MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

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Methodological guidelines
for practical classes,
self-dependent and calculator-graphical works
on the Subject

THEORETICAL MECHANICS
PART 2. KINEMATICS

(*for the first year full-time students specialty*
192 – Construction and Civil Engineer)

Kharkiv
O. M. Beketov NUUE
2019

Methodological Guidelines for practical classes, self-dependent and calculator-graphical works on the Subject «Theoretical Mechanics» (Part 2. Kinematics) (for the first year full-time students of the specialty 192 – Construction and Civil Engineer) O. M. Вeкetov National University of Urban Economy in Kharkiv ; com. : V. P. Shpachuk, A. I. Rubanenko, A. O. Garbuz and other. – Kharkiv : O. M. Beketov NUUE, 2019. – 31 p.

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Recommended by the department of Theoretical and Structural Mechanics, record N 10 on 25.06.2019

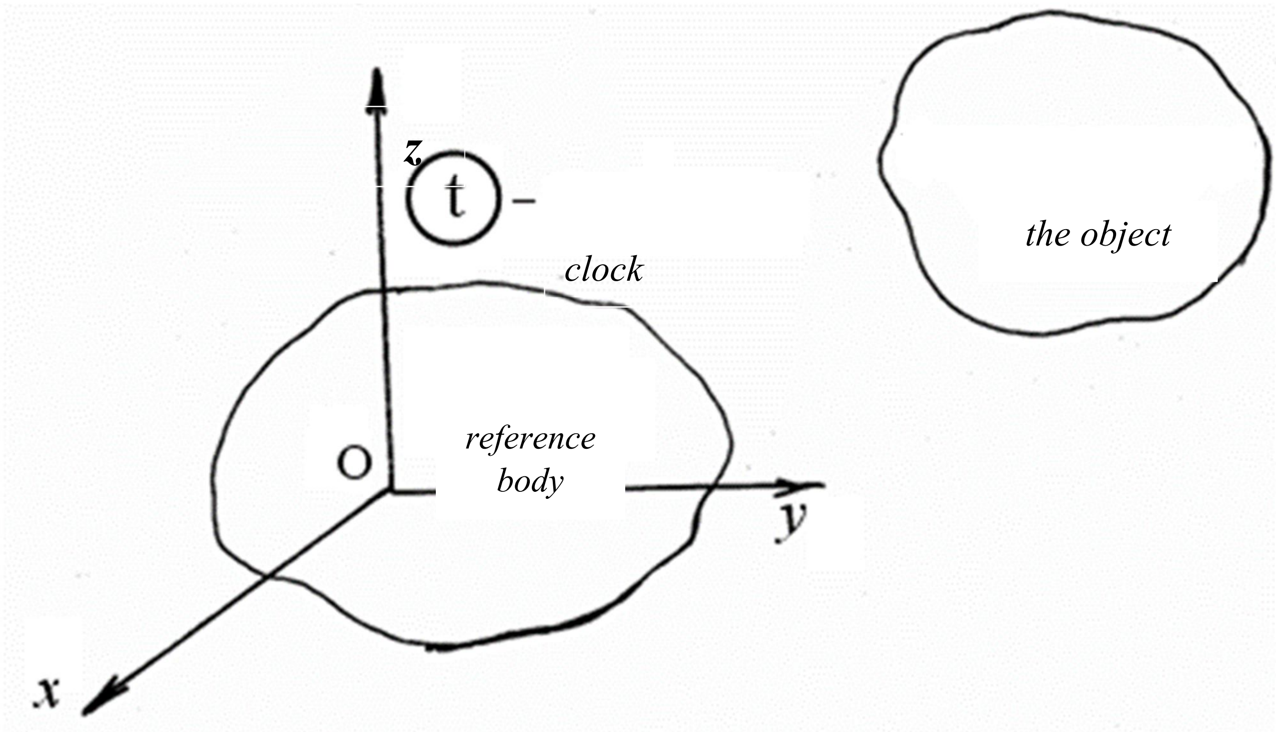
CONTENTS

KINEMATICS

Kinematics is the branch of mechanics, which studies motion as such, without acting forces. Kinematics deals only with geometrical aspect of the motion**.**

Motion is change in the location or position of an object in the time with respect to other body (*reference* body).

To study motion of any object we have to introduce at *reference system*. *Reference system* is the complex of the *coordinate system*, which is rigidly connected with the *reference body*, and the *clock*, which the time is calculated by.



Material point is the body dimension of which is substantially smaller than Earth radius.

1 Particle motion

The motion of a particle **is completely described** if the position of the particle is given as a function of the time.

Trajectory (Path) is the line along which particle moves. If path is a straight line, the motion is called **rectilinear**; if path is a curved line the motion is **curvilinear**.

Descriptions of particle motion. There are three main methods of the particle motion descriptions:

1. vector, 2. coordinate, 3. natural.

1.1 Vector method

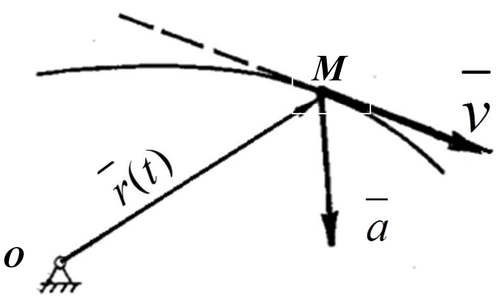
The position of a particle in three-dimensional space is given by its *vector-position* $\bar{r} = \bar{r}(t)$ - the law of motion. *Vector-position* $\bar{r}$ connects the origin of reference, the point ***O***, with a point ***M***, where the particle is situated. *Vector-position* is determined by its magnitude and direction. The motion is completely described when *vector-position* is known as a function of the time.

$\bar{V} = \dfrac{d\bar{r}}{dt}$ is *velocity of the particle* - vector value, which:

- *equals the first derivative* from *vector-position* with respect at time;
- is *directed* along tangent to the trajectory into particles moving;
- *characterizes* the variation of the particle position in time.

$$\bar{a} = \frac{d\bar{V}}{dt} = \frac{d^2\bar{r}}{dt^2} \text{ or}$$

$\bar{a} = \dot{\bar{V}} = \ddot{\bar{r}}$ is *acceleration of the particle* - vector value, which:



- *equals the first derivative* from *velocity*- with respect at time;
- is *directed* toward the center of curvature;
- *characterizes* the variation of the velocity magnitude in time.

1.2 Coordinate method: Rectangular Components

$x = x(t);\; y = y(t);\; z = z(t)$ – the law of motion (equation of particle motion in *coordinate form*), where *x, y, z* – coordinates of the particle.

Particle location is defined by the *vector-position*

$\bar{r} = r_x\bar{i} + r_y\bar{j} + r_z\bar{k}$,

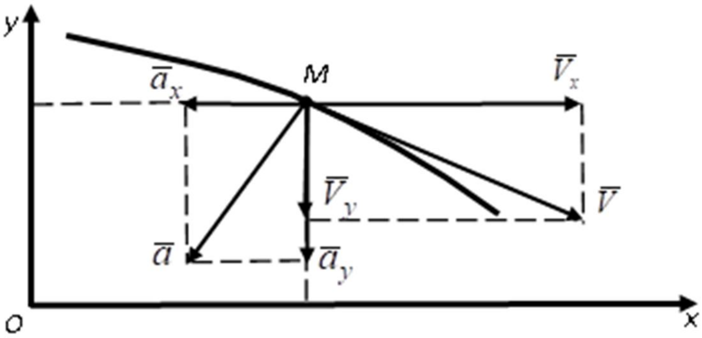
$\bar{v} = v_x\bar{i} + v_y\bar{j} + v_z\bar{k}$ - particle velocity

where v_x, v_y, v_z - coordinates components of particle velocity;

$\bar{i}, \bar{j}, \bar{k}$ - orts of coordinates axes.

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \text{ – magnitude of velocity;}$$

$v_x = \dfrac{dx}{dt} = \dot{x};\quad v_y = \dfrac{dy}{dt} = \dot{y};\quad v_z = \dfrac{dz}{dt} = \dot{z}$ – projections of vector-velocity on coordinate axis, represent the *first time derivatives* of *x = x(t), y = y(t), z = z(t)*, respectively .



$\overline{a} = a_x\overline{i} + a_y\overline{j} + a_z\overline{k}$ – particle's acceleration,

where a_x, a_y, a_z – coordinates components of particle acceleration;

$a_x = \dot{v}_x = \ddot{x}$; $a_y = \dot{v}_y = \ddot{y}$; $a_z = \dot{v}_z = \ddot{z}$ - projections of *vector-acceleration* on coordinate axis, represent, respectively, *the first time derivatives* of $v_x = v_x(t)$, $v_y = v_y(t)$, $v_z = v_z(t)$, or *the second time derivatives* of the functions $x = x(t)$, $y = y(t)$, $z = z(t)$.

1.3 Natural method

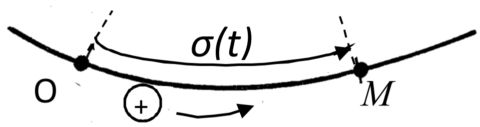
Natural method supposes that motion is completely described if *position of particle on its trajectory is given as function of the time*.

In a *natural method* are given characteristics of motion:

a) *trajectory* of the particle;

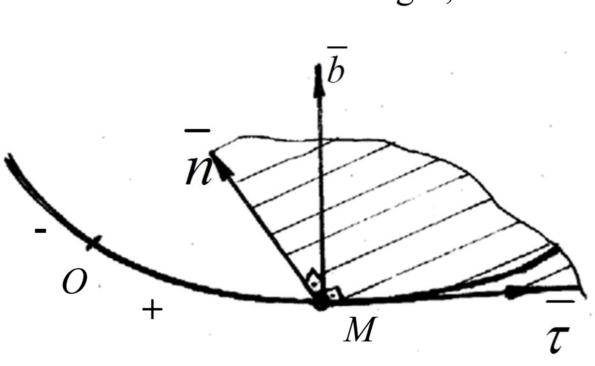
b) *start* and *direction* of the reference point of movement;

c) $\sigma = \sigma(t)$ is the *law of motion* (the equation of motion of a particle in its natural form),



where σ – curvilinear coordinate, which is counted along the trajectory

Natural coordinate system: the $\boldsymbol{\tau}$ axis is *tangent* to the trajectory and is positive in the direction of increasing $\boldsymbol{\sigma}$; the *normal axis* $\boldsymbol{n}$ is perpendicular to the $\boldsymbol{\tau}$ axis



with its positive sense *directed towards the center of curvature*;

the *bi-normal* axis $\boldsymbol{b}$ - complements the coordinate system to spatial, that is, perpendicular to the plane τMn ($\overline{b} = \overline{\tau} \times \overline{n}$),

where $\overline{\tau}, \overline{n}$ – the orts of the *tangent* and the *normal* axis;

$\overline{v} = \overline{v}_\tau = v_\tau \cdot \overline{\tau}$ – the particle's velocity of; $\overline{v}_\tau$ – component of the particles velocity which coincides with the *tangent* axis (the velocity vector);

$v_\tau = \dot{\sigma}$ – projections of velocity $\overline{V}$ on the *tangent a*xis; the particle's velocity $\boldsymbol{v}$ has a *direction that is always tangent* to the trajectory, and a *magnitude* is determined by taking the time derivative of the function $\boldsymbol{\sigma}$.

