B_1y + C_1z + D_1 = 0$  is the general equation of the first plane,

$A_2x + B_2y + C_2z + D_2 = 0$  is the general equation of the second plane,

$$\cos\varphi = \frac{|A_1 \cdot A_2 + B_1 \cdot B_2 + C_1 \cdot C_2|}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}}. \quad (2.19)$$

According to the task, we are given points:

$$A(160; 0; 0), B(100; 0; 160), C(100; 400; z_C), D(160; 50; 0),$$

$$E(x_E; 0; 200), F(0; 400; 200), K(-100; 0; 160)$$

and also planes  $KEF$  and  $BCFE$ . Let the angle between the slopes  $KEF$  and  $BCFE$  be denoted as  $\varphi$ , this will be the angle between the planes  $KEF$  and  $BCFE$ , is shown at the figure 1.8.

Points  $A, B, E, K$  belong to the one plane; points  $A, B, C, D$  also belong to the one plane; and points  $B, C, E, F$  also belong to the one plane.

Make the equation of the plane  $ABCD$ . Use the formula:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \quad (2.20)$$

Substitute the coordinates of the points  $A(160; 0; 0)$ ,  $B(100; 0; 160)$ ,  $D(160; 50; 0)$ , and get it:

$$\begin{vmatrix} x - 160 & y - 0 & z - 0 \\ 100 - 160 & 0 - 0 & 60 - 0 \\ 160 - 160 & 50 - 0 & 0 - 0 \end{vmatrix} = 0,$$

$$\begin{vmatrix} x - 160 & y & z \\ -60 & 0 & 160 \\ 0 & 50 & 0 \end{vmatrix} = 0.$$

Calculate the determinant, decomposing the elements of the third row:

$$-50(160(x - 160) + 60z) = 0,$$

$$8(x - 160) + 3z = 0,$$

$$8x + 3z - 1280 = 0.$$

We obtained the plane equation  $ABCD$ .



Find the coordinate  $z_C$  of a point  $C$ . Substitute the coordinates of a point  $C$  at the equation of a plane  $ABCD$ , and we get:

$$8 \cdot 100 + 3z - 1280 = 0,$$

$$3z = 480,$$

$$z = 160.$$

So,  $z_C = 160$ ,  $C(100; 400; 160)$ .

Compose the equation of the plane  $BCFE$ , using the formula (2.20) again, substitute at this equation the coordinates of points  $B(100; 0; 160)$ ,  $C(100; 400; 160)$  and  $F(0; 400; 200)$ :

$$\begin{vmatrix} x - 100 & y - 0 & z - 160 \\ 100 - 160 & 400 - 0 & 160 - 160 \\ 0 - 100 & 400 - 0 & 200 - 160 \end{vmatrix} = 0,$$

$$\begin{vmatrix} x - 100 & y & z - 160 \\ 0 & 400 & 0 \\ -100 & 400 & 40 \end{vmatrix} = 0.$$

Calculate the determinant, decomposing the elements of the second row:

$$400(40(x - 100) + 100(z - 160)) = 0,$$

$$2(x - 100) + 5(z - 160) = 0,$$

$$2x + 5z - 1000 = 0.$$

We obtained the plane equation  $BCFE$ .

Find the coordinate  $x_E$  of a point  $E$ . Substitute the coordinates of a point  $E$  at the equation of a plane  $BCFE$ , and we get:

$$2 \cdot x + 5 \cdot 200 - 1000 = 0, x = 0.$$

So,  $x_E = 0$ ,  $E(0; 0; 200)$ .

Compose the equation of the plane  $KEF$ , using the formula (2.20) again, substitute at this equation the coordinates of points  $E(0; 0; 200)$ ,  $F(0; 400; 200)$  and  $K(-100; 0; 160)$ :

$$\begin{vmatrix} x + 100 & y - 0 & z - 160 \\ 0 + 100 & 0 - 0 & 200 - 160 \\ 0 + 100 & 400 - 0 & 200 - 160 \end{vmatrix} = 0,$$

$$\begin{vmatrix} x + 100 & y & z - 160 \\ 100 & 0 & 40 \\ 100 & 400 & 40 \end{vmatrix} = 0.$$

Calculate the determinant, decomposing the elements of the second row:

$$5(40y - 400(z - 160)) + 2(400(x + 100) - 100y) = 0,$$

$$2(x + 100) - 5(z - 160) = 0,$$

$$2x - 5z + 1000 = 0,$$

we get the equation of a plane  $KEF$ .

Calculate the cosine of the angle  $\varphi$  between two planes  $KEF$  and  $BCFE$  using the formula (2.19):

$$\cos \varphi = \frac{|2 \cdot 2 + 0 \cdot 0 + (-5) \cdot 5|}{\sqrt{2^2 + 0^2 + (-5)^2} \cdot \sqrt{2^2 + 0^2 + 5^2}} = \frac{|-21|}{29} \approx 0,72.$$

Thus,  $\varphi \approx 44^\circ$ .

