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ELECTRICAL ENGINEERING
IN CONSTRUCTION

Tutorial

2-nd edition, corrected and enlarged

Edited by V. M. Okhrimenko

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The material of the 1st edition of the textbook "Електротехніка у будініцтві" (approved by the Ministry of education and science of Ukraine as a textbook for students of construction specialties of higher educational institutions, letter № 1/11-5516 from 22 June 2010) supplemented with the theme "Synchronous electric machines" and issues "Electrical measurements" and "DC machines". Basic facts from the theory of electrical circuits are given, transformers and electrical machines, basic electronics and electric drive were considered. Describes the electrical equipment of construction areas, enterprises of construction industry, engineering systems of buildings.

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CONTENT

INTRODUCTION

Modern construction production is characterized by a wide use of various equipment for the mechanization and automation of technological processes. Construction cranes, elevators, lifts, welding machines, lighting systems and other equipment is characterized by the use of electrical energy to realize their functional purpose. The relative density of electrical equipment as part of the engineering systems of modern buildings constantly widen. In these conditions, the engineer must understand the physics of electrical equipment, its characteristics, usage features, selection criteria and rules for safe exploitation.

By curriculums of construction specialties the study of discipline "Electrical engineering in construction" are considered. Part of the topics of this course are traditionally considered in several disciplines in electrotechnical specialties. This is the questions of the theory of electrical circuits, electrical machines, industrial electronics, electric drive. But the issues of electrical equipment of construction areas, engineering systems of buildings and the enterprises of construction industry are reflected only in the special literature.

The publication of this textbook aims to concentrate in a single source information on all subjects of discipline "Electrical engineering in construction". The contents of the book and the method of exposition of material is aimed primarily at students of building specialties. The book is intended also for engineering and technical personnel who are not specialists in the field of electrical engineering, but in practice solve the problems of the use and exploitation of electrical equipment and want to get an idea about the basic principles of its work. In addition, most sections of the book can be useful to students of technical specialties of educational institutions in the study of disciplines, in program of which issues of electrical engineering, electrical machines and electronics are included.

The practical part of the tutorial presents the tasks with example of their decisions and choices of initial data for independent decisions.

This edition of the textbook is prepared by a team consisting:

- A. Y. Achkasov – chapters 9, 17, 18, 22, section VII;
- V. A. Lushkin – chapters 1, 7, 12, 19, 20, section VII;
- V. M. Okhrimenko – chapters 3, 5, 6, 11, 13, 16, 21, general editing;
- T. B. Voronkova – chapters 2, 4, 7, 8, 10, 14, 15.

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SECTION I.
DC ELECTRICAL CIRCUIT

The first electrical instruments operated on direct current (DC). However, with growing needs for electrical energy failure to transmit over long distances has become evident being the main disadvantage of electrical power of DC.

Modern production, transmission, distribution and use of electricity are carried out mainly by the alternating current (AC) AC instruments. The widespread use of this power has been favoured by the opportunity of economic transfer of large amounts of electrical energy from the place of its production (power plants) to the places of its consumption (collectors). Such factors as comparative simplicity and economic efficiency of generators, motors and other AC instruments have been of crucial importance as well.

However, despite the dominant position of the alternating current in modern life, many consumers need DC. These are consumers, installation of which cannot operate on alternating current (e.g., electrochemical). In some branches of engineering installations of DC provide higher technical and economic indicators than those of alternating current.

DC instruments are also widely used at construction sites and enterprises of the construction industry, in transport, electrical vehicles, aerospace instruments, aircrafts, vehicles, automation systems and computer engineering.

1 BASIC TERMS AND CONCEPTS

Key concepts: electrical circuit, subcircuit, sources, consumers, electric current, electromotive force (EMF), voltage, direct current (DC), alternating current (AC), linear element, nonlinear element, unbranched circuit, branched circuit, resistance, inductance, capacity, mutual inductance, passive element, active element, connection scheme of circuit, equivalent scheme of circuit, active and passive two-terminal network, series and parallel connection, contour, node, branch.