The tangential component of acceleration

$v = | \ v_\tau \ |$ – magnitude of velocity;

$$\overline{a} = \overline{a}_\tau + \overline{a}_n = a_\tau \cdot \overline{\tau} + a_n \cdot \overline{n}$$ – particle acceleration,

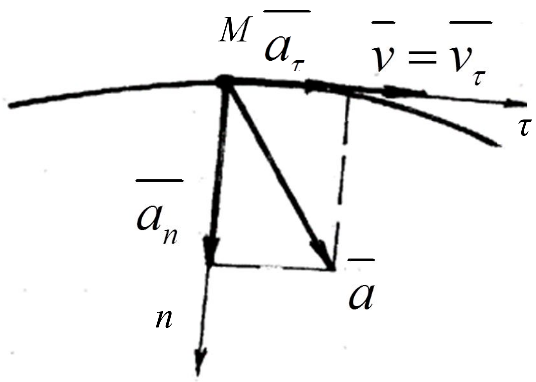
where $\overline{a}_\tau, \overline{a}_n$ – components of the acceleration of the particle to the axes $\boldsymbol{\tau}$ and $\boldsymbol{n}$;

$a_\tau = \dot{v}_\tau = \ddot{\sigma}$ – the tangential component of acceleration on the $\boldsymbol{\tau}$, is the result of the time rate of change in the magnitude of velocity; This component acts in the positive $\boldsymbol{\sigma}$ direction.

$a_n = \dfrac{v^2}{\rho}$ – the normal component of acceleration is the result of the time rate of change in the direction of the velocity. This component is always directed towards the center of curvature of the trajectory, i.e., along the positive $\boldsymbol{n}$ axis***;*** where $\boldsymbol{\rho}$ – radius of curvature of the trajectory at a given point.

The acceleration of the particle is the time rate of change in velocity

$a = \sqrt{a_\tau^2 + a_n^2}$ – magnitude of acceleration.



The conformity between the *natural* and *coordinate* methods:

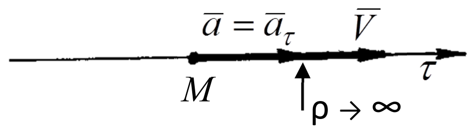
$$a_\tau = \frac{V_x a_x + V_y a_y + V_z a_z}{V};$$

$$a_n = \sqrt{\frac{(V_x a_y - V_y a_x)^2 + (V_z a_x - V_x a_z)^2 + (V_y a_z - V_z a_y)^2}{V_x^2 + V_y^2 + V_z^2}};$$

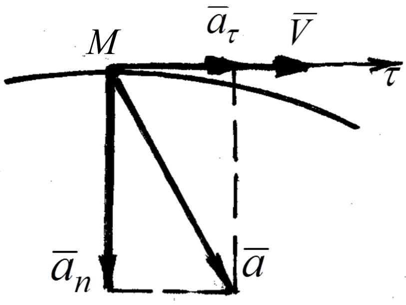
$$\rho = \frac{\overline{V}^2}{a_n} = \frac{V_x^2 + V_y^2 + V_z^2}{a_n}.$$

1.4 Special cases

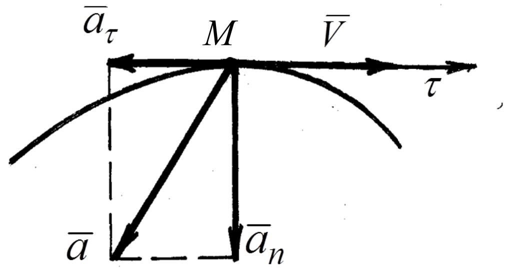
1) *Rectilinear* (straight-line) *motion*: $\bar{a}_n = 0$, $\bar{a} = \bar{a}_\tau$, the radius of curvature is $\rho = \infty$



2) *Curvilinear motion:* $\bar{a}_n \neq 0$, $\bar{a} = \bar{a}_\tau + \bar{a}_n$.



Accelerated motion, directions $\bar{v}$ and $\bar{a}_\tau$ coincide, signs of projections $\bar{v}$ and $\bar{a}_\tau$ are same.



Decelerated motion (particle is slowing down), directions $\bar{v}$ and $\bar{a}_\tau$ are opposite, signs of projections $\bar{v}$ and $\bar{a}_\tau$ are different.

3) *Uniform motion*: $a_\tau = const \neq 0$ (*uniformly accelerated* motion , if v_τ and a_τ have the same signs and *uniformly decelerated* – if have the difference one).

$$v_\tau = a_\tau \cdot t + v_{o\tau},$$

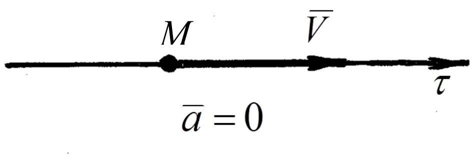
$$\sigma = a_\tau \frac{t^2}{2} + v_{o\tau} \cdot t + \sigma_0,$$

where $\sigma_0, v_{o\tau}$ – initial coordinate and particle velocity;

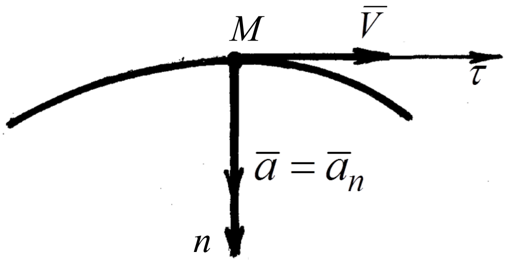
4) *Uniform motion*: $\bar{a}_\tau = 0$, in this case $\bar{a} = \bar{a}_n$,

$$v_\tau = v_{o\tau} = const,$$

$$\sigma = v_{o\tau} \cdot t + \sigma_0,$$



Uniform rectilinear motion



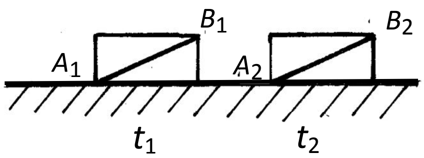
Uniform curvilinear motion

2 Kinematics of the rigid body

2.1. Translation motion of the rigid body

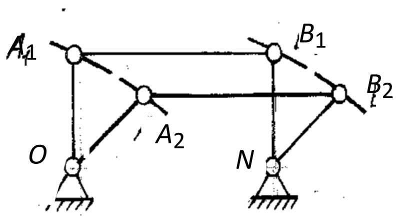
Translation is the type of motion, in which every line segment on the body remains parallel to its original direction during the motion.

1)



– the body slides along a straight surface, the trajectory of the points is a straight line ;

2)

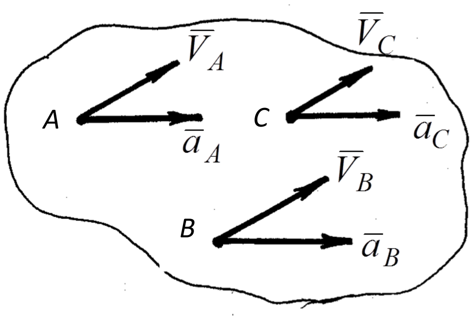


– coupling rod *AB* when the rods of *OA* and *NB* (*OA* = *NB*) rotate, the trajectory of the points - *the circle*.

Low of ***translation***:

i=1,2,3

Theorem: *If body moves translationally all the points have at time **t** the same*



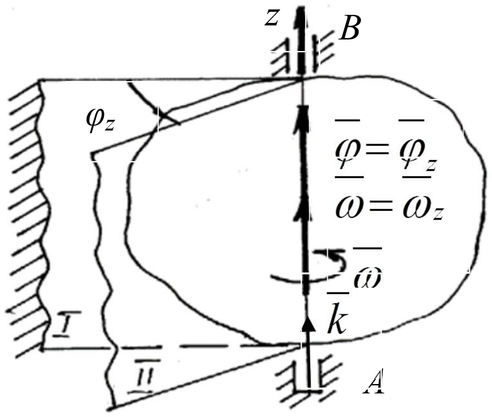
$$\overline{v}_A = \overline{v}_B = \overline{v}_C = \ldots$$
$$\overline{a}_A = \overline{a}_B = \overline{a}_C = \ldots$$

velocity and acceleration relative to some reference.

The translational motion of a rigid body can be determined by studying the motion of only one of its points (often it is the center of masses).

2.2 Rotation about a Fixed Axis

A rotation is such motion of a rigid body when *one line of the body or of an extension of the body remains fixed*. The fixed line is called the ***axis of rotation***. A rotating body has one degree of freedom.



In the figure:

I – fixed plate,

II – a plate, which rigidly connected the body and rotates with it.

The angular position measured from a fixed reference plate *I* to *II* – $\boldsymbol{\varphi_z}$ is *angle rotation* of the body.

$\varphi_z = \varphi(t)$ – the *law of rotation* (equation of the body rotation about fixed axis ***z***). Since motion is about a fixed axis, the direction of $\boldsymbol{d\varphi}$ is always along this axis. *Angle* $\boldsymbol{\varphi}$ measured in degrees or radians: [φ] = *rad*.

$\bar{\omega} = \dot{\bar{\varphi}}$ – the vector of *angular velocity ω – omega* (the time rate of change in the angular position). It's directed along the axis of rotation ***z*** in the direction from which the rotation of the body is visible *counter clockwise*,

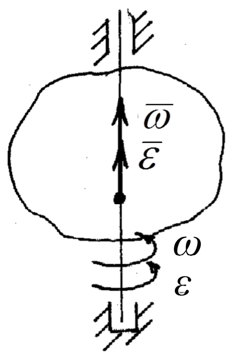
$$\bar{\omega} = \bar{\omega}_z = \omega_z \cdot \bar{k},$$

where $\bar{k}$ – ort of axis *z*; $\omega_z = \dot{\varphi}_z$ – projection angular velocity on the axis *z*; $\omega = |\ \omega_z\ |$ – magnitude of angular velocity.

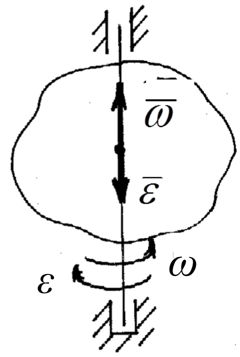
Angular velocity measured in radian per second: $[\omega] = \dfrac{rad}{s} = s^{-1}$

$\bar{\varepsilon} = \dot{\bar{\omega}} = \ddot{\bar{\varphi}}$ – the *angular acceleration* ε (*epsilon*) measures the time rate of change of the angular velocity. The magnitude of this vector is first derivative from angle of rotation.

Angular acceleration measured in radian per square second: $[\varepsilon] = \dfrac{rad}{s^2} = s^{-2}$.



Accelerated rotation
directions $\bar{\omega}$ *and* $\bar{\varepsilon}$ are the same



Decelerated rotation
directions $\bar{\omega}$ *and* $\bar{\varepsilon}$ are opposite

Constant Angular Acceleration

1) *Uniform rotation*: $\varepsilon_z = const \neq 0$ (*uniformly accelerated* motion, if ω_z and ε_z have the same signs and *uniformly decelerated* –have the different ones).

$$\omega_z = \varepsilon_z \cdot t + \omega_{0z},$$

$$\varphi_z = \varepsilon_z \frac{t^2}{2} + \omega_{0z} \cdot t + \varphi_{0z}$$

where $\varphi_{0z}, \omega_{0z}$ – initial *angle of rotation* and *angular velocity*.