**Answer:** the angle between the slopes of the attic roof is approximately equal  $44^\circ$ .

Task 12. To calculate the approximate increase in the volume of a cylindrical column with the height  $H$  and a radius  $R$  on which the ball layer plaster  $\Delta R$  was applied, it is necessary to determine the relation between the volume of the column and its geometric measurements.

According to the task we have a cylinder, which characterizes the shape of the column.  $H$  is a cylinder height,  $R$  is a cylinder radius,  $V$  is a cylinder volume. Since the radius  $R$  of the cylinder changes during the application of plaster, and its height  $H$  is not changeable (constant), then the volume  $V$  is a function that respect of a variable  $R$ .

Use the formula to find the volume of the cylinder:

$$V(R) = \pi R^2 H \quad (2.21)$$

Denote  $\Delta R$  as a change of a cylinder radius  $R$ , and  $\Delta V$  as a change of a cylinder volume after its radius change on  $\Delta R$ .

To solve the task, it is necessary that all values are measured in the same units. Transform everything in meters. Then we get

$$H = 3,5 \text{ m}, R = 20 \text{ cm} = 0,2 \text{ m}, \Delta R = 0,5 \text{ cm} = 0,005 \text{ m}.$$

If the radius of the cylinder changes to  $\Delta R$ , then its initial volume  $V(R)$  is incremented  $\Delta V$ . Thus,

$$\Delta V \approx V'(R) \cdot \Delta R \quad (2.22)$$

Substitute the value  $\Delta R = 0,005 \text{ (m)}$  at the formula (2.22), and get:

$$\Delta V \approx 0,005 \cdot V'(R).$$

Find the derivative of a function  $V(R)$ :

$$V'(R) = (\pi R^2 H)' = 2\pi R H,$$

when  $H = 3,5$  m then we get:

$$V'(R) = 7\pi R.$$

Calculate now it  $\Delta V$  if  $R = 0,2$  m:

$$\Delta V \approx 0,005 \cdot V'(0,2) = 0,005 \cdot 7\pi \cdot 0,2 = 0,007\pi \text{ (m}^3\text{)}.$$

**Answer:** the volume of the cylindrical column, when applying the plaster, will increase by  $\Delta V \approx 0,007\pi \text{ (m}^3\text{)}.$

Task 13. Light intensity (c) is a physical quantity that characterizes the amount of light energy transmitted in a certain direction per unit time. Unit of measurement in the SI system: candela (*kd*).

Illumination (I) is a light quantity that determines the amount of light that falls on a certain surface area of the body. Units of measurement in the SI system: lux (*lk*).

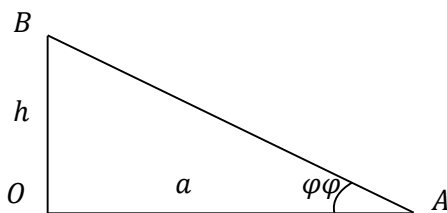


Figure 2.5 – Scheme of the light device location

Let it be  $a = 2\text{ m}$ , denote the distance between two points  $B$  (location point of the light bulb) and  $A$  (points on the plane in which the illumination should be the greatest), as  $r = AB$ ; the angle of incidence of the beam relative to the illumination plane as  $\angle OAB = \varphi$  (Figure 2.5); illumination of the plane as  $I$ ; the power of the light source as  $c$ .

The triangle  $\triangle OAB$  is a right-angled triangle,  $\angle AOB = 90^\circ$ . The luminance at a point  $A$  is given by a formula:

$$I = \frac{c \cdot \sin \varphi}{r^2}. \quad (2.23)$$

According to Pythagoras' theorem we have:

$$AB^2 = AO^2 + OB^2 \Leftrightarrow r^2 = a^2 + h^2 \Leftrightarrow r = \sqrt{a^2 + h^2} \quad (2.24)$$

$$\sin \angle BAO = \frac{OB}{AB} \Leftrightarrow \sin \varphi = \frac{h}{r}. \quad (2.25)$$

Substitute the ratio (2.24) and (2.25) at the formula (2.23), and we get:

$$I = \frac{c \cdot h}{\sqrt{(a^2 + h^2)^3}}.$$

We get a function  $I$ , that respect to variables  $h$  and  $c$ . Let the power of light  $c$  be a fixed variable (constant), the illumination  $I$  will be a function of the one variable  $h$ . To determine at what height  $h$  do you need to hang a light bulb so that the illumination  $I$  of points that are at a distance from the column  $a = 2\text{ m}$  was the maximum, you need to find the maximum point of the function  $I(h)$ , where  $0 < h < +\infty$ .

