MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

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Methodological guidelines
for practical classes,
self-dependent and calculator- graphical works
on the Subject

"STRENGTH of MATERIALS"

(CALCULATION of STATICALLY INDETERMINATE FRAME by FORCE METHOD)

(for the second year full-time students specialty
192 – Construction and civil engineering)

Kharkiv
O. M. Beketov NUUE
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INTRODUCTION

Strength of Materials is one of the most important disciplines which makes the foundations of the future specialist of the civil engineering in the field of structural calculation and their elements for strength, stiffness and durability of machines and structures.

The course of Strength of Materials is studied by students after learning the courses of higher mathematics and theoretical mechanics.

For the design of structures, it is necessary to learn the theoretical and practical methods of their calculation, which ensure the reliability of the structure, and its cost-effectiveness. In the conditions of exploitation of structures there is a constant problem of their calculation for high loads. Such calculations and knowledge can be obtained by studying the course Strength of Materials.

When acquiring a course on Strength of Materials, the most effective method is the students independent solving problems and control tasks. The Guidelines are used for students independent work on the theme “Calculation of a statically indeterminate frame by the force method”. It contain theoretical statements and variants of problems for control work and an example of its application.

Each student is given out a problem to perform the calculation and graphical work, and in order to eliminate possible questions when performing home control work, practical classes are held to analyses the basic positions of homework.

After receiving a note on the supervisory work, the student must correct the mistakes indicated by the teacher, make the necessary corrections, even if the work has been approved.

If the work is not approved, correct the same or in a separate drawing and resubmit the work for reconsideration. Independence in the execution of calculated and graphical work is of paramount importance for mastering program material. Detailed instructions for completing and design of the problem are given below.

1 THE FORCE METHOD FOR STATICAL INDETERMINATE FRAME

The work is executed in accordance with the personal code of the student, on which from the table the output and the frame scheme are chosen. Works carried out without following instructions, are not approved and returned without consideration.

It is recommended that you submit your work for review immediately after completion so that the reviewer's comments can be taken into account and corrected.

Basic information of the Force Method

Statically indeterminate structures are the structures for which *all* reactions and internal forces cannot be determined simply using equilibrium equations. *Redundant constraints* (or *excess*) are constraints, which are not necessary for geometrical unchangeability of a given structure.

Degree of redundancy, or statically indeterminacy, equals to the number of redundant constraints which elimination leads to the new geometrically unchangeable and statically determinate structure. *Degree of statically redundancy* is the difference between the number of constraints and number of independent equilibrium equations which can be written for a given structure.

Primary unknowns represent *reactions* (forces and/or moments), which arise in redundant constraints. That method is called the *Force Method*. Unknown *internal* forces also may be treated as primary unknowns. Primary system (principal or released structure) is such structure, which is obtained from the given one by eliminating redundant constraints and replacing them by primary unknowns.

The following procedure may be recommended for analysis of statically indeterminate structures by the superposition principle:

1. Determine the degree of statical indeterminacy.

2. Choose the redundant unknowns; their number equals to degree of statical indeterminacy.

3. Construct the statically determinate structure (primary structure) by eliminating all redundant constraints.

4. Replace the eliminated constraints by primary unknowns. These unknowns present reactions of eliminated constraints.

5. Form the compatibility equations; their number is equal to degree of statical indeterminacy.

Each compatibility equation should be presented in terms of given loads and primary unknowns.

6. Solve the system of equations with respect to primary unknowns.

7. Since reactions of the redundant constraints are determined, then the computation of all remaining reactions and analysis of the structure may be performed as for the statically determinate structure.

Canonical equations of force method offer a unified procedure for analysis of statically indeterminate structures of different types. The word "canonical" indicates that these equations are presented in standard, or in an orderly fashion form. Very important is that canonical equations of the force method may be presented in a matrix form. Thus, this set of equations is a first bridge between classical analytical methods and numerical ones.

The Concept of Unit Displacements

Analysis of any statically indeterminate structure by the force method begins with determination of degree of statical indeterminacy. Primary system is obtained by elimination of redundant constraints and replacing them by reactions of these constrains. *Primary unknowns* X_i represent reactions (forces or moments) in eliminated redundant constrains.

Let us consider a simple redundant structure, such as clamped-pinned beam. The number of redundant constraints is n = 4 - 3 = 1. Assume that the right rolled support is the redundant one. Thus the reaction of this constraint, X_1, is a *primary unknown*. Given and primary systems are shown in Figure 1.