1.1 Electrical circuit and its elements

Electrical circuit is a combination of devices intended for generation, transmission, transformation and use of electrical energy. Individual devices included in the circuit are called **the elements of an electrical circuit.** Part of the electrical circuit containing the selected elements in it is called **a subcircuit**.

Elements of a circuit intended for generation of electrical power are called **the sources of power** or **sources of electrical energy**, and the elements that use electrical energy are called **the electrical power receivers**.

In the power sources the electrical energy is obtained by conversion from other types of energy: mechanical, chemical, heat, light, etc.

In the receivers, on the contrary, the electrical energy is converted into other forms of energy: mechanical in electrical motors, chemical in batteries, heat in a variety of heaters and furnaces, radiant in the illuminators, etc.

Transmitting elements of the circuit are the link between the sources and receivers of electrical energy. Apart from electrical wires, various devices for monitoring, controlling and transforming (transformers, rectifiers, etc.), in which electrical energy is brought to the point where it becomes convenient to transmit over distances and distribute between receivers also belong to this group.

1.2 Classification of electric current, EMF and voltages

When the electrical circuit is closed and it has a source of electrical energy, then the movement of electric charge carriers is observed. This movement is called **electric current**. The magnitude or strength of the electric current is determined by the amount of electricity (charge) flowing through the cross-section per a unit of time.

The electric current whose magnitude and direction remain unchanged is called **direct current** (fig. 1.1, *a*) and denoted by a capital letter *I*. If for *t* seconds *q* coulombs of electric charge have passed, then DC power

$$I = q/t. \tag{1.1}$$

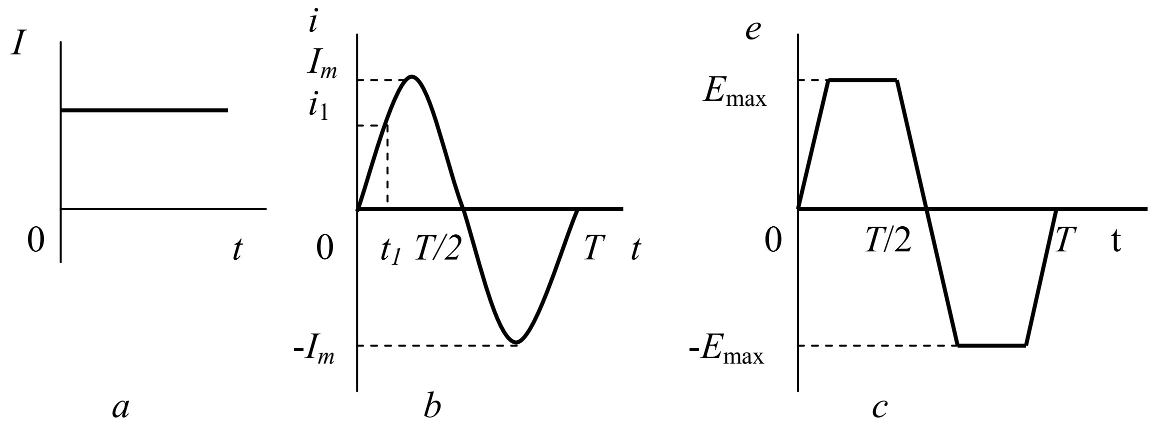


Figure 1.1: Various forms of the curves of direct and alternating currents (*a* and *b*) and EMF (*c*)

Then the charge, which carries a direct current for a time t,

$$q = I \cdot t\ . \tag{1.2}$$

The electric current whose magnitude or direction does not remain constant is called the **changing** or **alternating current**. Values of the alternating current at a given moment of time are called the instantaneous values and denoted by the lowercase letter *i*. The current *i* is associated with the charge *q* and the time *t* proportion

$$i = dq/\,dt\ . \tag{1.3}$$

For the period of time from *0* to *t* changing current transfers charge

$$q = \int_0^t i \cdot dt. \tag{1.4}$$

The main electrical unit in the International system of units (SU) is the unit of current, the Ampere (A). The charge in this system is measured in Ampere-seconds (A·S), or Coulombs (CL). Charge 1 *CL* corresponds to the charge of $6.29 \cdot 10^{18}$ electrons. If the current in 1 A a charge equal to the charge of $6.29 \cdot 10^{18}$ electrons passes through the cross-section of the conductor for 1 second.