Find the function derivative  $I(h)$ :

$$\begin{aligned}
 I'(h) &= \left( \frac{c \cdot h}{\sqrt{(a^2 + h^2)^3}} \right)' = \\
 &= \frac{c \cdot h'(a^2 + h^2)^{3/2} - c \cdot h((a^2 + h^2)^{3/2})'}{(a^2 + h^2)^3} = \\
 &= c \cdot \frac{(a^2 + h^2)^{3/2} - h \cdot \frac{3}{2}(a^2 + h^2)^{1/2} \cdot 2h}{(a^2 + h^2)^3} = \\
 &= c \cdot \frac{a^2 + h^2 - 3h^2}{(a^2 + h^2)^{5/2}} = \\
 &= c \cdot \frac{a^2 - 2h^2}{(a^2 + h^2)^{5/2}}.
 \end{aligned}$$

Find the critical points of the function  $I(h)$ :

$$\begin{aligned}
 I'(h) = 0 &\Leftrightarrow c \cdot \frac{a^2 - 2h^2}{(a^2 + h^2)^{5/2}} = 0 \Leftrightarrow \\
 a^2 - 2h^2 &= 0; \quad h^2 = \frac{a^2}{2}; \\
 h &= \pm \frac{a}{\sqrt{2}},
 \end{aligned}$$

whereas  $0 < h < +\infty$ , then  $h = -\frac{a}{\sqrt{2}}$  does not satisfy the condition of the task, thereby  $h = \frac{a}{\sqrt{2}}$  is the only one critical point of the function  $I(h)$ .

We will study the critical point to the extreme. Consider the interval  $0 < h < \frac{a}{\sqrt{2}}$ . There are  $c > 0$  and the denominator of the derivative is also a positive number, and the

numerator  $a^2 - 2h^2 > 0$  is also a positive number at this interval. So, the derivative is  $I'(h) > 0$  at the interval  $0 < h < \frac{a}{\sqrt{2}}$ .

Consider the interval  $h > \frac{a}{\sqrt{2}}$ . There are  $c > 0$  and the denominator of the derivative is also a positive number, and the numerator is  $a^2 - 2h^2 < 0$  at this interval. So, the derivative also is  $I'(h) < 0$  at the interval  $h > \frac{a}{\sqrt{2}}$ .

Hence the function  $I(h)$  at the point  $h = \frac{a}{\sqrt{2}}$  reaches a maximum if  $0 < h < +\infty$ .

So, the illumination  $I(h)$  will be maximal if the lighting device is hung at a height  $h = \frac{a}{\sqrt{2}} (m)$ .

According to the task  $a = 2 m$ , so  $h = \frac{2}{\sqrt{2}} \approx 1,41 (m)$ .

**Answer:** height of a column with the lighting device should be  $h \approx 1,41(m)$ .

Task 14. The curvature of the curve on the plane given by the equation  $y = f(x)$ , could be found by the formula

$$k = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}. \quad (2.26)$$

Before we start to solve this task, we should select the coordinate system and do it as it's shown at the figure 2.6.

Let the length of the span be  $18 m$ , and the deflection of the rope coincides with high accuracy with the value  $\frac{3(e^3 + e^{-3} - 2)}{2} m$ .

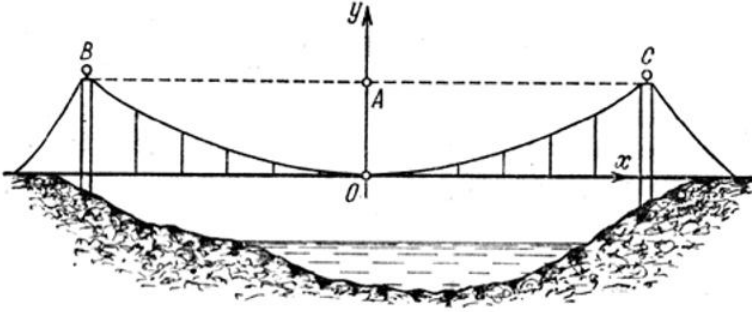


Figure 2.6 – Coordinates system

Denote a point  $O$  as the lowest point of the chain line, points  $B$  and  $C$  are points bound the line. Point  $A$  is a point of intersection of a straight line  $BC$  and perpendicular, drawn from the point  $O$ . The point  $O$  in the selected coordinate system point has coordinates  $(0; 0)$ . And these segments have a length

$$OA = \frac{3(e^3 + e^{-3} - 2)}{2} (m),$$

$$BA = AC = 9(m),$$

then points  $B$  and  $C$  have coordinates:

$$B \left( -9; \frac{3(e^3 + e^{-3} - 2)}{2} \right), C \left( 9; \frac{3(e^3 + e^{-3} - 2)}{2} \right).$$

If  $x = 0$ ,  $y = a$ , then the graph of the chain line will be given by the equation

$$y = \frac{a \left( e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)}{2} - a, \quad (2.27)$$

where  $a$  is a positive unknown number. Find it from the condition that the line passes through the points  $O$ ,  $B$  and  $C$ .