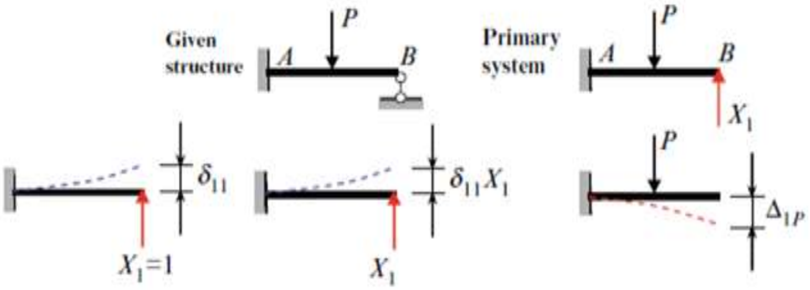


Figure 1 – Simple redundant structure. The idea of the force method and the concept of unit displacement

The compatibility condition may be written in the following form

$$y_B = y_{B(P)} + y_{B(X_1)} = 0, \tag{1}$$

where $y_{B(P)}$ is displacement of point B in primary system due to given load P, and $y_{B(X_1)}$ is displacement of point B in primary system due to primary unknown $R_B = X_1$.

Displacement $y_{B(X_1)}$ caused by unknown X_1 may be presented as

$$y_{B(X_1)} = \delta_{11} \cdot X_1, \tag{2}$$

where δ_{11} is the displacement in direction 1 (first index) caused by the force $X_1 = 1$ (second index). Coefficient δ_{11} is the *unit displacement* since it is caused by *unit* primary unknown $X_1 = 1$. The term $\delta_{11}X_1$ is the displacement in the direction of the eliminated constraint 1 caused by the *actual* primary unknown X_1. If displacement in direction 1 caused by given load $y_{B(P)}$ is denoted as Δ_{1P}, then (2) may be rewritten in the following form

$$\delta_{11}X_1 + \Delta_{1f} = 0. \tag{3}$$

General case of canonical equations. The canonical equations of the force method for a statically indeterminate structure with n redundant constraints are written as follows

$$\begin{aligned}
&\delta_{11}X_1 + \delta_{12}X_2 + \ldots + \delta_{1n}X_n + \Delta_{1f} = 0\\
&\delta_{21}X_1 + \delta_{22}X_2 + \ldots + \delta_{2n}X_n + \Delta_{2f} = 0\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\delta_{n1}X_1 + \delta_{n2}X_2 + \ldots + \delta_{nn}X_n + \Delta_{nf} = 0
\end{aligned} \tag{4}$$

All coefficients δ_{ik} of canonical equations represent a *displacement* of the primary structure due to *unit* primary unknowns; these coefficients are called the *unit displacements*.

Physical meaning of the canonical equations. The left part of the i th equation presents the total displacement along the direction of unknown X_i due to action of all real unknowns X_k as well as applied load. Total displacement of the primary structure in directions of eliminated restrictions caused by primary unknowns and applied load equals zero. In this case, the difference between the given and primary structures is vanished.

Calculation of Coefficients and Free Terms of Canonical Equations

Computation of coefficients and free terms of canonical equations presents significant and very important part of analysis of any statically indeterminate structure.

For their calculation, any methods can be applied. The graph multiplication method is best suited for beams and framed structures. For this, it is necessary in primary system to construct bending moment diagrams $\bar{M}_1, \bar{M}_2, ..., \bar{M}_n$ due to unit primary unknowns X_i , $i = 1, ..., n$ and diagram M_P due to given load. Unit displacements and loaded terms are calculated by Mohr's formula

$$\delta_{ik} = \sum \int \frac{\bar{M}_i \cdot \bar{M}_k}{EI} dx; \quad \Delta_{if} = \sum \int \frac{\bar{M}_i \cdot M_f}{EI} dx, \tag{5}$$

where δ_{ik} is a *displacement* of the primary structure due to *unit* primary unknowns (*unit displacements),*

Δ is *virtual deflection* (horizontal, vertical, angle of rotation),

M_f is cross-sectional bending moment of load,

$\bar{M}$ is cross-sectional bending moment of unit load.

Mohr's integral can be calculated graphically- analytically on *Vereshchagin's formula*

$$\Delta = \sum \frac{\varpi \cdot y_C}{EI}, \tag{6}$$

where Δ is linear deflection or angle of rotation;

ω is non-linear diagram of bending moment area;

y_C is *ordinate linear diagram* under center of gravity area ω.