2) *Equable rotation*: $\varepsilon_z = 0$

$$\omega_z = \omega_{0z} = const$$

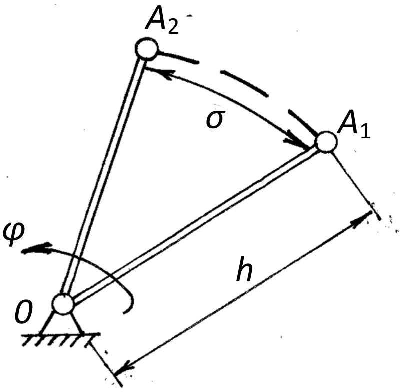
$$\varphi_z = \omega_{0z} \cdot t + \varphi_{0z}.$$

The *angle of rotation* can be expressed through the *number of rotation N* of the body for the full period:

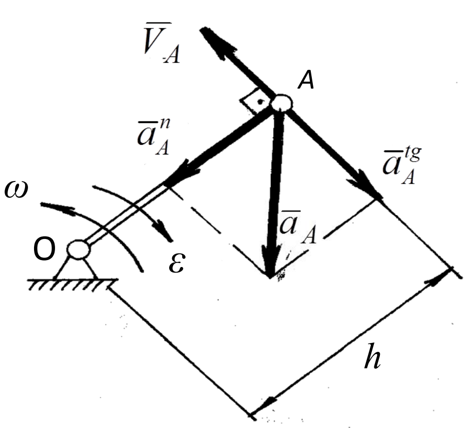
$$\varphi = 2\pi N.$$

Angular velocity in the technique is often given to *the number of rotations per minute*: $\omega = \frac{2\pi n}{60} = \frac{\pi\ n}{30}$.

Formulas to determine **the kinematic values** of any point in the body:



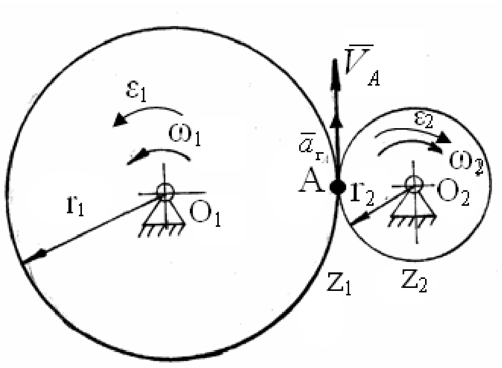
$\sigma = \varphi \cdot h$ is curvilinear coordinate of point the circle;
h – distance from the point A to the axis of rotation;
$v_A = \omega \cdot h$ – magnitude of linear velocity point A, vector $\overline{v}_A \perp h$ and directed from any point on the axis of rotation to the point on curvilinear with the angular velocity ω;
$a_A^{tg} = \varepsilon \cdot h$ – *the tangential component of acceleration* point A; vector $\overline{a}_A^{tg} \perp h$ and has the same direction of curvilinear of the angular acceleration ε;
$a_A^n = \omega^2 \cdot h$ – *the normal component of acceleration* point *A,* vector $\overline{a}_A^n$ has the direction of *A* towards ***O***, the center of the circular trajectory

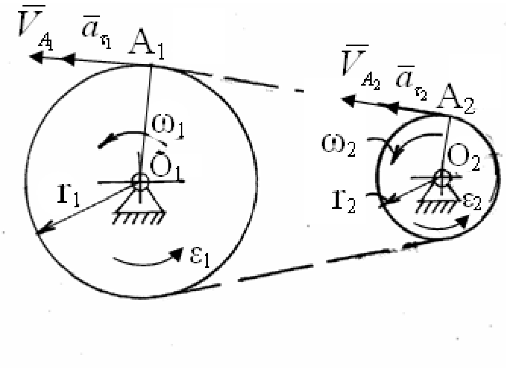


$$\overline{a}_A = \overline{a}_A^{tg} + \overline{a}_A^n;$$

$a_A = \sqrt{(a_A^{tg})^2 + (a_A^n)^2}$ – magnitude of acceleration point *A*.

2.3 Transmission of rotational motion





For gear transmission

$$v_{A_1} = v_{A_2} = v_A$$

$$\omega_1 r_1 = \omega_2 r_2.$$

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}.$$

For belt transmission

$$a_{\tau_1} = a_{\tau_2} = a_{\tau A}.$$

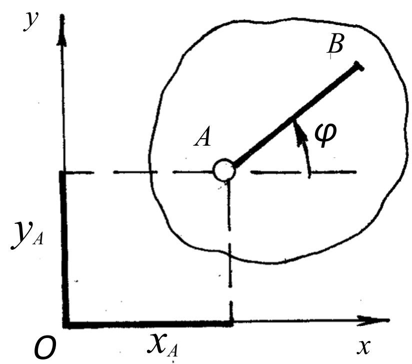
$$\varepsilon_1 r_1 = \varepsilon_2 r_2.$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{r_2}{r_1}.$$

2.4 General plane motion of a rigid body

A rigid body executes *plane motion* when all parts of the body move parallel to planes. It undergoes a combination of translation and rotation. The *translation* occurs *within a reference plane*, and the *rotation* occurs about *an axis perpendicular to the reference plane*.

Position of the figure at a given time is determined by the coordinates of its arbitrary point *A*, considered as *the pole*, and the *angle of rotation* φ of the figure around the pole.



Equation of plate figure motion:

$$x_A = f_1(t); \quad y_A = f_2(t); \quad \varphi = \varphi(t),$$

where *A* – point, chosen as a pole or the base point.

The first two equations characterize *the translational motion of the figure*, in which all the points of the figure move in the same way as the pole, and the third - the rotational motion around the pole.

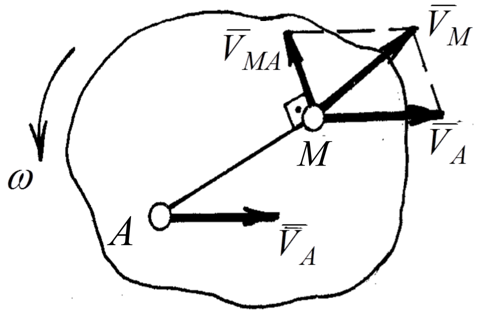
Angular velocity and angular acceleration of the rotational movement of the figure are independent of the pole choice.

Theorem on the velocity of points of a plane figure: *The linear velocity of any point M of the figure at its plane motion is equal to the geometric sum of the velocity of the pole A and the velocity of the point M in its rotational motion around the pole.*

$\overline{v}_M = \overline{v}_A + \overline{v}_{MA}$.

where $v_{MA} = \omega \cdot MA$,

where ω – angular velocity of plate figure; and velocity $\overline{V}_{MA} \perp MA$.

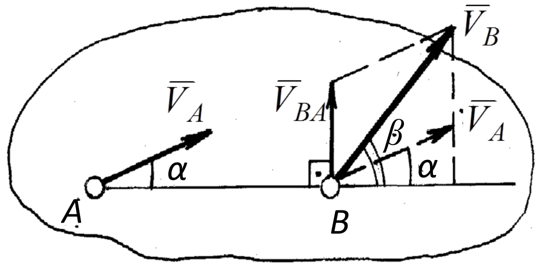


The Theorem of velocity vectors on the projections: *The projections of the two points' velocities onto the line connecting the points are equal*:

$\mathrm{Pr}oj_{AB}(\overline{V_A}) = \mathrm{Pr}oj_{AB}(\overline{V_B})$

or $V_A \cos\alpha = V_B \cos\beta$

where α i β – angles between $\overline{V}_A$ and $\overline{V}_B$ and direction segment AB.

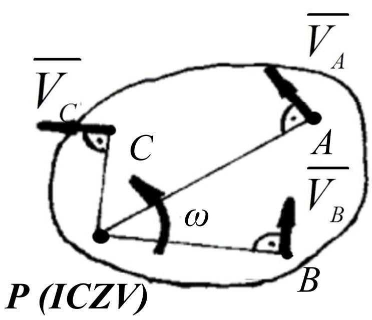


The plate motion of the body at moment can be considered as a rotational motion around the instantaneous center of rotation, or instantaneous center of zero velocity (*ICZV*).

The instantaneous center of zero velocity (*ICZV*) is the point of a plate figure, which velocity at the time is zero.

Determination of the position of the *ICZV* with use of the angular velocity of the figure and the velocity of any point of the figure:

1.



In general, the ***ICZV*** lies at the intersection of the perpendiculars put from two points of the plane figure to their velocities.

The angular velocity ω of a plate figure at each instant time is equal to the ratio of the velocity of any point of the figure to its distance from the ***ICZV***:

$$\omega = \frac{V_A}{AP} = \frac{V_B}{BP} = \frac{V_C}{CP}.$$

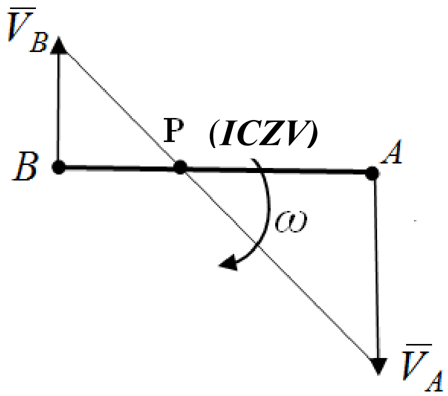
Therefore, the velocity of any point of a plane figure at each time is equal to the product of the angular velocity ω at a distance from the given point to the ***ICZV***:

$V_C = \omega \cdot CP \quad (\overline{V}_B \perp BP);$

$V_B = \omega \cdot BP \quad (\overline{V}_C \perp CP)$

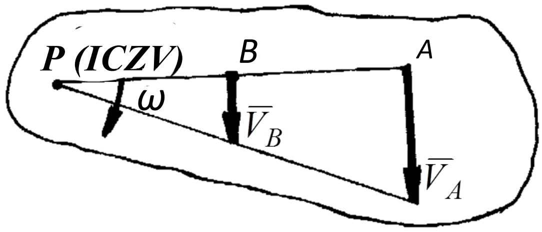
2. if $\overline{V}_A \| \overline{V}_B, \overline{V}_A \perp AB$

a)



$$\omega = \frac{V_A}{AP} = \frac{V_B}{BP}$$

b)

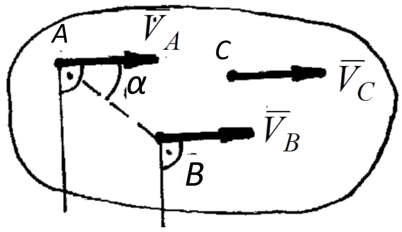


$$\omega = \frac{V_A}{AB + BP} = \frac{V_B}{BP}$$

3. if $\overline{V}_A \| \overline{V}_B, \quad \alpha \neq \pi / 2$

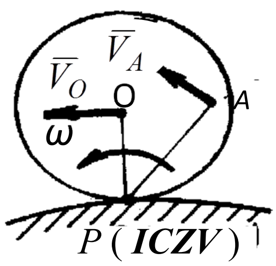
$$\omega = \frac{V_A}{\infty} = 0,$$

$$\overline{V}_A = \overline{V}_B = \overline{V}_C.$$



P (ICZV)→∞

This is a case of instantaneous translation motion.