Substitute the coordinates of the point  $C$  at the equation (2.27):

$$\frac{3(e^3 + e^{-3} - 2)}{2} = \frac{a\left(e^{\frac{9}{a}} + e^{-\frac{9}{a}}\right)}{2} - a,$$

multiplied by 2 both parts of the equality, we obtain:

$$3(e^3 + e^{-3} - 2) = a(e^3 + e^{-3} - 2),$$

so, we get the value of  $a$

$$a = 3.$$

Therefore, in the selected coordinate system, the equation of the chain line will have the form

$$y = \frac{3\left(e^{\frac{x}{3}} + e^{-\frac{x}{3}}\right)}{2} - 3.$$

To find the curvature by formula (2.26), we find the first and second derivatives of the given function  $y = f(x)$

$$y' = \left( \frac{3\left(e^{\frac{x}{3}} + e^{-\frac{x}{3}}\right)}{2} - 3 \right)' = \frac{3}{2} \left( \frac{1}{3} e^{\frac{x}{3}} - \frac{1}{3} e^{-\frac{x}{3}} \right) = \frac{1}{2} \left( e^{\frac{x}{3}} - e^{-\frac{x}{3}} \right);$$

$$y'' = \left( \frac{1}{2} \left( e^{\frac{x}{3}} - e^{-\frac{x}{3}} \right) \right)' = \frac{1}{2} \left( \frac{1}{3} e^{\frac{x}{3}} + \frac{1}{3} e^{-\frac{x}{3}} \right) = \frac{1}{6} \left( e^{\frac{x}{3}} + e^{-\frac{x}{3}} \right).$$

Find the curvature of the chain line using the previously mentioned formula (2.26):

$$\begin{aligned}
k &= \frac{\left| \frac{1}{6} \left( e^{\frac{x}{3}} + e^{-\frac{x}{3}} \right) \right|}{\left( 1 + \left( \frac{1}{2} \left( e^{\frac{x}{3}} - e^{-\frac{x}{3}} \right) \right)^2 \right)^{3/2}} = \frac{1}{6} \cdot \frac{e^{\frac{x}{3}} + e^{-\frac{x}{3}}}{\left( 1 + \frac{e^{\frac{2x}{3}} - 2 + e^{-\frac{2x}{3}}}{4} \right)^{3/2}} = \\
&= \frac{1}{6} \cdot \frac{e^{\frac{x}{3}} + e^{-\frac{x}{3}}}{\left( \frac{e^{\frac{2x}{3}} + 4 + e^{-\frac{2x}{3}}}{4} \right)^{3/2}} = \frac{1}{6} \cdot \frac{e^{\frac{x}{3}} + e^{-\frac{x}{3}}}{\left( \frac{\left( e^{\frac{x}{3}} + e^{-\frac{x}{3}} \right)^2}{4} \right)^{3/2}} = \frac{8}{6} \cdot \frac{e^{\frac{x}{3}} + e^{-\frac{x}{3}}}{\left( e^{\frac{x}{3}} + e^{-\frac{x}{3}} \right)^3} = \\
&= \frac{4}{3 \left( e^{\frac{x}{3}} + e^{-\frac{x}{3}} \right)^2}. \tag{2.28}
\end{aligned}$$

To draw a graph of the obtained function, we will use the platform MS Excel.

To do this, divide the interval  $[-9; 9]$  into segments with a step 0,5. We enter the obtained formula in the field of curvature values  $k$ .

Repeat this procedure for each argument value using a key combination (Ctrl+C – Ctrl+V) (look at the Figure 2.7).