Simpson's formula:

$$\Delta = M \times \bar{M} = \frac{l}{6}\left(M_l \bar{M}_l + 4M_m \bar{M}_m + M_r \bar{M}_r\right) \tag{7}$$

where l is length of section

Construction of internal force diagrams. Solution of (4) is the primary unknowns X_i , $i = 1,\dots, n$. After that the primary system may be loaded by determined primary unknowns and given load. Internal forces may be computed as for usual statically determinate structure. However, the following way allows once again an effective use of the bending moment diagrams in primary system. The final bending moment diagram M_P may be constructed by formula:

$$M_P = \bar{M}_1 \cdot X_1 + \bar{M}_2 \cdot X_2 + \dots + \bar{M}_n \cdot X_n + M_P. \tag{8}$$

Procedure for analysis The following procedure provides analysis of statically indeterminate beams and frames using the canonical equations of the force method:

1. Provide the kinematical analysis and define the degree of statically indeterminacy n of a structure.

2. Choose the primary system and replace the eliminated redundant constraints by corresponding primary unknowns Xi , $i = 1,\dots n$.

3. Formulate the canonical equations of the force method.

4. Apply the successive unit forces $X_1 = 1; X_2 = 1; \dots ; X_n = 1$ to primary system and for each unit primary unknown construct corresponding bending moment diagrams $\bar{M}_1, \bar{M}_2, \dots, \bar{M}_n$.

5. Calculate the unit coefficients δ_{ik}.

6. Construct the bending moment diagram M_P due to applied load in primary system and calculate the load terms Δ_{iP} of (4).

7. Solve the system of equations with respect to primary unknowns $X_1, X_2, \dots, X_n$.

8. Construct the bending moment diagrams by (6), next compute the shear and construct corresponding shear force diagram, and lastly compute axial forces and construct the corresponding axial force diagram.

9. Having internal force diagrams, calculate the reactions of supports. Other way is consider primary system subjected to determined primary unknowns and given load and provide computation of all internal forces by definition; this way is less effective.

10. Provide the static control for all structure (or any its part).

11. Provide the kinematical control (2) for displacements of an entire structure in direction of primary unknowns.

Intermediate checking of computation These verifications are recommended to be performed *before* solving canonical equations for determining primary unknowns X_i , i.e., on the steps 5 and 6 of the algorithm above. For control of unit displacements and free terms, it is necessary to construct *summary* unit bending moment diagram

$$\sum M = \bar{M}_1 + \bar{M}_2 + ... + \bar{M}_n .$$

The following types of controls are suggested as follows:

(a) Row verification of unit displacements: Multiply summary unit bending moment diagram $\sum M$ on a primary bending moment diagram $\sum \bar{M}_i$:

$$\frac{\sum \bar{M} \times \bar{M}_i}{EI} = \sum \int (\bar{M}_1 + \bar{M}_2 + ... + \bar{M}_n) \cdot \bar{M}_i \frac{dx}{EI} = \delta_{i1} + \delta_{i2} + ... + \delta_{in} .$$

The result of this multiplication equals to the sum of unit displacements of the *i*-equation.

2 CALCULATION AND GRAPHICAL WORK

Problem

Frame scheme chosen by figure 2, (for teacher’s instructions), numeric data – by table 1, figure 2.

According to the table. 1 take only the load data which is shown in the given scheme.

For a given frame, it is necessary to:

– find out the degree of static indeterminately;

– choose of the primary system;

– write down the canonical equations of the force method;

– draw unite and load moment diagrams for the primary system;

– calculate coefficients and free terms of canonical equations (unit and loaded (term) displacement);

– solve system of canonical equations; draw internal forces diagram;

– perform static and kinematic verification of the solution obtained.

Table 1 – Output data for problem

	Number variants (schemes)								
	1	2	3	4	5	6	7	8	9
a, m	4	5	6	6	8	5	6	4	6
P, kN	20	10	30	50	20	10	10	20	30
q, kN/m	10	15	10	5	20	5	10	10	5

	Number variants (schemes)								
	10	11	12	13	14	15	16	17	18
a, m	5	3	4	6	6	8	5	6	4
P, kN	20	20	10	30	5	20	10	10	20
q, kN/m	10	10	2	5	1	1	2	5	10