The forms of curves of changing currents are very diverse. Those known as periodic currents are dominant. ***Periodic current*** is a current, the instantaneous values of which are repeated at regular intervals of time. The smallest period of time, after which the instantaneous values of current are repeated, is called the ***period*** and is denoted by T. The number of periods in one second is called the ***frequency*** of the periodic current. Frequency is measured in Hertz (Hz) and is denoted by f. Frequency and period are related in the following way

$$f = 1/T. \tag{1.5}$$

The current changing in accordance with the harmonic law, is called ***sinusoidal current*** (fig. 1.1, *b*). A sinusoidal current of industrial frequency of 50 Hz is called ***alternating current***, although in the theory of circuits every changing current refers to AC.

If the law of change in the instantaneous values of periodic current (voltage) is different from the harmonic, the current (voltage) is called ***non-sinusoidal*** (fig. 1.1, *c*).

The greatest value of the sinusoidal current is called the **amplitude** and denoted by capital letter I with subscript m (I_{m}). The highest value of non-sinusoidal current is denoted by a lowercase letter i with subscript *max* ($i_{\max}$).

Direct or alternating currents occur in electrical circuits under the action of an **electromotive force** (EMF) induced in the sources in the process of converting any form of energy into electrical energy in them. EMF and voltage (similar to currents) in accordance with the law of change of their instantaneous values are called constant, variable, sinusoidal and non-sinusoidal. Constant EMF and voltage are denoted by capital letters E and U, and changing EMF and voltage are denoted by lowercase letters e and u. The unit of measurement of electromotive force and voltage is the **volt** (V).

A set of values that characterize the impact on circuit EMF and the resulting impact of the voltages and currents, determines the mode of operation of the circuit and its components.

1.3 Elements of electrical circuits and their graphic pictures

The electric circuit depends on its character and it is called: "**the electrical circuit of DC**", "**electric circuit of alternating current**". In some cases there may be further clarification: "**electrical circuit of sinusoidal (non-sinusoidal) current**".

There are definitions of elements of circuits: "DC electric machine, AC electric machine ", "DC source", "receiver AC", etc.

The elements of the circuits and circuits which are composed of them are divided in accordance with such characteristics, expressing, for example, the dependence of current on applied voltage $I(U)$ – ampere-volt characteristic (AVC). Examples of such characteristics are shown in figure 1.2. Circuit elements, AVC which is linear, is called **linear elements**. The nonlinear characteristics are **nonlinear elements**.

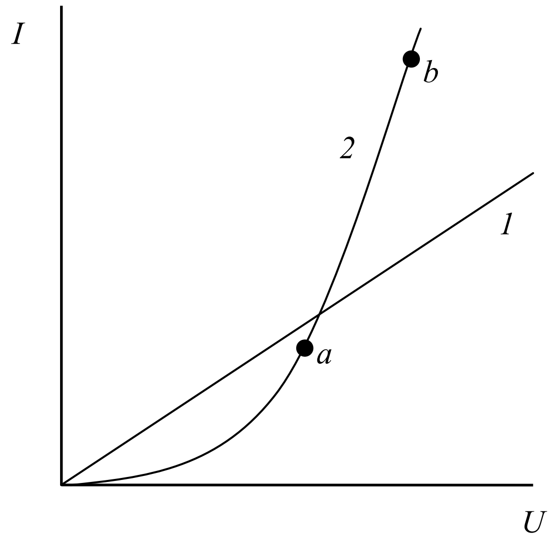


Figure 1.2: Characteristics of the elements of circuit: 1 – linear, 2 – nonlinear

The electrical circuits of DC and AC currents are distinguished also by the way of connection of these elements. There may be ***unbranched and branched circuits***, on the number of sources of electrical energy – circuits with one or more sources of electrical energy. There are other names of circuits.