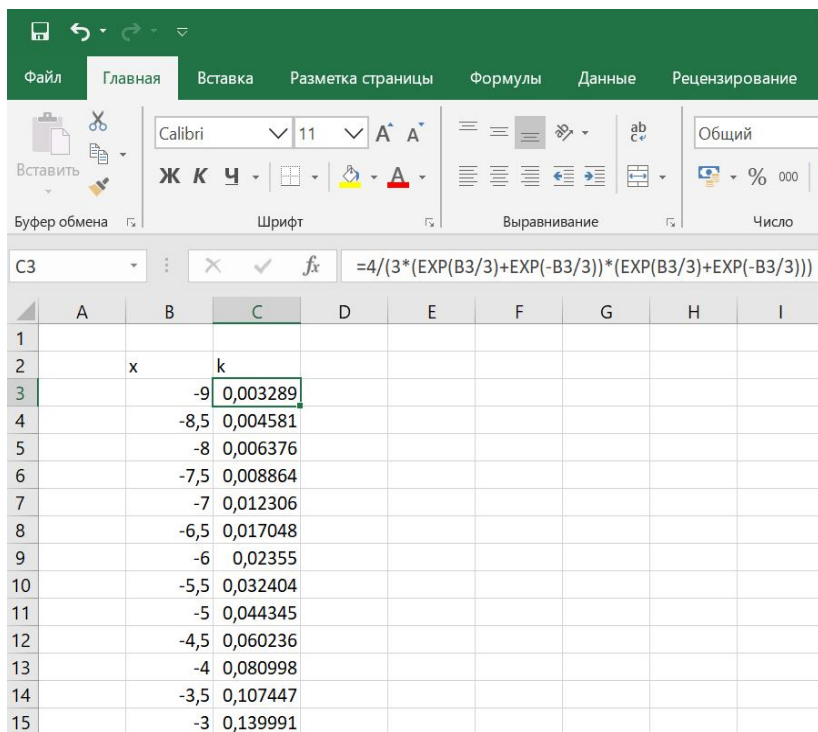


Figure 2.7 – Calculation of values of curvature of a chain line due to the MS Excel platform

Based on the results of the calculation, we obtain the relation of the curvature  $k$  to the coordinate  $x$  (Figure 2.8) and draw the obtained function (Figure 2.9).

2	x	k			
3		-9	0,003289	22	0,5 0,324243
4		-8,5	0,004581	23	1 0,298877
5		-8	0,006376	24	1,5 0,262149
6		-7,5	0,008864	25	2 0,220121
7		-7	0,012306	26	2,5 0,178173
8		-6,5	0,017048	27	3 0,139991
9		-6	0,02355	28	3,5 0,107447
10		-5,5	0,032404	29	4 0,080998
11		-5	0,044345	30	4,5 0,060236
12		-4,5	0,060236	31	5 0,044345
13		-4	0,080998	32	5,5 0,032404
14		-3,5	0,107447	33	6 0,02355
15		-3	0,139991	34	6,5 0,017048
16		-2,5	0,178173	35	7 0,012306
17		-2	0,220121	36	7,5 0,008864
18		-1,5	0,262149	37	8 0,006376
19		-1	0,298877	38	8,5 0,004581
20		-0,5	0,324243	39	9 0,003289
21		0	0,333333		

Figure 2.8 – The calculated values of the chain line curvature

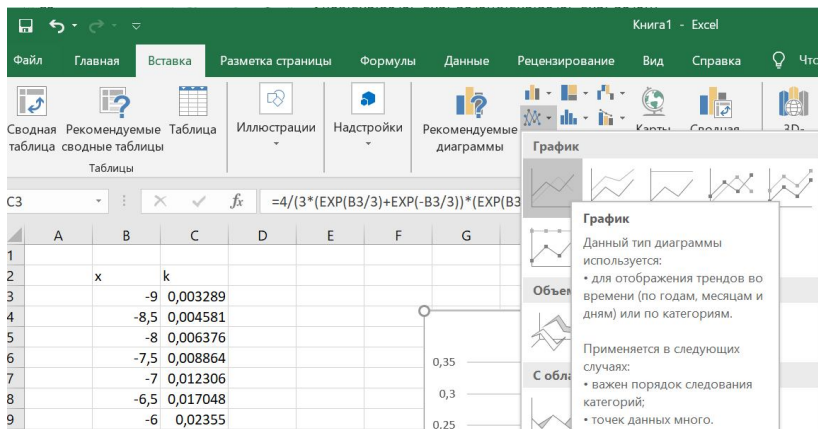


Figure 2.9 – Graphing a function using a platform MS Excel

The graph of curvature versus coordinate is presented at the figure 2.10.

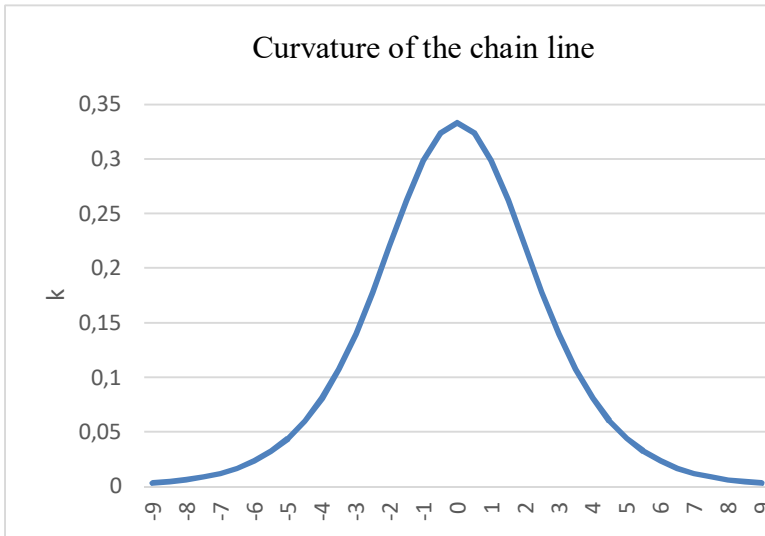


Figure 2.9 – Curvature of the chain line

Task 15. **Cash flow** is any movement of cash between participants in cash circulation.