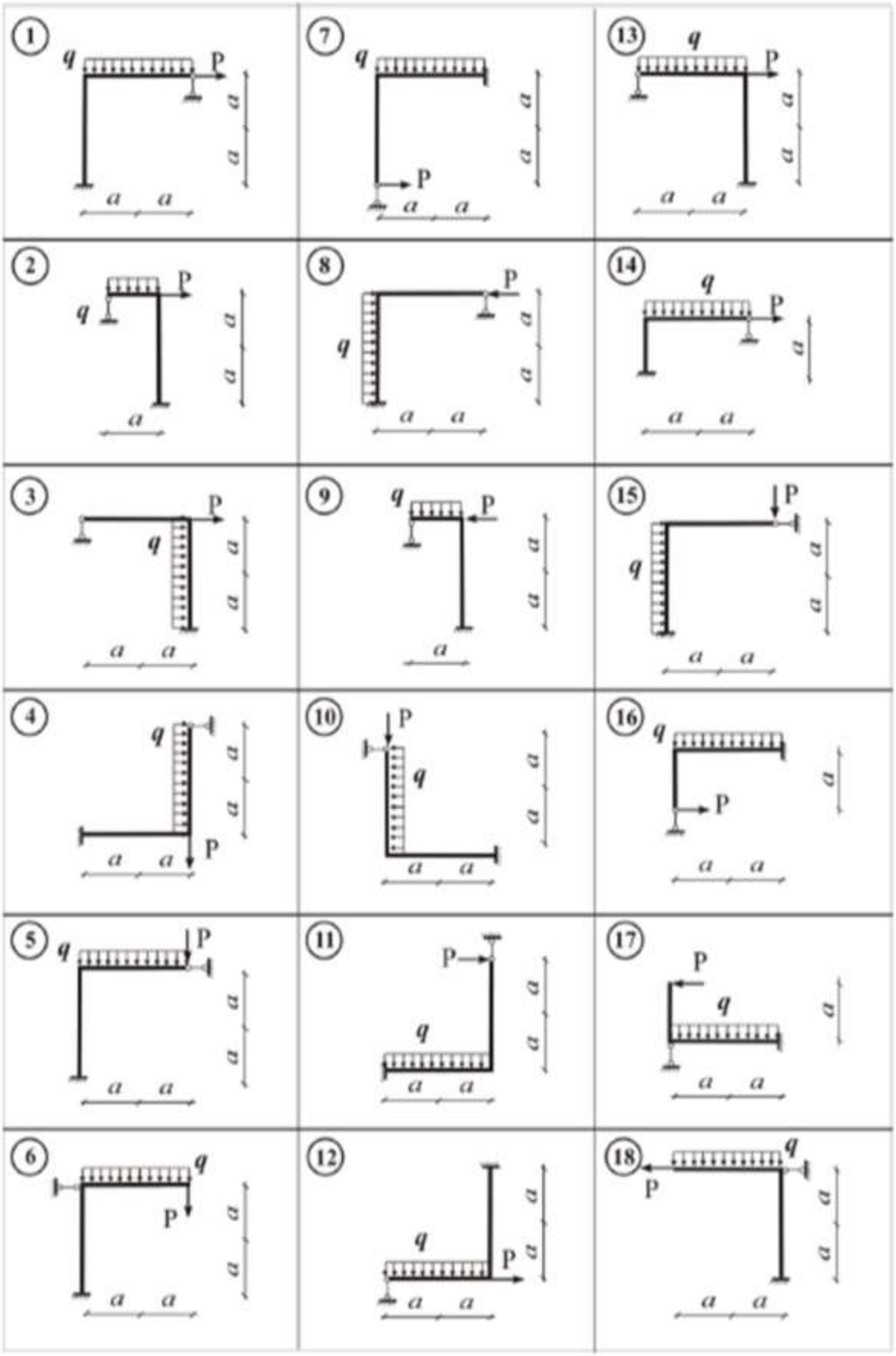


Figure 2 – Frame schemes

3 EXAMPLE OF CALCULATIONS

Problem

Static indeterminately are systems which reactions cannot be established by equilibrium equations only.

Output data:

For the frame shown in Figure *3a* to determine constrained reactions and to draw a diagram ***Q*** and ***M***, and longitudinal forces ***N***.

Solution: The procedure for the calculation of frames by force is as follows:

1. *Degree of redundancy*, or statically indeterminacy.