Electrical circuit consisting of linear elements, is called a **linear circuit**. An electrical circuit that includes at least one nonlinear element, is called the **nonlinear circuit**. Calculations of modes of nonlinear circuits are more cumbersome, so, for simplicity, they execute on the linear sections of the characteristics of nonlinear elements (section *a*-*b* of characteristic 2 in fig. 1.2).

Receivers of electric energy, as the elements of an electrical circuit, have the properties to absorb electrical energy from the circuit and convert it into other forms of energy (*irreversible process*), to create their magnetic and electric field energy which can be accumulated and, under certain conditions, to return to the circuit (*reversible process*). To characterize these properties, it is introduced the concept of the parameters of the element. There are the following parameters of the element of circuit: resistance, inductance and capacitance.

Resistance (R) is a parameter that characterizes a property of the element to absorb energy from the electric circuit and convert it into other forms of energy (heat or light). It is known that the power (P, p) conversion of electrical energy is proportional to the square of the current (I^2, i^2), so the value of this parameter of resistance is determined by the ratio $R = P/I^2$ for DC and $R = p/i^2$ for AC. The unit of resistance is Ohm.

The properties of the element to create their own magnetic field (field of induction), and to contain electric current is characterized by the parameter of inductance L. The ***inductanc*e** is a coefficient of proportionality between the current (I, i) and flux linkage (ψ, ψ_t) of this device: $\psi = L{\cdot}I$ or the $\psi_t = L{\cdot}i$. It is called the coefficient of self-inductance and is measured in Henry (H).

The mutual inductance parameter M characterizes the property of the first element with a current i_1 to generate a magnetic field, partly linked to the coils w_2 of the second element. Flux linkage ψ_{21} of the second element (the first index), due to the current i_1 of the first element (the second index), referred to as flux linkage of mutual induction of the second element. The parameter of ***mutual inductance*** M is the coefficient of proportionality between the current of first element and created by this current flux linkage of second element: $\psi_{21} = M \cdot I_1$ or the $\psi_{21t} = M \cdot i_1$.

There is a similar connection between flux linkage of first element, caused by the current of the second element: $\psi_{12} = M \cdot I_2$ or $\psi_{12t} = M \cdot i_2$.

Capacity (C) is a parameter that characterizes a property of the element to accumulate charges or actuate them to the electric field. This parameter is a coefficient of proportionality between voltage and battery of element: $q = C \cdot U$.

In the general case, ***any real device*** has all three parameters R, L and C.

The main property of the source of electrical energy is the ability to create and maintain the potential difference at some subcircuits, as well as initiate and maintain an electric current in a closed circuit is characterized by its ***electromotive force*** (EMF). The magnitude of the EMF (E, e) is equal to the energy that acquires a positive charge in the amount of 1 coulomb, moving under the action of external forces on the source. If during the time dt by the AC power source the charge passes $dq = i \cdot dt$, then developed by a source of energy $dW = e \cdot dq = e \cdot i \cdot dt$ and instantaneous power $p = dW/dt = e \cdot i$. For constant current source developed energy and power are respectively equal to: $W = E \cdot I \cdot t$ and $P = E \cdot I$.

The passage of current through the source is accompanied by a loss of energy inside source for heating. These losses are characterized by the ***parameter resistance*** R. Therefore, the parameter resistance along with the EMF is the most important parameter of the source. In some cases, the AC power sources also consider the option of inductance L.

The elements of the electric circuit, which can be described using the parameters R, L, M, and C, are called ***passive***. The term "passive" emphasizes that such elements cannot serve its purpose without exposure to extraneous sources.

The circuit elements, which are used in addition to the passive parameters and EMF or current are call ***active***. The active elements include all sources of electrical energy and some receivers, with the description of processes which cannot be limited by passive parameters (batteries when charging, the DC motors and others).