**Discounted cost** is the amount (sum) that must be used at a given time to obtain the expected amount in the future at a market interest rate.

According to the task  $I(t)$  is a rate of the cash flow,  $p$  is an interest rate,  $t$  is a time.

Denote  $\Pi(t)$  as a discounted cost of a cash flow.

The discounted cost of a cash flow is found by the formula:

$$\Pi(t) = \int_0^T I(t) \cdot e^{-p \cdot t} dt, \quad (2.29)$$

where  $[0; T]$  is a considering time interval.

Let in our case be  $T = 25$ (years),  $I(t) = -a_2 t^2 + a_1 t + a_0 = -t^2 + 25t + 10$ ,  $p = 5\%$ , then:

$$\Pi(t) = \int_0^{25} (-t^2 + 25t + 10) \cdot e^{-0,05 \cdot t} dt =$$

use the integration by parts:

$$\int_a^b u \cdot dv = u \cdot v|_a^b - \int_a^b v \cdot du,$$

then we have:

$$\begin{aligned} & \left| \begin{array}{l} u = -t^2 + 25t + 10; \quad du = (-2t + 25)dt; \\ dv = e^{-0,05 \cdot t} dt; \quad v = -20 \cdot e^{-0,05 \cdot t} \end{array} \right| \\ &= -20(-t^2 + 25t + 10) \cdot e^{-0,05 \cdot t} \Big|_0^{25} + \\ &+ 20 \int_0^{25} (-2t + 25) e^{-0,05 \cdot t} dt = \end{aligned}$$

once again apply the integration by parts:

$$\begin{aligned} & \left[ \begin{array}{l} u = -2t + 25 \quad du = -2dt \\ dv = e^{-0,05 \cdot t} dt \quad v = -20e^{-0,05 \cdot t} \end{array} \right] \\ &= -20(-2t + 25) e^{-0,05 \cdot t} \Big|_0^{25} - \end{aligned}$$

$$\begin{aligned}
& -20 \cdot 20(-2t + 25)e^{-0,05 \cdot t} \Big|_0^{25} - 20 \cdot 20 \cdot 2 \int_0^{25} e^{-0,05t} dt = \\
& = -20(-t^2 + 25t + 10)e^{-0,05 \cdot t} \Big|_0^{25} - \\
& -20 \cdot 20(-2t + 25)e^{-0,05 \cdot t} \Big|_0^{25} + 20 \cdot 20 \cdot 2 \cdot 20e^{-0,05 \cdot t} \Big|_0^{25} = \\
& = 20(t^2 - 25t - 10 + 40t - 500 + 800)e^{-0,05 \cdot t} \Big|_0^{25} = \\
& = 20(t^2 + 15t + 290)e^{-0,05 \cdot t} \Big|_0^{25} = \\
& = 20e^{-1,25}(625 + 375 + 290) - 20 \cdot 290 \approx 1591,7 \text{ (mln. UAH)}.
\end{aligned}$$

**Answer:** the discounted cost of a cash flow for the construction of a research center is approximately  $\Pi(t) \approx 1591,7$  million UAH.

Task 16. By the condition of the task, the exhibition pavilion has the shape of a parabolic cylinder with measurements  $a = 8$  m,  $b = 25$  m,  $h = 10$  m. Denote  $V$  is the exhibition pavilion volume,  $S$  is its cover area, and  $S_1$  is an area of a cross section. The cross section of the pavilion will be the part of the parabola that is bounded by a straight line. Choose the coordinate system as shown at the figure 2.10.

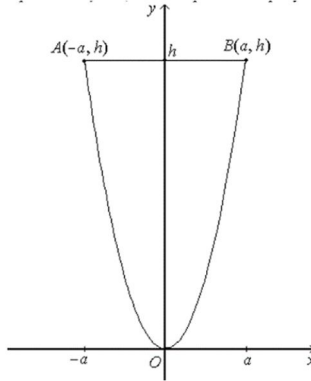


Figure 2.10 – Schematic representation of the exhibition pavilion

From the figure 2.10 we see that the construction volume of the pavilion can be found by the formula

$$V = S_1 \cdot b,$$

where  $S_1$  is an area of a figure bounded by the parabolas  $AOB$  and a straight line  $y = h$ .