1. The wheel rolls without sliding on a stationary surface. In this case, the position of *ICZV* is at the point of contact of the figure with the surface

$$\omega = \frac{V_0}{OP}, \; V_A = \omega \cdot AP.$$

Theorem about the points' acceleration of a plane figure:
*The **acceleration** of points of the figure, executing plane motion, defined as the geometric sum of the **acceleration** pole **A** in a fixed coordinate system and the **acceleration** of point **M** in its rotation around the pole A* :

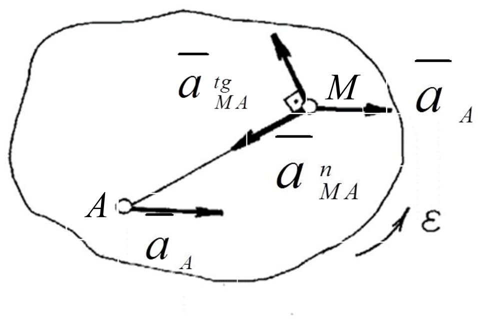
$$\overline{a}_M = \overline{a}_A + \overline{a}_{MA},$$

or because

$$\overline{a}_{MA} = \overline{a}^n_{MA} + \overline{a}^{tg}_{MA},$$

$$\overline{a}_M = \overline{a}_A + \overline{a}^n_{MA} + \overline{a}^{tg}_{MA}, \text{ where } a^n_{MA} = \omega^2 \cdot MA, \; a^{tg}_{MA} = \varepsilon \cdot MA,$$

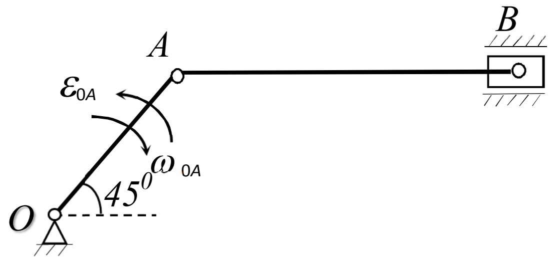
while $\overline{a}^{tg}_{MA} \perp MA$ and directed by the arrow ε, $\overline{a}^n_{MA}$ directed along the line MA from point M to point A.



2.5 Kinematical analysis of plate mechanism

When solving problems it is necessary to consistently consider the movement of individual parts of the mechanism and calculate each of them (to determine the speed and acceleration of the point, which belongs simultaneously to the considered and incremental crank). Calculation starts with a crank, the movement of which is given. If the crank performs a rotational motion, (it has a fixed point, for example, in the form of a fixed hinge) then any of its points moves in a circle and determine the velocity and acceleration of this point.

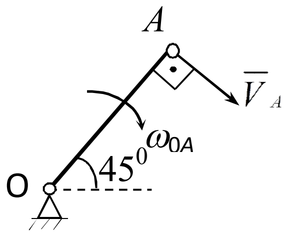
Problem 1. The plate mechanism consists of a crank *OA*, a connecting rod *AB* with a slider *B*. Here *OA* = 20 cm, *AB* = 40 cm. The crank *OA* has at this time the angular velocity ω_{OA} = 2 rad/s and the angular acceleration ε_{OA} = 5 rad/s^2. For a given position mechanism it is necessary to determine the velocity and acceleration of the point B, and the angular velocity and acceleration of the connecting rod *AB*.



Solution:

1. Determine velocity of point *B*.

   At first, consider the crank *OA*, which motion is given.

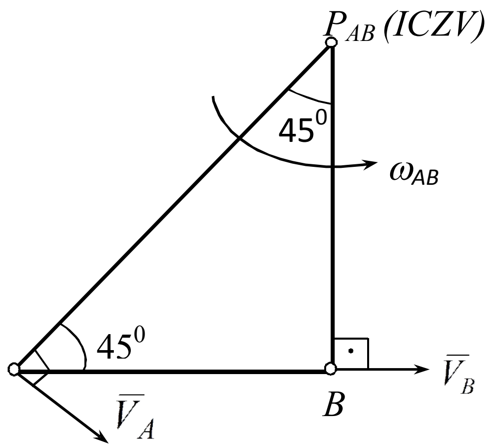


It performs rotational motion about a stationary point *O*, so the velocity of point *A* is determined by the formula

$V_A = \omega_{OA} \cdot OA = 2 \cdot 20 = 40$ (cm/s).

Vector $\overline{V}_A \perp OA$ and has direction of the rod rotation (by "arrow" ω_{OA}).

2) Consider the connecting rod *AB*. It performs a general plane motion, so at first it is necessary to draw its *ICZV*.



The velocity of point A is determined when calculating the rod *OA*. Point *B* belongs to a slider so the line of its velocity $\overline{V}_B$ is parallel to the direction of slider motion. We draw the *ICZV* (point) as the point of intersection of the perpendiculars to the velocity $\vec{V}_A$ and velocity of the point B, which is horizontal (directed in the tangent to the trajectory).

Angular velocity of rod *AB* equals:

$$\omega_{AB} = \frac{V_A}{AP_{AB}} = \frac{V_A}{AB / \cos 45^0} = \frac{V_A \cdot \cos 45^0}{AB} = \frac{40 \cdot 0,7}{40} = 0,7 \left(\frac{rad}{s}\right)$$

and is directed according to how the vector "rotates" around the *ICZV* (P_{AB}).

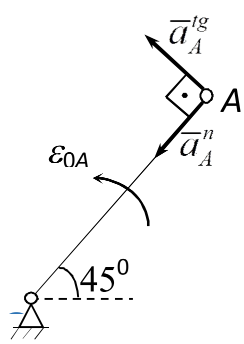
Velocity of point *B*

$$V_B = \omega_{AB} \cdot BP_{AB} = 0,7 \cdot 40 = 28 \text{ (cm/s)}$$

and directed in accordance to the "arrow" ω_{AB} .

2. Determine further acceleration of the point *B*.

1) Consider the crank *OA*. It performs rotational motion; therefore the modules of components of acceleration of point *A* are determined by formulas:



$a_A^n = \omega_{OA}^2 \cdot OA = 4 \cdot 20 = 80$ (cm/s^2),

$a_A^{tg} = \varepsilon_{OA} \cdot OA = 5 \cdot 20 = 100$ (cm/s^2).

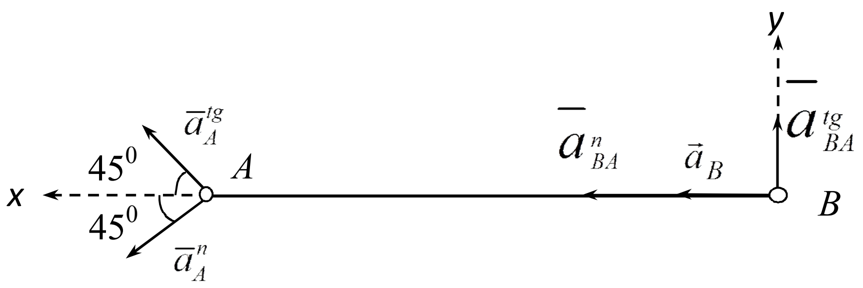
We show the directions of the components: the vector $\overline{a}_A^n$ is directed along the rod from point *A* to the center of rotation *O*, the vector $\overline{a}_A^{tg} \perp OA$ directed in accordance to the "arrow".

2) Consider the connecting rod AB. It performs a general plane motion, so the acceleration of point B is determined by the formula:

$$\overline{a}_B = \overline{a}_A + \overline{a}_{BA} = \overline{a}_A^n + \overline{a}_A^{tg} + \overline{a}_{BA}^n + \overline{a}_{BA}^{tg}$$

The component magnitude $\overline{a}_{BA}^n$ is calculated using the formula:

$$a_{BA}^n = \omega_{AB}^2 \cdot BA = 0,7^2 \cdot 40 = 20 \text{ (cm/s}^2\text{)}.$$



The vector $\overline{a}_{BA}^n$ of directions along the segment BA from point B to pole A. The magnitude of the component $\overline{a}_{BA}^n$ cannot be calculated yet, so ε_{AB} unknown. In this case, we will depict the vector $\overline{a}_{BA}^{tg} \perp BA$ and point in any direction (for example, upwards). Since the trajectory of point B is a straight line, then the vector $\overline{a}_B$ must have direction along this line. Let's depict it, for example, to the left of point B.

Choose the axis of the coordinates x, y (for example, x-axis along the rod to the left, axis y in - upwards) and design a vector equation for determining the acceleration to these axes:

$$x: \quad a_B = a_A^n \cdot \cos 45^0 + a_A^{tg} \cdot \cos 45^0 + a_{BA}^n,$$

$$y: \quad 0 = -a_A^n \cdot \sin 45^0 + a_A^{tg} \cdot \sin 45^0 + a_{BA}^{tg}.$$

In the resulting system of algebraic equations unknown magnitude are accelerations a_B and a_{BA}^{tg}. From the first equation we find acceleration of point B:

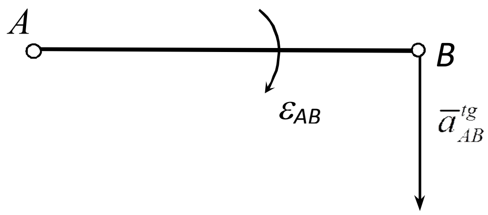
$$a_B = 80 \cdot 0,7 + 100 \cdot 0,7 + 20 = 146 \text{ (cm/s}^2\text{)}.$$

Since $a_B > 0$, then, the vector $\overline{a}_B$ corresponds to the actual direction of acceleration of point B. From the second equation we find a_{BA}^{tg}:

$$a_{AB}^{tg} = a_A^n \cdot \sin 45^0 - a_A^{tg} \cdot \sin 45^0 = 56 - 70 = -14 \text{ (cm/s}^2\text{)}.$$

The sign "-" -- means that in fact the vector $\overline{a}_{BA}^{tg} \perp BA$ is directed downwards. Angular acceleration

$$\varepsilon_{AB} = \frac{|a_{BA}^{tg}|}{BA} = \frac{14}{40} \approx 0,35 \left(\frac{\text{rad}}{\text{s}^2}\right)$$

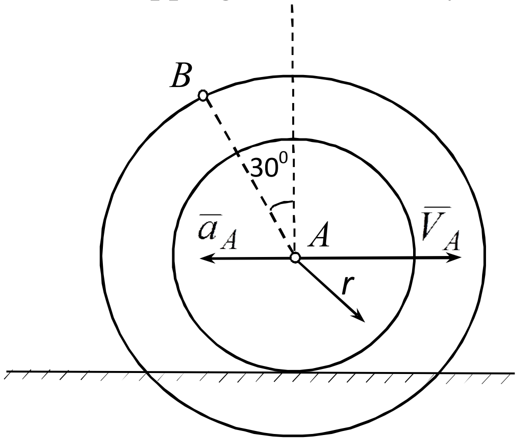


and directed clockwise (as the real vector $\bar{a}_{BA}^{tg}$ "rotates" around pole A).

Answer: $\omega_{AB} = 0,7 \ \dfrac{\text{rad}}{\text{s}}$; $\quad V_B = 28$ cm/s;

$\varepsilon_{AB} = 0,35 \ \dfrac{\text{rad}}{\text{s}^2}$; $\quad a_B = 146$ cm/s^2.

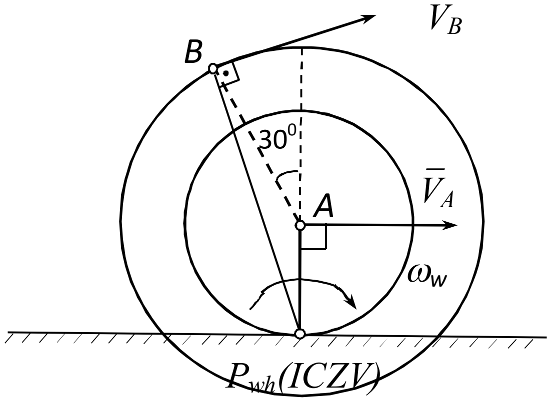
Problem 2. The wheel A consists of two rigidly fastened discs. Wheel of radius ***r*** rolls without slipping on a stationary surface.



Here AB = 15 cm, r = 10 cm. The center of wheel A has at this time velocity V_A = 28 cm/s and acceleration $a_A = 146$ cm/s^2. For a given position of the wheel to determine its angular velocity and acceleration, as well as the velocity and acceleration of point B.

Solution:

1. Determine velocity of point B.



A wheel rolling without slipping on a stationary surface performs a general plane motion. *ICZV* (point P_{wh}) is at the point of contact with a stationary surface. Angular wheel velocity

$$\omega_{wh} = \frac{V_A}{AP_{wh}} = \frac{28}{10} = 2,8 \ \left(\frac{\text{rad}}{\text{s}}\right.$$

and directed clockwise (according to the "rotation" of the vector around the *IZCV* (point P_{wh}).