Elements of a circuit that has only one parameter is called the ideal. The ideal source of EMF has only the parameter E (the magnitude of the EMF – fig. 1.3, *a*) ideal current source – only parameter J (the magnitude of the current – fig. 1.3, *c*), ideal inductive element (ideal inductive coil) – only parameter L (fig. 1.3, *d*), ideal capacitive element (ideal capacitor) – only parameter C (fig. 1.3, *e*), only one parameter of resistance R has a resistance element (resistor) (fig. 1.3, *f*). In the general case, any real passive device has three parameters R, L, C.

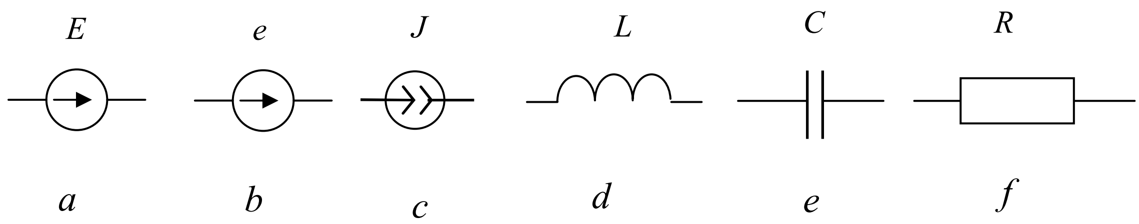


Figure 1.3: The graphic symbols of ideal elements: source EMF of constant current (*a*); source of EMF of the alternating current (*b*), source voltage (*c*),

inductance (*d*), capacitance (*e*), resistor (*f*)

Sources of electrical energy are divided into sources of EMF and current sources, the equivalent scheme of which is shown in figure 1.4. The properties of the source of electrical energy is determined by its current-voltage (external) characteristic. It is the dependence of the output voltage from the voltage *U*(*I*).

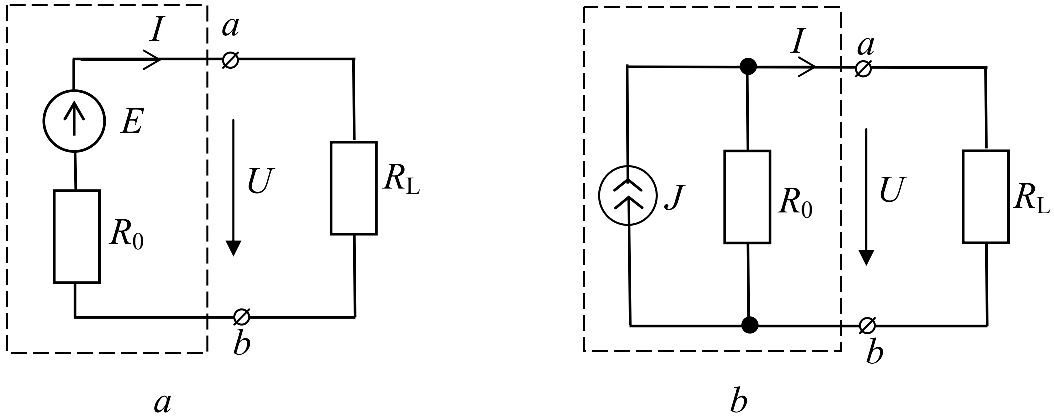


Figure 1.4: Equivalent scheme of sources of EMF (*a*) and current (*b*)

If the internal resistance in the scheme of the circuit of the electric power source R_0 is small in comparison with the load resistance R_L, it leads the inequality $R_0 \cdot I << E$. In this case, the voltage between the terminals of the source of electrical energy practically does not depend on the current, i.e., $U = E = const$, and the source is called a ***source of EMF***.

Source with low internal resistance can be replaced by idealized model in which $R_0 = 0$. Such a source is called an ***ideal source of EMF*** with one parameter $E = U$. The voltage on output connection terminals of an ideal source of EMF does not depend on the current, and its external characteristics has the form of a straight line $U = E = const$ (fig. 1.5, *a*).