The area of the figure is calculated by the formula

$$S = \int_a^b (y_2(x) - y_1(x)) dx. \quad (2.30)$$

But before that we need to write the equation of the parabola that passes through the points  $A(-8; 10)$  and  $B(8; 10)$ .

Write the equation of a parabola symmetric about the y-axis with a vertex at the origin and according to the selected coordinate system

$$x^2 = 2py$$



where  $p$  is a parabolas parameter. Define it from the condition that the line passes through, for example, a point  $A(-8; 10)$ :

$$(-8)^2 = 2 \cdot p \cdot 10; \quad p = \frac{64}{20} = \frac{16}{5}.$$

We get finally

$$x^2 = 2 \cdot \frac{16}{5} \cdot y; \quad \text{or} \quad y = \frac{5}{32} x^2.$$

Calculate the area of the figure by the formula (2.30):

$$\begin{aligned} S &= \int_{-8}^8 \left( 10 - \frac{5}{32} x^2 \right) dx = \left( 10x - \frac{5}{32} \frac{x^3}{3} \right) \Big|_{-8}^8 = \\ &= 10 \cdot 8 - \frac{5}{32} \frac{8^3}{3} - 10 \cdot (-8) + \frac{5}{32} \frac{(-8)^3}{3} = \frac{320}{3} \approx 106,3 \text{ (m}^2\text{)}. \end{aligned}$$

The coverage area of the exhibition pavilion will be found by the formula

$$S_{\Pi} = l \cdot b, \quad (2.31)$$

where  $l$  is a length of arc of a parabolas  $y = \frac{5}{32} x^2$  between two points  $A(-8; 10)$  and  $B(8; 10)$ , that is calculated by the formulas

$$l = \int_a^b \sqrt{1 + (y'(x))^2} dx, \quad (2.32)$$

and  $b$  is a length of the pavilion.

Calculate the length of the arc by the formula (2.32). After substitution we obtain an integral that can be calculated using trigonometric substitution or integration by parts. We chose double integration by parts (an inverse integral):

$$y = \frac{5}{32} x^2 y' = \frac{5}{16} x;$$

$$\begin{aligned}
l &= \int_{-8}^8 \sqrt{1 + \frac{25}{256} x^2} dx = \left[ \begin{array}{l} u = \sqrt{1 + \frac{25}{256} x^2} \\ dv = dx \\ \frac{25}{256} x dx \\ du = \frac{\frac{25}{256} x dx}{\sqrt{1 + \frac{25}{256} x^2}} \\ v = x \end{array} \right] = \\
&= x \sqrt{1 + \frac{25}{256} x^2} \Big|_{-8}^8 - \int_{-8}^8 \frac{\left( \frac{25}{256} x^2 + 1 \right) - 1}{\sqrt{1 + \frac{25}{256} x^2}} dx = \\
&= 8\sqrt{29} - \int_{-8}^8 \sqrt{1 + \frac{25}{256} x^2} dx + \int_{-8}^8 \frac{dx}{\sqrt{1 + \frac{25}{256} x^2}};
\end{aligned}$$

the second term coincides with the initial integral, then let into the denotation as  $l = \int_{-8}^8 \sqrt{1 + \frac{25}{256} x^2} dx$ , and we get

$$\begin{aligned}
2l &= 8\sqrt{29} + \frac{16}{5} \ln \left| \frac{5}{16} x + \sqrt{1 + \frac{25}{256} x^2} \right| \Big|_{-8}^8; \\
l &= 4\sqrt{29} + \frac{16}{5} \ln \left| \frac{\sqrt{29}+5}{\sqrt{29}-5} \right| \approx 36,08(m).
\end{aligned}$$

Thus, the coverage area of the exhibition pavilion is equal to

$$S_{\Pi} = 36,08 \cdot 25 = 902 \text{ (m}^2\text{)}.$$

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## APPENDIX A

### Standard values of trigonometric functions

Table A.1 – Values of trigonometric functions

Angel value $\alpha$		Function			
degr	rad	$\sin \alpha$	$\cos \alpha$	$\operatorname{tg} \alpha$	$\operatorname{ctg} \alpha$
1	2	3	4	5	6
$0^\circ$	0	0	1	0	doesn't exist
$30^\circ$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$45^\circ$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$60^\circ$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$90^\circ$	$\frac{\pi}{2}$	1	0	doesn't exist	0
$180^\circ$	$\pi$	0	-1	0	doesn't exist
$270^\circ$	$\frac{3\pi}{2}$	-1	0	doesn't exist	0
$360^\circ$	$2\pi$	0	1	0	doesn't exist