The segment BP_{wh} can be determined from the triangle ABP_{wh} by the law of cosines:

$$BP_{wh} = \sqrt{AB^2 + AP_{wh}^2 - 2AB \cdot AP_{wh} \cdot \cos 150^0} =$$

$$= \sqrt{15^2 + 10^2 - 2 \cdot 15 \cdot 10 \cdot (-0,85)} = \sqrt{580} \approx 24 \quad \text{(cm)},$$

So $V_B = \omega_{wh} \cdot BP_{wh} = 2,8 \cdot 24 \approx 67,2$ (cm/s). Vector $\overline{V}_B \perp BP_{wh}$ and has direction according to the clockwise ω_{wh}.

2. Determine acceleration point *B*.

The acceleration of the point B in the wheel, which performs the general plane motion, is determined by the formula:

$$\overline{a}_B = \overline{a}_A + \overline{a}_{BA} = \overline{a}_A + \overline{a}_{BA}^n + \overline{a}_{BA}^{tg}.$$

Since the trajectory of point *B* is unknown (it has no restrictions on other bodies), its acceleration will be find in the form of two unknown components, directed along the coordinate axes:

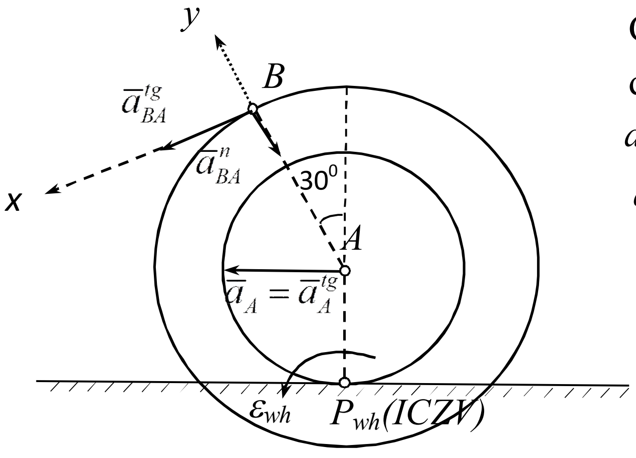
$$\overline{a}_{Bx} + \overline{a}_{By} = \overline{a}_A + \overline{a}_{BA}^n + \overline{a}_{BA}^{tg}$$

Angular acceleration of the rolling wheel is determined by the formula:

$$\varepsilon_{кол} = \frac{d\omega_{wh}}{dt} = \frac{d(V_A / AP_{wh})}{dt} = \frac{1}{AP_{wh}}\frac{dV_A}{dt} = \frac{a_A^{tg}}{AP_{wh}} = \frac{a_A}{AP_{wh}} = \frac{146}{10} = 14,6\left(\frac{rad}{s^2}\right),$$

where $\overline{a}_A^{tg} = \overline{a}_A$, since the trajectory of point *B* is a straight line.

Direction of ε_{wh} is counter clockwise (as the vector $\overline{a}_A^{tg}$ "rotates" around the *ICZV* counter clockwise).



Component magnitudes $\overline{a}_{BA}^n$ and $\overline{a}_{BA}^{tg}$ are determined by the formula:

$a_{BA}^n = \omega_{wh}^2 \cdot BA = 2,8^2 \cdot 15 = 117,6$ (cm/s^2),

$a_{BA}^{tg} = \varepsilon_{wh} \cdot BA = 14,6 \cdot 15 = 219$ (cm/s^2).

The vector $\overline{a}_{BA}^n$ is directed along the segment *BA* from point *B* to pole *A*, and the vector $\overline{a}_{BA}^{tg} \perp BA$ is directed in accordance to the "arrow" $\boldsymbol{\varepsilon_{wh}}$. We construct the *xy* coordinate axis and take the projection of a vector equation for determining the acceleration $\overline{a}_B$ on these axes:

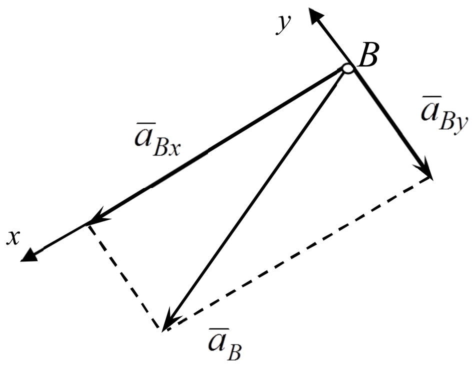
x: $a_{Bx} = a_A \cdot \cos 30^0 + a_{BA}^{tg} = 146 \cdot 0,85 + 219 = 343,1$ (cm/s^2);

y: $a_{By} = a_B \cdot \sin 30^0 - a_{BA}^{tg} = 146 \cdot 0,5 - 117,6 = -44,6$ (cm/s^2).

Acceleration magnitude of point *B* is determined by the formula

$$a_B = \sqrt{a_{Bx}^2 + a_{By}^2} = \sqrt{(343,1)^2 + (-44,6)^2} \approx 346 \text{ (cm/s}^2\text{)}.$$

Since, $a_{Bx} > 0$, $a_{By} \lhd 0$ then, the acceleration component $\overline{a}_{Bx}$ of point B is directed towards the positive direction of the axis x, and the component $\overline{a}_{By}$ – towards the negative direction of the axis y. The vector $\overline{a}_B$ is depicted as a diagonal of a rectangle constructed on the components $\overline{a}_{Bx}$, $\overline{a}_{By}$ as on the sides.

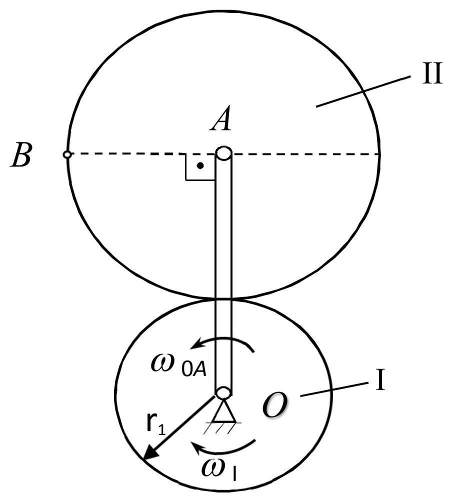


Answer:

$\omega_{wh} = 2,8\ \dfrac{\text{rad}}{\text{s}}$; $V_B = 67,2$ cm/s;

$\varepsilon_{wh} = 14,6\ \dfrac{\text{rad}}{\text{s}^2}$; $a_B = 346$ cm/s^2.

Problem 3. The plate mechanism consists of the crank OA and two gear wheels I and II, connected with the crank OA.

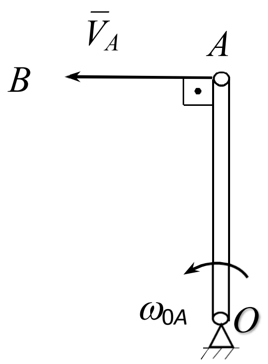


Here OA = 30 cm , r_I = 10 cm. Crank OA rotates about axis O and has angular velocity in this moment $\omega_{OA} = 3$ rad/s and angular acceleration $\varepsilon_{OA} = 0$. Wheel I rotates about axis O with constant angular velocity $\omega_I = 6$ rad/s. For a given position of the mechanism to determine the velocity and acceleration of the point B, and the angular velocity and acceleration of wheel II.

Solution:

1. Determine velocity of point B.

At first, consider the crank OA, which motion is given.

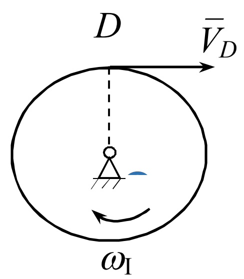


It performs rotational motion around a fixed point O, so the velocity of point A is determined by the formula:

$$V_A = \omega_{OA} \cdot OA = 3 \cdot 30 = 90 \text{ (cm/s)}.$$

Vector $\overline{V}_A \perp OA$ and directed counter clockwise (by “arrow” ω_{OA}).

2. Consider wheel I, the motion of which is given. Wheel I performs

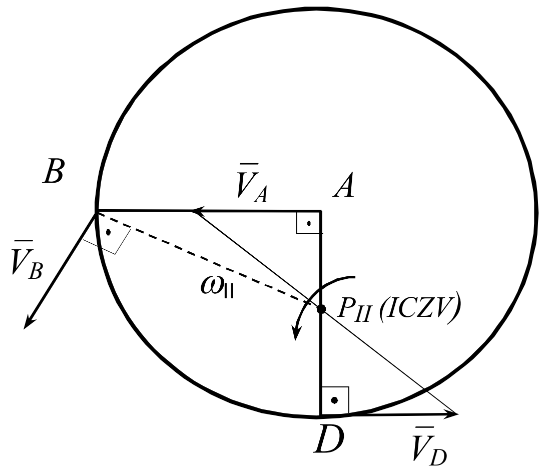


rotation motion around a fixed point O. Denote touch point of the wheel I and II as letter D, then its velocity equals

$$V_D = \omega_I \cdot OD = 6 \cdot 10 = 60 \text{ (cm/s)}.$$

Vector $\overline{V}_D \perp OD$ and directed clockwise the wheel I.

3. Consider wheel II. It performs a general-plane motion. At first, it is necessary to draw the pole of *ICZV*. To draw the *ICZV* we connect



ends of velocity vectors $\overline{V}_A$ and $\overline{V}_D$. Position of *ICZV* is at the intersection of this segment with a line passing through points A and D. The angular velocity of the wheel V_A is related to velocity V_D by ratio:

$$\omega_{II} = \frac{V_A}{AP_{II}} = \frac{V_D}{DP_{II}}.$$

Considering, $DP_{II} = AD - AP_{II} = (OA - r_I) - AP_{II} = 20 - AP_{II}$,
the right side of the relation ω_{II} can be considered as an equation with respect to the segment AP_{II}:

$$\frac{V_A}{AP_{II}} = \frac{V_D}{20 - AP_{II}}.$$

Solve this equation and determine AP_{II}:

$$V_A \cdot (20 - AP_{II}) = V_D \cdot AP_{II},$$
$$V_A \cdot 20 = AP_{II} \cdot (V_A + V_D),$$
$$AP_{II} = \frac{V_A \cdot 20}{V_A + V_D} = \frac{90 \cdot 20}{90 + 60} = \frac{1800}{150} = 12 \text{ (cm)}.$$

So

$$\omega_{II} = \frac{V_A}{AP_{II}} = \frac{90}{12} = 7{,}5 \ (rad/s).$$

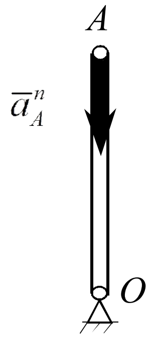
Angular velocity ω_{II} is directed according to how the vector is $\overline{V}_A$ “rotates” about *ICZV* (P_{II}). Velocity point B is

$$V_B = \omega_{II} \cdot BP_{II} = \omega_{II} \cdot \sqrt{AB^2 + AP_{II}^2} = 7{,}5 \cdot \sqrt{20^2 + 12^2} = 7{,}5 \cdot \sqrt{544} =$$
$$= 7{,}5 \cdot 4 \cdot \sqrt{34} \approx 30 \cdot 5{,}83 \approx 175 \ \text{(cm/s)}$$

And is directed according to the "arrow" ω_{II}.