If in the scheme of circuit the internal resistance of the source of electrical energy is many times greater than the load resistance ($R_0 >> R_L$), its current $I = E/(R_0 + R_L) \approx E/R_0 = J = const$ does not depend on the load resistance and the source is called a ***current source***.

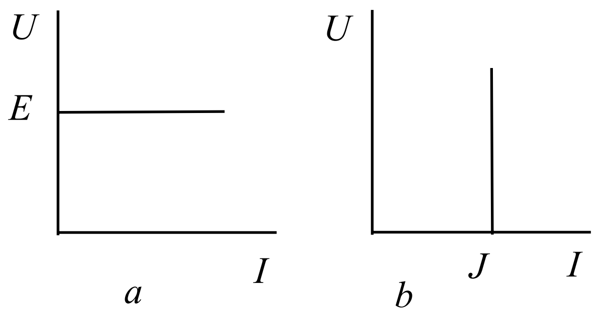


Figure 1.5: External features:
a – an ideal source of EMF;
b – ideal current source.

Source with a high internal resistance, you can replace by the idealized model, where $R_0 = \infty$ and $E = \infty$ and for which the true expression $E/R_0 = J$. This source is called an ideal current source with one parameter *J*. Current of the current source does not depend on the voltage at its output connection terminals, and its external characteristics has the form of a straight line $I = J = const$ (fig. 1.5, *b*).

Picture of an electrical circuit using conventional pictures of its elements is called a ***circuit scheme***. Elements of electrical circuits in the schemes portray using conventional graphic symbols. Schemes allow you to get a visual representation of the structure of the electric circuit, the join order of its elements.

Figure 1.6 as an example, here is the connection scheme of the simplest electrical DC circuit consisting of a source (battery) E with internal resistance R_0, ammeter PA, switch S and receiver (light bulbs HL).

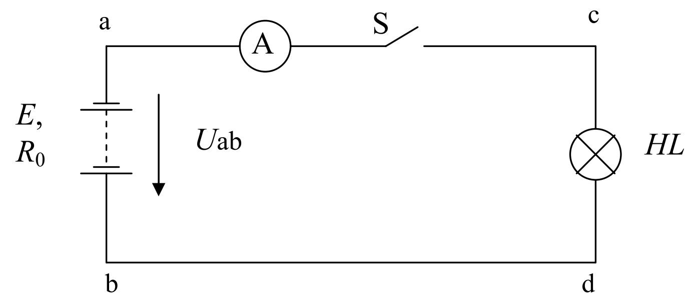


Figure 1.6: Connection scheme of the simplest electrical circuit

The study of the properties of the electric circuit and its elements, taking into account all parameters is very difficult and to simplify the analysis the real circuit is presented by its model – the set of ideal elements.

A graphic image of a circuit using the ideal elements, parameters of which are the parameters of the replaced elements , called the ***equivalent scheme of circuit***.

The ideal elements introduced into the equivalent scheme in order to account for phenomena that characterize their parameters and have a significant impact on physical processes in the circuit. The parameters of the real elements that do not have a significant impact on physical processes are not taken into account. For example, to account for the irreversible process of energy absorption of element of circuit in the equivalent scheme enter the resistive element. Inductive and capacitive elements can be added in the equivalent scheme in the case when you want to take into account the influence of the fields.

The same element of the circuit can be represented in a different equivalent schemes depending on the purposes for which designed this scheme. For example, the inductive coil in a DC circuit to account for heat is presented in the equivalent scheme only by one resistive element. But in the study of physical processes in the coil with the changeable currents, it is represented by series-connected resistive and inductive ideal elements. And in the case of a work coil in high frequency circuits, for it form the equivalent scheme of resistive, inductive and capacitive elements.

When developing design and construction documents for electrical equipment apply electric schemes, which, in contrast to the equivalent schemes, are performed in strict accordance with working for this moment standards, for example standards ***USDD*** – Unified system for design documentation. In the development of standards on rules of graphic documents take into account the recommendations of international organizations: ISO (international organization for standardization), IEC (international electrotechnical Commission) and other.