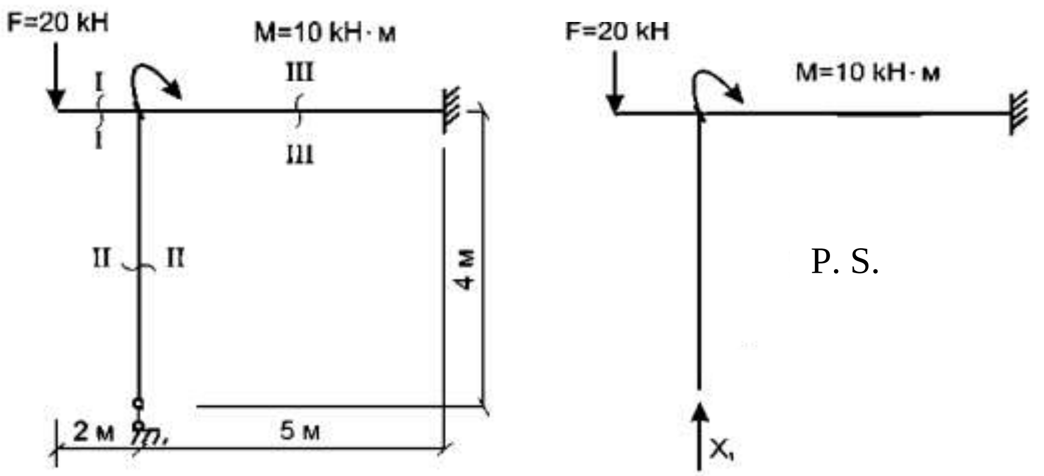
Determine the degree of statically indeterminacy.

So, for the frame shown in fig. *3 a*, by the formula:

$$\boldsymbol{n} = 4\text{-}3 = 1.$$

The given frame is *one statically indeterminate*. Choose the redundant unknowns; their number equals to degree of statically indeterminacy.

2. Choosing the *primary system*. Construct the statically determinate system (*primary structure*) by eliminating all redundant constraints (Fig. 3b). Change the eliminated constraints by primary unknowns. These unknowns present reactions of eliminated constraints.



a *b*

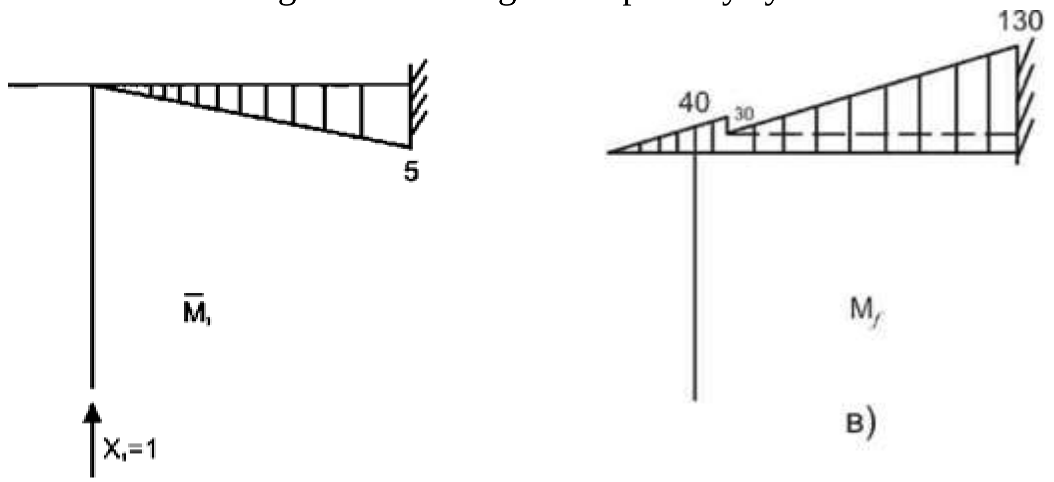
Figure 3 – Frame scheme: *a* – given scheme, *b* – primary system

3 Canonical equations of force method.

The compatibility equation is called the *canonical equation of the force method* for any structure with one redundant constraint; the free term Δ_{1F} is called the loaded term (loaded displacement, free term). The solution of the canonical equation allows us to calculate the primary unknown X_1.

$$\delta_{11} X_1 + \Delta_{1f} = 0 .$$

4 To draw diagrams of bending moments in primary system from unit forces and given loads. Bending moments diagram in primary system is shown on fig. 4.



a)

Figure 4 – Bending moments diagrams in primary system (*a*) from unit forces and given loads (*b*)

5 Calculate unit and loaded displacements.

We determine the displacements of the equation by the method of Mohr using the Vereshchagin rule:

$$\delta_{11} = \frac{1}{EI} \cdot \frac{5 \cdot 5}{2} \cdot \frac{2}{3} \cdot 5 = \frac{41,67}{EI};$$

$$\Delta_{1f} = -\frac{1}{EI} \cdot \left(\frac{100 \cdot 5}{2} \cdot \frac{2}{3} \cdot 5 + \frac{5 \cdot 5}{2} \cdot 30\right) = -\frac{1208,33}{EI}.$$