4. Determine acceleration of point B.

1) Consider the crank OA. It performs rotation motion,



therefore the magnitudes of acceleration components of point A are determined by formulas:

$$a_A^n = \omega_{OA}^2 \cdot OA = 3^2 \cdot 30 = 270 \text{ (cm/s}^2\text{)},$$

$$a_A^{tg} = \varepsilon_{OA} \cdot OA = 0.$$

Vector $\overline{a}_A^n$ directed along the rod from point A to the center of rotation O.

2) Acceleration of point B to wheel II, which performs a general-plane motion, is determined by the formula:

$$\overline{a}_B = \overline{a}_A + \overline{a}_{BA} = \overline{a}_A^n + \overline{a}_A^{tg} + \overline{a}_{BA}^n + \overline{a}_{BA}^{tg}.$$

Since the trajectory of point B is unknown (it has no connections to other bodies), its acceleration will be determine in the form of two unknown components $\overline{a}_{Bx}, \overline{a}_{By}$, directed along the axes of the coordinates x, y:

$$\overline{a}_{Bx} + \overline{a}_{By} = \overline{a}_A^n + \overline{a}_A^{tg} + \overline{a}_{BA}^n + \overline{a}_{BA}^{tg}.$$

The angular acceleration of the wheel II will be determined using the ratio

$$\frac{V_A}{AP_{\text{II}}} = \frac{V_D}{DP_{\text{II}}} = \frac{V_A + V_D}{AD}.$$

Then

$$\varepsilon_{\text{II}} = \frac{d\omega_{\text{II}}}{dt} = \frac{d\left(\dfrac{V_A + V_D}{AD}\right)}{dt} = \frac{1}{AD}\frac{d\left(V_A + V_D\right)}{dt} = \frac{1}{AD}\left(\frac{dV_A}{dt} + \frac{dV_D}{dt}\right) =$$

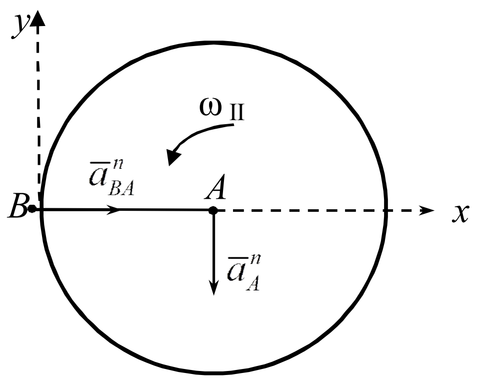
$$= \frac{1}{AD}\left(a_A^{tg} + a_D^{tg}\right) = 0.$$

In this formula $a_D^{tg} = \varepsilon_{\text{I}} \cdot OD = \dfrac{d\omega_{\text{I}}}{dt} \cdot OD = 0$, since under the problem $\omega_{\text{I}} = const$.

Components of magnitudes $\overline{a}_{BA}^n$ and $\overline{a}_{BA}^{tg}$ are determined by the formula:

$$a_{BA}^n = \omega_{\text{II}}^2 \cdot BA = 7{,}5^2 \cdot 10 = 56{,}25 \cdot 10 = 562{,}5 \text{ (cm/s}^2\text{)},$$

$$a_{BA}^{tg} = \varepsilon_{\text{II}} \cdot BA = 0.$$



The vector $\overline{a}_{BA}^n$ has direction along the segment V_A from point B to pole A.

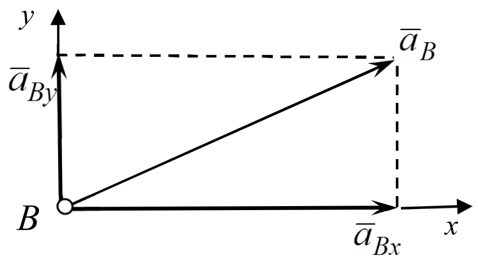
Draw the coordinate axis xy and find the projection of a vector equation for determining the acceleration $\overline{a}_B$ on these axes:

x: $a_{Bx} = a_{BA}^n = 562{,}5$ (cm/s^2);

y: $a_{By} = a_A^n = 270$ (cm/s^2).

Magnitude of point B acceleration determined by the formula

$$a_B = \sqrt{a_{Bx}^2 + a_{By}^2} = \sqrt{(562{,}5)^2 + (270)^2} \approx 607 \text{ (cm/s}^2\text{)}.$$



Since $a_{Bx} > 0, a_{By} > 0$, then components of point B acceleration are directed towards the positive direction of the x and y axes, and the vector $\overline{a}_B$ is represented as a diagonal of the rectangle, drawn on components a_{Bx}, a_{By} as on a sides.

Answer: $\omega_{II} = 7{,}5$ rad/s; $V_B = 67{,}2$ cm/s,

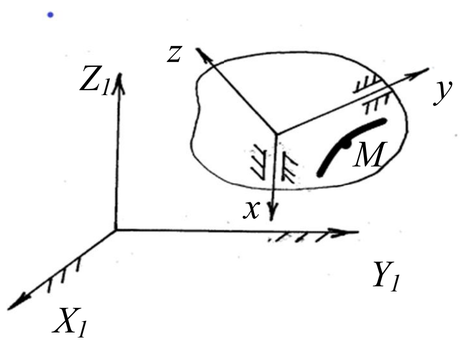
$\varepsilon_{II} = 0$; $a_B = 607$ cm/s^2.

3 Compound motion of a point

The compound motion of a point is the motion in which point M takes part in two or more motions simultaneously.

Absolute motion is the motion of point M according to a fixed coordinate system x, y, z.

Relative motion is the motion of point M relative to the moving coordinate system $X_1Y_1Z_1$.



Transport (bulk) motion is the motion of the moving reference with respect to the fixed reference.

Theorem on adding velocities: The absolute velocity of a point equals the geometric sum of velocities in the transport and relative motions:

$$\overline{v}_a = \overline{v}_e + \overline{v}_r,$$

$\overline{v}_a$ – absolute velocity; $\overline{v}_e$ – transport velocity; $\overline{v}_r$ – relative velocity.

Coriolis's theorem on addition accelerations: the absolute acceleration of a point equals the geometric sum of acceleration in the transport motion $\overline{a}_e$, the acceleration in relative motion $\overline{a}_r$ and the acceleration of the Coriolis $\overline{a}_c$:

$$\overline{a} = \overline{a}_e + \overline{a}_r + \overline{a}_c;$$

Coriolis's acceleration vector is expressed by the formula

T$\overline{a}_c = 2(\overline{\omega}_e \times \overline{v}_r)$.

he

magnitude of the Coriolis's acceleration vector is equal to

$$a_c = 2\omega_e V_r \sin(\overline{\omega}_e \,\hat{,}\, \overline{v}_r),$$

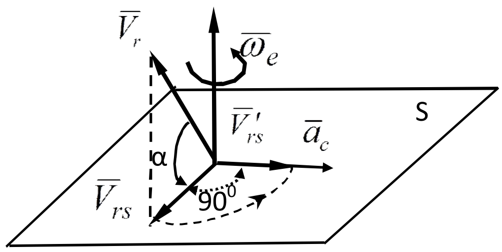
where ω_e – the angular velocity of the transport rotational motion.

Coriolis acceleration is zero if:

1) the transport motion is translation ($\omega_e = 0$);
2) the relative velocity $\overline{V}_r$ at the moment is zero ($v_r = 0$);
3) the relative velocity vector $\overline{V}_r$ at the moment is parallel to the vector of the angular velocity of the transport rotational motion ($\sin(\overline{\omega}_e \,\hat{,}\, \overline{v}_r) = 0$).

To determine the direction of Coriolis's acceleration can be used either the rule of the vector product, or the rule of Zhukovsky. According to Zhukovsky's rule, the direction of the vector is determined in such order:

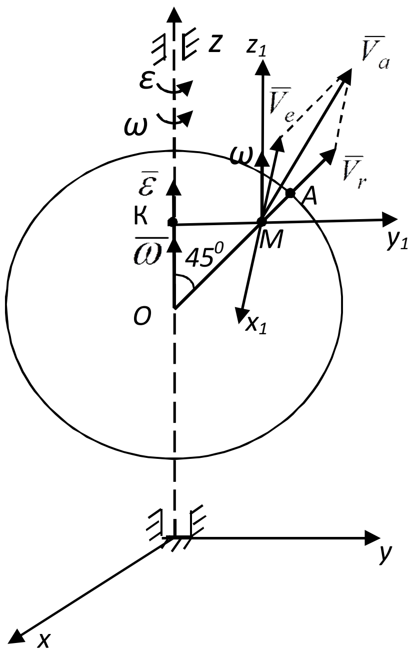
1) it is necessary to take projection of a vector of relative velocity $\overline{V}_r$ on a plane S perpendicular to the axis of transport rotation (perpendicular to the vector $\overline{\omega}_e$);
2) get the projection $\overline{V}_{rs}$ and turn in this plane by 90 degrees in the direction of transport rotation (in direction of curve-line arrow ω_e);
3) 3) the direction of the Coriolis's acceleration $\overline{a}_c$ coincides with the direction of the vector $\overline{V}'_{rs}$.



magnitude $a_c = 2\omega_e V_r \sin\alpha$.

Problem 4. The disk rotates around a vertical diameter with angular acceleration $\varepsilon = 2t$ (rad/s^2)., the point M moves with a constant acceleration a = 2 (cm/s^2) along the radius of the OA inclined to the axis of rotation at an angle of 45°. At the time t = 0, the angular velocity of the disk and the relative velocity of the point of the disk center are zero, i. e. $\omega_0 = 0$, $V_{r0} = 0$. Determine the absolute velocity and absolute acceleration of the point M at the moment t = 1 s.

Solution: For a fixed reference system, we take the ground, and for moving – a rotating disk. Transport motion (motion of the disk relative to the ground) is rotational. Angular velocity of the disk



$$\omega = \int \varepsilon dt = \int 2tdt = t^2 + C,$$

where $C = 0$, as $\omega_0 = 0$.

So $\omega = t^2$.

For absolute velocity of point M we get:

$$\overline{V}_a = \overline{V}_e + \overline{V}_r .$$

The relative velocity is directed along the radius of the OA and equals $V_r = 2t$. The transport velocity is directed perpendicularly to the drawing from us in the direction of disk rotation is equal to

$$V_e = \omega \cdot MK = \omega \cdot s \cdot \sin 45^0 ,$$

where $s = (a \cdot t^2)/2$. So $V_e = t^4 / \sqrt{2}$.

Since between the velocity vectors $\overline{V}_e$ and $\overline{V}_r$ the angle of 90^0, then

$$\overline{V}_a = \sqrt{V_e^2 + V_r^2} = \sqrt{4t^2 + t^3 / 2}$$

At the moment $t = 1$ s

$$V_a = \sqrt{4,5} \approx 2,12 \text{ cm/s}.$$

Consider that the transport motion is rotational, we have

$$\overline{a}_a = \overline{a}_r + \overline{a}_e + \overline{a}_c .$$

Relative acceleration is directed along the radius of the OA and equals $a_r = a = 2$ (cm / s). The transport acceleration of the point M consists of two components: rotating and centripetal acceleration, i.e.

$$\overline{a}_e = \overline{a}_e^{tg} + \overline{a}_e^{cp}$$

Transportable centripetal acceleration is directed along the MK (from the point M to the point K) and is numerically equal to

$$a_e^{cp} = \omega^2 \cdot MK = \omega^2 \cdot s \cdot \sin 45^0$$

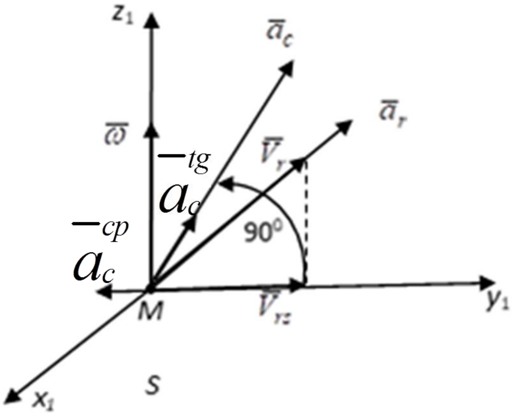
or $a_e^{cp} = \dfrac{t^6\sqrt{2}}{2}\dfrac{\text{cm}}{\text{s}^2}.$

Transport rotational acceleration is directed perpendicularly to the drawing from us in the direction of the "arrow" ε.

$$a_e^{tg} = \varepsilon \cdot MK = 2t \cdot s \cdot \sin 45^0 = t^3\sqrt{2}\,\frac{\text{cm}}{\text{s}^2}.$$

at t = 1 s, $a_e^{cp} = \sqrt{2/2}\,\frac{\text{cm}}{s^2}$, $a_e^{tg} = \sqrt{2}\,\frac{\text{cm}}{s^2}$.