It is distinguished an ***electrical principles schemes***, ***structural***, ***functional***, and ***assembly***.

As an example in figure 1.7 it is shown the equivalent scheme AC circuit consisting of a source of variable electromotive force *e*, capacitor *C*, lighting lamps *HL* and the rheostat *R*. On example of this scheme let's consider the names of the individual sections and the entire circuit.

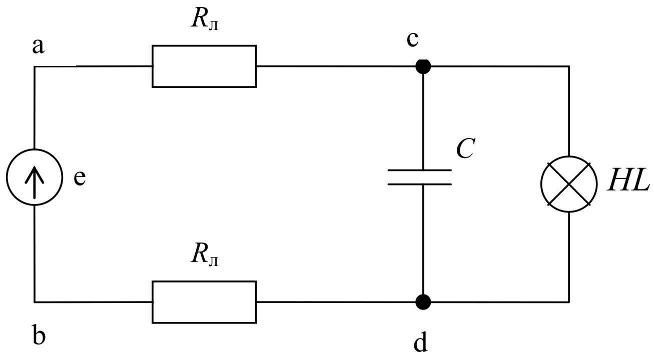


Figure 1.7: Scheme of circuit of AC current

The power source forms an ***internal subcircuit***, and the receiver together with the connecting wires – the ***external subcircuit*** or an external circuit. Connection terminals (poles) *a* and *b* of the source to which you attach an external circuit, referred to as ***output connection terminals*** (poles) of the source. Connection terminals *a* and *b* are both input connection terminals of the external circuit. In some cases, the external circuit is divided into two characteristic sections: power line (connecting wires) and the electricity consumers (receivers). Figure 1.7 sections *a-c* and *b-d* form a transmission line with resistance $2{\cdot}R_l$, and section *c-d* with connected capacitor and lamp HL is the collector. In this case, the connection terminals *c-d* are input connection terminals of the collector.

The connection terminals of the external circuit by means of which it attached to the wires coming from the source, called the ***input connection terminals*** (poles) of the external circuit.

Part of a circuit that has two poles, is called two-terminal network. There are the ***two-terminal networks active*** (containing sources) and ***passive*** (not containing sources).

The connection of circuit elements, in which in all sections the same current passes, called a ***series connection***. Any closed path across multiple sections, called the ***contour of the electric circuit***. The circuit scheme of which is shown in figure 1.6 is a single-loop circuit consisting of series-connected elements.

The subcircuit, along which at any point in time, the current has the same value, called the ***branch***, and the connection of three or more branches – ***node***.

Connection in which two or more branches connected to one pair of nodes, referred to as a parallel connection. Scheme figure 1.7 has two nodes c and d which are connected to the two receivers (the lamp and the capacitor) connected in parallel. Parallel connected sections are under the same tension. Electrical circuit with series and parallel branches are also called branched circuit. Branched circuit is multi-circuit. Contours that differ from each other in at least one branch, are independent. The scheme in figure 1.7 is a double-circuit.

1.4 The positive directions of the currents, EMF and voltages

For the unambiguous description of the processes taking place in any element of the circuit, it is necessary to know not only the values of current and voltage, but also their direction at a given moment of time. Indeed, if it is a graph of current (see fig. 1.1, *b*) or equation, it can be argued that the current through the half-period changes its direction on the return and the value of it for the moment it is positive, but the direction of the current in the element at this point is unknown. To answer this question, one of the two possible directions of current in the element take for basic and indicate it on the scheme by the arrow. The ***arrows*** placed on the schemes ***indicate the positive directions*** of EMF, voltages and currents, i.e. such directions, for which values are denoted by positive values.

If, for example, at a given point of time the values are positive: $E > 0$, $U > 0$, $I > 0$, $e > 0$, $u > 0$, $i > 0$, then the actual direction in this time coincide with the directions indicated in the scheme by arrows. If the values of these values is negative, then their actual directions are opposite to the directions indicated on the scheme by arrows.