## APPENDIX B

### **The relationship between trigonometric functions of the same argument**

$$\sin^2 x + \cos^2 x = 1; \quad \operatorname{tg} x \cdot \operatorname{ctg} x = 1;$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x}; \quad 1 + \operatorname{tg}^2 x = \frac{1}{\cos^2 x};$$

$$\operatorname{ctg} x = \frac{\cos x}{\sin x}; \quad 1 + \operatorname{ctg}^2 x = \frac{1}{\sin^2 x}.$$



## APPENDIX C

Table C.1 – Table of elementary function derivatives

	Name of Function	Derivative
1	Constant	$C' = 0$
2	Powerful function	$(u^a)' = a u^{a-1} \cdot u'$
2a	Variable $x$	$x' = 1$
2b	$\sqrt{u}$	$(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u'$
2c	$\frac{1}{u}$	$\left(\frac{1}{u}\right)' = -\frac{1}{u^2} \cdot u'$
3	Indicative function	$(a^u)' = a^u \ln a \cdot u'$
3a	Exponenta	$(e^u)' = e^u \cdot u'$
4	Logarithmic function	$(\log_a u)' = \frac{1}{u \ln a} \cdot u'$
4a	Natural Logarithm	$(\ln u)' = \frac{1}{u} \cdot u'$
5	Sine	$(\sin u)' = \cos u \cdot u'$
6	Cosine	$(\cos u)' = -\sin u \cdot u'$
7	Tangent	$(\operatorname{tg} u)' = \frac{1}{\cos^2 u} \cdot u'$

Continued Table C.1

	Name of Function	Derivative
8	Cotangent	$(ctg\ u)' = -\frac{1}{\sin^2 u} \cdot u'$
9	Arcsine	$(\arcsin u)' = \frac{1}{\sqrt{1-u^2}} \cdot u'$
10	Arccosine	$(\arccos u)' = -\frac{1}{\sqrt{1-u^2}} \cdot u'$
11	Arctangent	$(arctg\ u)' = \frac{1}{1+u^2} \cdot u'$
12	Arccotangent	$(arcctg\ u)' = -\frac{1}{1+u^2} \cdot u'$

## APPENDIX D

### Indefined integrals table

Table D.1 – The standard integrals

1	$\int 0 du = C$	5	$\int \sin u du = -\cos u + C$
2	$\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + C$	6	$\int \cos u du = \sin u + C$
2a	$\int du = u + C$	7	$\int \frac{du}{\cos^2 u} = \operatorname{tg} u + C$
2б	$\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$	8	$\int \frac{du}{\sin^2 u} = -\operatorname{ctg} u + C$
2B	$\int \frac{du}{u^2} = -\frac{1}{u} + C$	9	$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
3	$\int \frac{du}{u} = \ln u  + C$	10	$\int \frac{du}{\sqrt{u^2 + b}} = \ln \left  u + \sqrt{u^2 + b} \right  + C$
4	$\int a^u du = \frac{a^u}{\ln a} + C$	11	$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C$
4a	$\int e^u du = e^u + C$	12	$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left  \frac{u-a}{u+a} \right  + C$

Continued TableD.1

Supplementary integrals			
1	$\int \frac{du}{\sin u} = \ln \left  \operatorname{tg} \frac{u}{2} \right  + C$	2	$\int \frac{du}{\cos u} = \ln \left  \operatorname{tg} \left( \frac{u}{2} + \frac{\pi}{4} \right) \right  + C$
3	$\int \operatorname{tg} u \, du = -\ln  \cos u  + C$	4	$\int \operatorname{ctg} u \, du = \ln  \sin u  + C$
5	$\int sh u \, du = chu + C$	6	$\int chu \, du = sh u + C$
7	$\int \frac{du}{ch^2 u} = th u + C$	8	$\int \frac{du}{sh^2 u} = -cth u + C$
9	$\int \sqrt{u^2 \pm a^2} \, du = \frac{1}{2} u \sqrt{u^2 \pm a^2} \pm \frac{1}{2} a^2 \ln \left  u + \sqrt{u^2 \pm a^2} \right  + C$		
10	$\int \sqrt{a^2 - u^2} \, du = \frac{1}{a} u \sqrt{a^2 - u^2} + \frac{1}{2} a^2 \arcsin \frac{u}{a} + C$		
11	$\int e^{au} \sin bu \, du = \frac{-b e^{au} \cos bu + a e^{au} \sin bu}{a^2 + b^2} + C$		
12	$\int e^{au} \cos bu \, du = \frac{a e^{au} \cos bu + b e^{au} \sin bu}{a^2 + b^2} + C$		

## APPENDIX E

### **The example of the design of the calculation and graphics tasks title page**

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*За авторською редакцією*

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