To determine the Coriolis's acceleration, the conditionally vector $\overline{\omega}$ transferred parallel to itself in the point M. Acceleration $\overline{a}_c$ will be directed perpendicularly to the plane $(\overline{\omega}, \overline{V}_r)$ in the direction from which the rotation $\overline{\omega}$ to $\overline{V}_r$ a smaller angle has a direction against the clockwise movement, that is, in this case perpendicular to the plane of the drawing from the observer. Magnitude



$$a_c = 2\omega \cdot V_r \cdot \sin 45^0 = 2t^3\sqrt{2}\quad \frac{\text{cm}}{s^2}$$

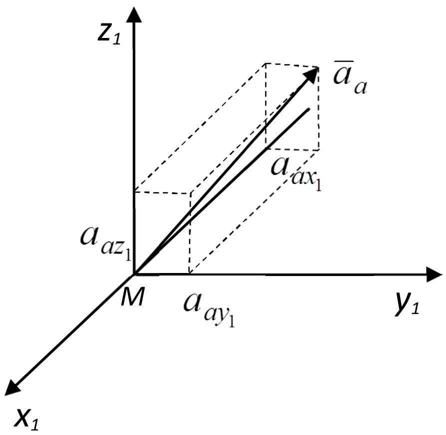
At $t = 1s$ $a_c = 2\sqrt{2}\,\frac{\text{cm}}{\text{s}^2}$.

To determine the numerical value of the absolute acceleration and hence the compilation of vectors $\overline{a}_r, \overline{a}_e^{cp}, \overline{a}_e^{tg}, \overline{a}_c$, we make three mutually perpendicular axes X_1, Y_1, Z_1. If we take projections of all components of absolute acceleration on these coordinate axes, we obtain the projections of absolute acceleration of the point M

$$a_{ax_1} = -a_e^{tg} - a_c = -\sqrt{2} - 2\sqrt{2} = -3\sqrt{2}\,,$$

$$a_{ay_1} = -a_e^{cp} + a_r \cdot \cos 45^0 = -\frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} = \frac{\sqrt{2}}{2},$$

$$a_{az_1} = a_r \cdot \cos 45^0 = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$



We construct a vector of absolute acceleration taking into account certain projections on the axis. By the magnitude of the absolute acceleration, the point M will be:

$$a_a = \sqrt{a_{ax_1}^2 + a_{ay_1}^2 + a_{az_1}^2} = \sqrt{18 + \frac{1}{2} + 2} = \sqrt{20{,}5} = 4{,}5\,\frac{\text{cm}}{\text{s}^2}.$$

Answer: the absolute velocity of the point M V_a = 2,12 cm / s; the absolute acceleration of the point M a_a = 4,5 cm / s^2.

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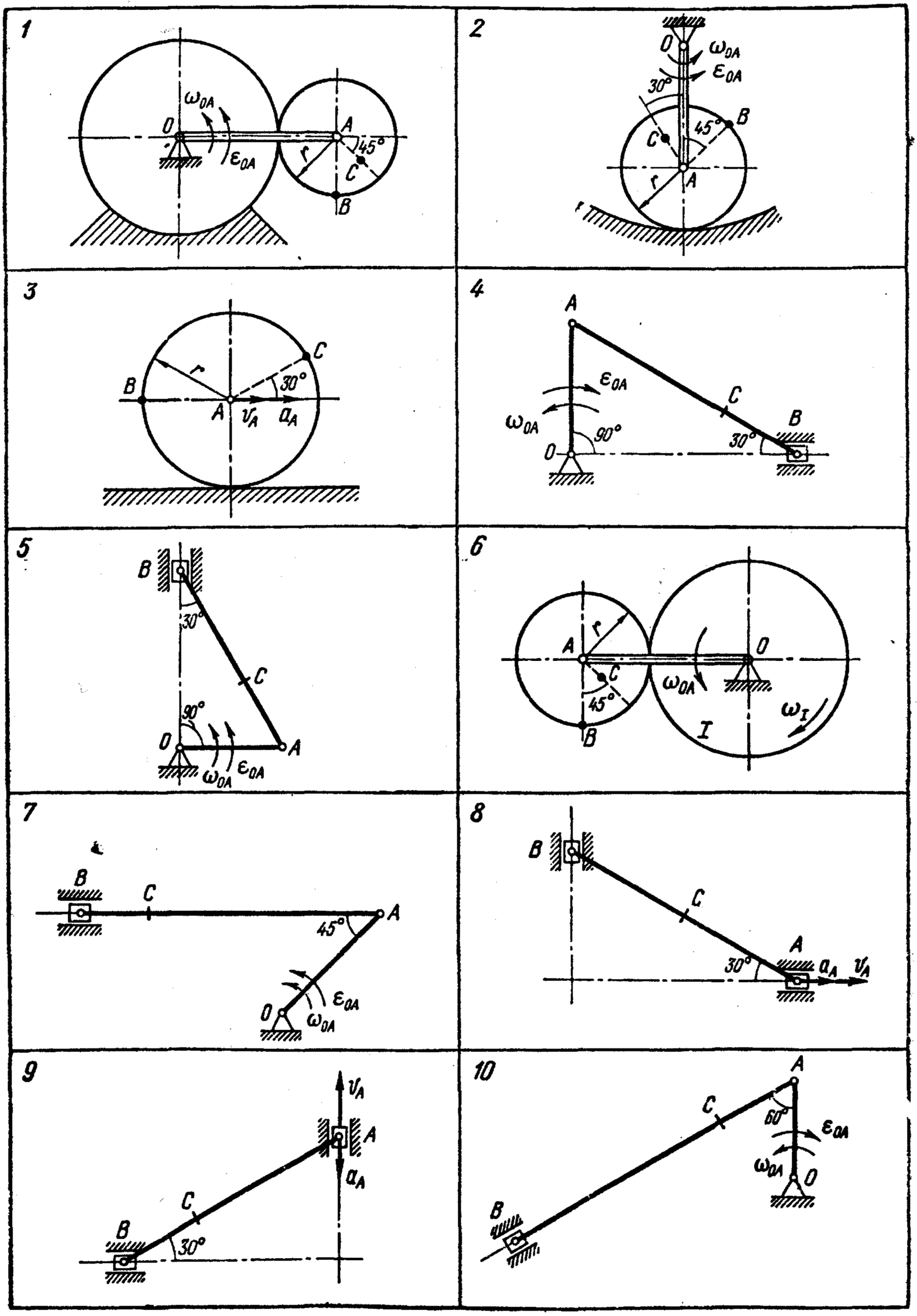
APPENDIX

Output dates for work

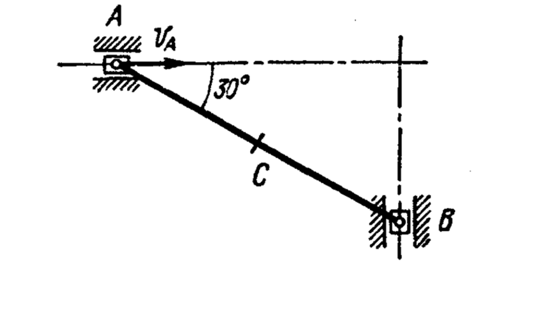
"Kinematical analysis of plate mechanism"

Determine for a given position of the mechanism velocity and acceleration of points *B* and *C*, as well as angular velocity and angular acceleration of the link, which belongs to the point *C*. Schemes of mechanisms are shown in the following pages, and the required size and kinematic parameters are presented in the table, where $\omega_{OA}, \varepsilon_{OA}$ - angular velocity and angular acceleration of the crank *OA* for a given position of the mechanism; ω_{I} - the angular velocity of the wheel I ; V_A, a_A - velocity and acceleration of point *A*. Rolling of wheels is carried out without slipping. The method of solving problems is considered in section 2.5.

Number of variant	Dimensions , cm				ω_{OA}, rad/s	ω_{I}, rad/s	ε_{OA}, rad/s^2	v_A, cm/s	a_A, cm/s^2
	OA	*r*	*AB*	*AC*					
1	40	15	-	8	2	-	2	-	-
2	30	15	-	8	3	-	2	-	-
3	-	50	-	-	-	-	-	50	100
4	35	-	-	45	4	-	8	-	-
5	25	-	-	20	1	-	1	-	-
6	40	15	-	6	1	1	0	-	-
7	35	-	75	60	5	-	10	-	-
8	-	-	20	10	-	-	-	40	20
9	-	-	45	30	-	-	-	20	10
10	25	-	80	20	1	-	2	-	-
11	-	-	30	15	-	-	-	10	0
12	-	-	30	20	-	-	-	20	20
13	25	-	55	40	2	-	4	-	-
14	45	15	-	8	3	12	0	-	-
15	40	15	-	8	1	-	1	-	-
16	55	20	-	-	2	-	5	-	-
17	-	30	-	10	-	-	-	80	50
18	10	-	10	5	2	-	6	-	-
19	20	15	-	10	1	2,5	0	-	-
20	-	-	20	6	-	-	-	10	15
21	30	-	60	15	3	-	8	-	-
22	35	-	60	40	4	-	10	-	-
23	-	-	60	20	-	-	-	5	10
24	25	-	35	15	2	-	3	-	-
25	20	-	70	20	1	-	2	-	-
26	20	15	-	10	2	1,2	0	-	-
27	-	15	-	5	-	-	-	60	30
28	20	-	50	25	1	-	1	-	-
29	12	-	35	15	4	-	6	-	-
30	40	-	-	20	5	-	10	-	-


1
O
A
ωOA
εOA
r
45°
C
B
2
O
ωOA
εOA
30°
B
45°
C
A
r
3
C
r
B
30°
A
vA
aA
4
A
εOA
C
ωOA
B
90°
30°
O
5
B
30°
C
90°
O
A
ωOA
εOA
6
A
r
O
C
45°
ωOA
ωI
B
I
7
B
C
A
45°
εOA
O
ωOA
8
B
C
A
30°
aA
vA
9
vA
A
aA
C
B
30°
10
A
C
60°
εOA
ωOA
O
B

11



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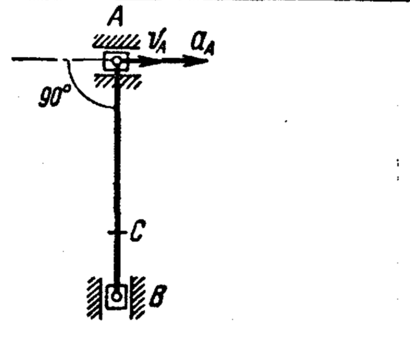