Substituting the obtained displacements into *canonical equation* and reducing them to *EI*, we obtain:

$$41,67 X_1 - 1208,33 = 0$$

Determine from this equation:

$$X_1 = \frac{1208,33}{41,67} = 28,99 \approx 29$$

When drawn the corrected diagrams M_1 (Fig. 5*a*), we summarize with the diagram M_f. obtained in this way is shown in Fig. 5*b*:

$$M = \overline{M}_1 \cdot X_1 + M_f$$

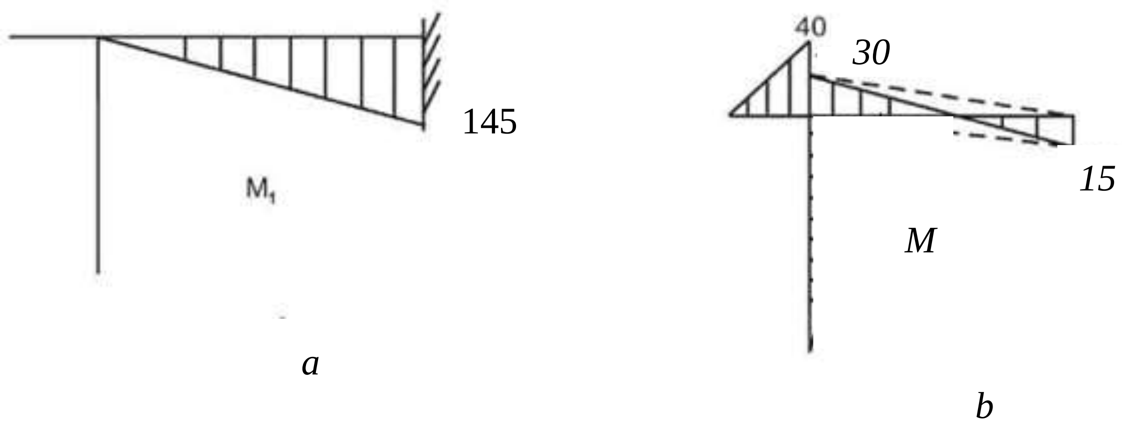


Figure 5 – The corrected diagrams *M_1* (*a*) and
the final diagram of moments *M (b)*

6. We draw diagrams of internal forces. Diagram Q is drawn in Figure 6 *a*

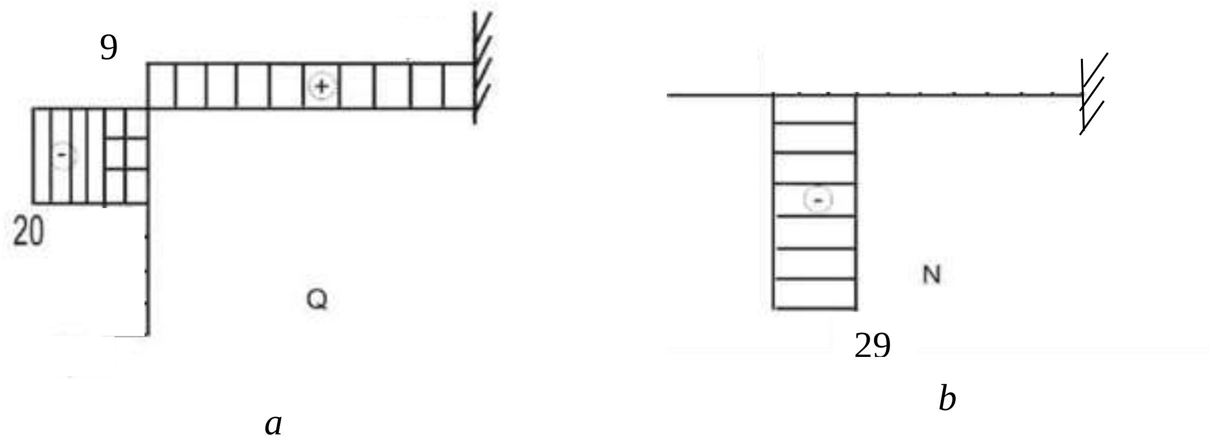


Figure 6 – Shear forces Q and longitudinal forces N diagrams in given system

Longitudinal forces: $N_I = 0$; $N_{II} = -29$ *kN*. Draw the diagram N.

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