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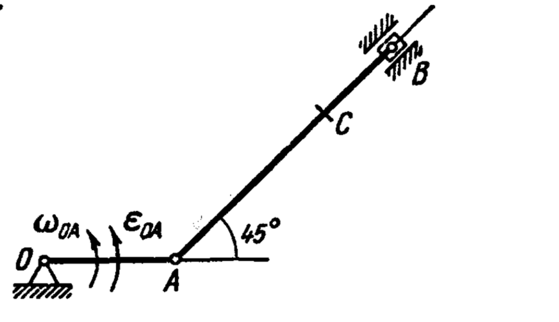

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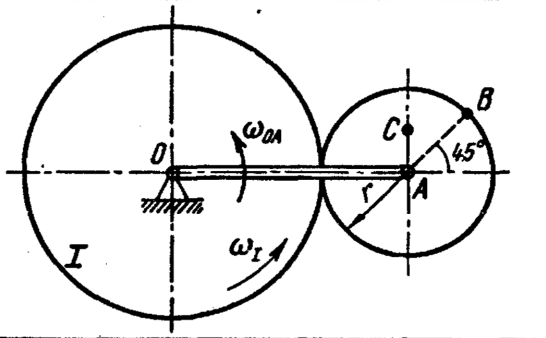

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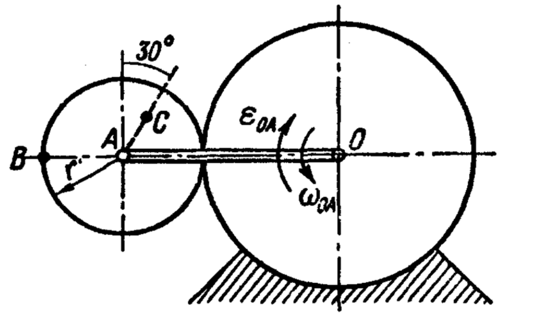

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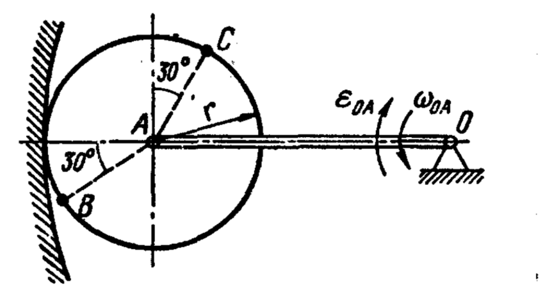

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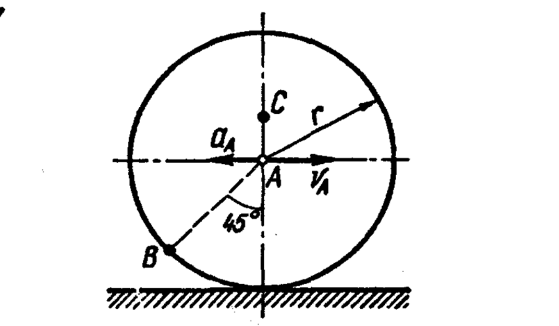

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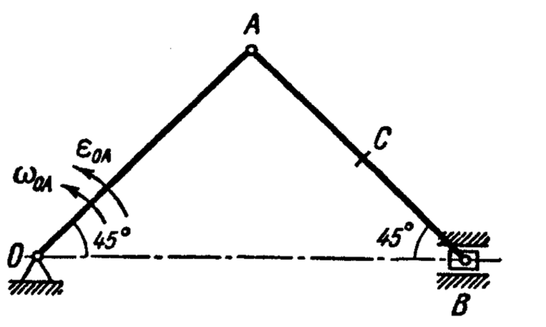

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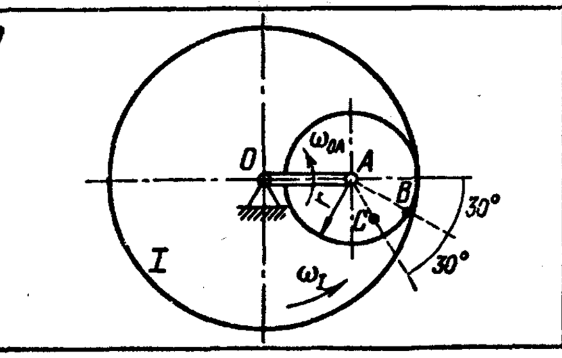

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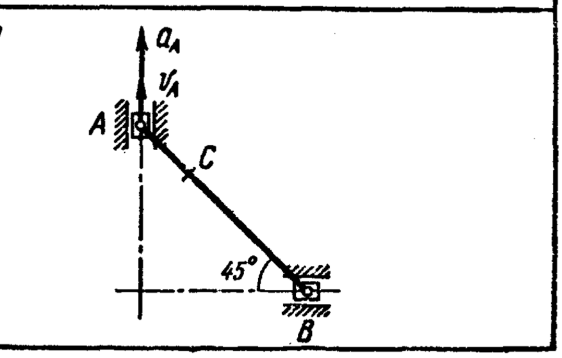

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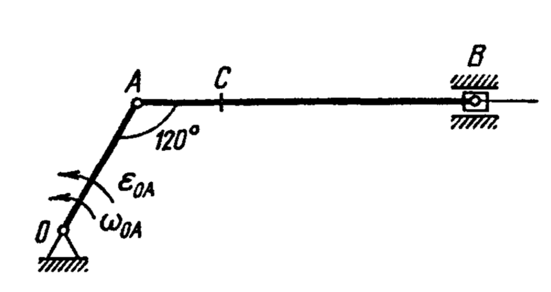

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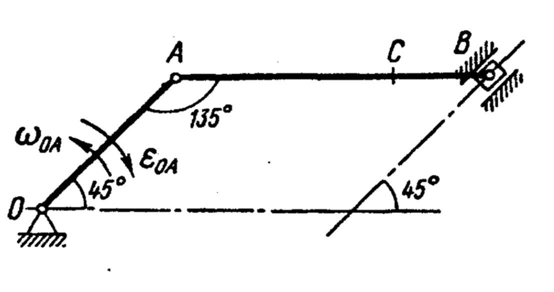

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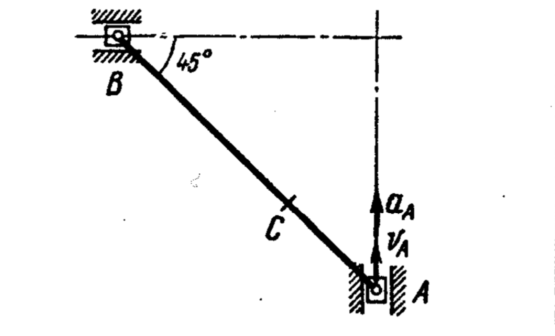

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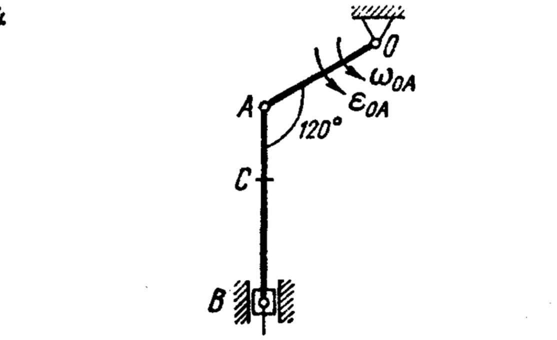

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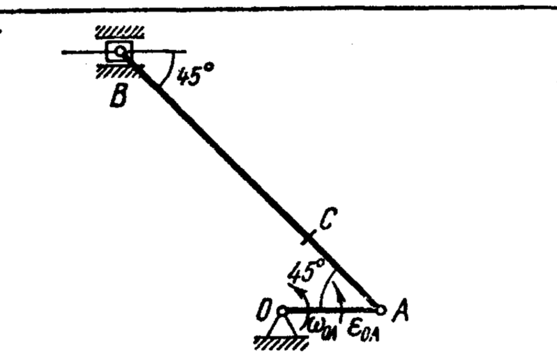

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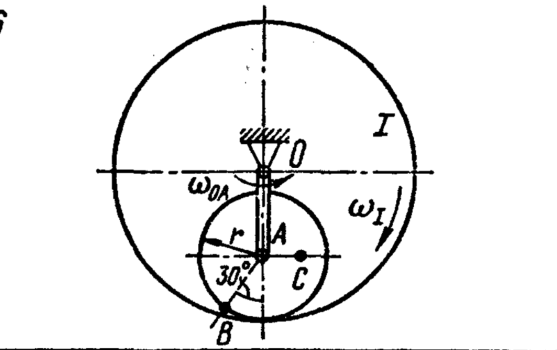

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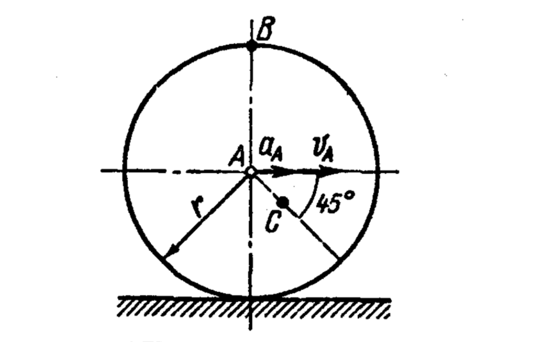

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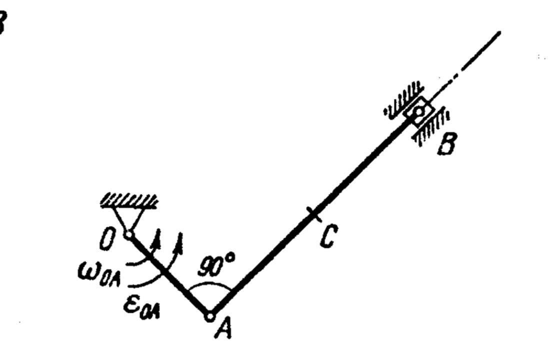

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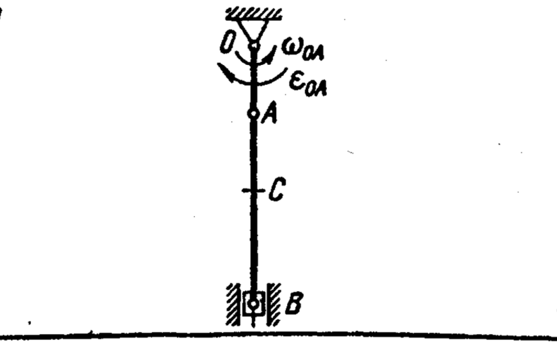

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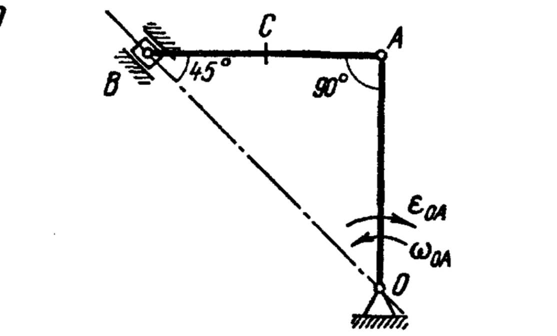

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Методичні рекомендації
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(Частина 2 Кінематика)

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Відповідальний за випуск *Н. В. Середа*
За авторською редакцією
Комп'ютерне верстання *А. О. Гарбуз*

План 2019, поз. 172 М

Підп. до друку 09.07.2019. Формат 60 × 84/16
Друк на ризографі. Ум. друк. арк. 1,1
Тираж 50 пр. Зам. №

Видавець і виготовлювач:
Харківський національний університет
міського господарства імені О. М. Бекетова,
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Свідоцтво суб'єкта видавничої справи:
ДК № 5328 від 11.04.2017.