Consider the examples of circuits and explain the value placed on their schemes arrows. Figure 1.8 shows the schemes of connections of two of the simplest circuits: DC (*a*) and variable (*b*) currents, consisting of an ideal power sources with EMF *E* and *e* and electric lamps *HL*, directly connected to the external connection terminals of the sources.

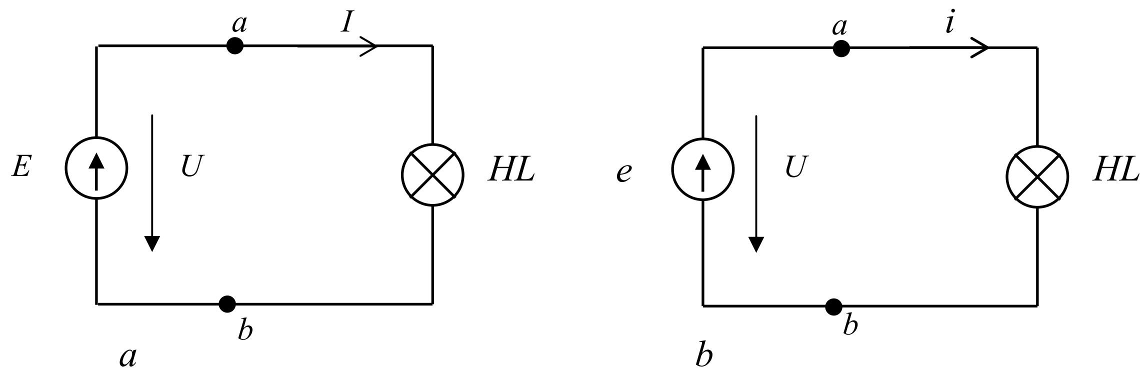


Figure 1.8: Connections schemes simple circuit of DC (a) and AC (b) currents with the ideal power sources

In figure 1.9 it is given equivalent schemes of these circuits, in which an electric lamp presents a resistive element with a resistance *R*. In figure 1.10 it is shown graphs of EMF of power sources.

From physics it is known that the direction of movement of the positive charges is accepted as a positive direction of the current, the direction of external forces to the positive charge is taken as the positive direction of the EMF, the direction of decreasing potential is considered to be the positive direction of the voltage. Since the positive charges inside the source are moving in the direction of the action of external forces, and in the receiver they are moving in the direction of decreasing potential, the positive directions of the current and the EMF of source, the current and voltage of the receiver are the same.

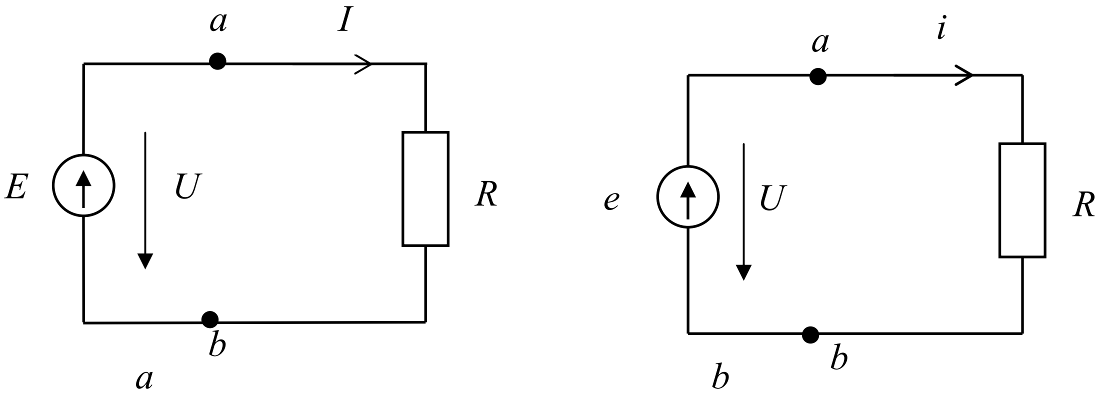


Figure 1.9: Equivalent scheme of simple circuit of DC (*a*) and AC (*b*) currents with the ideal power sources

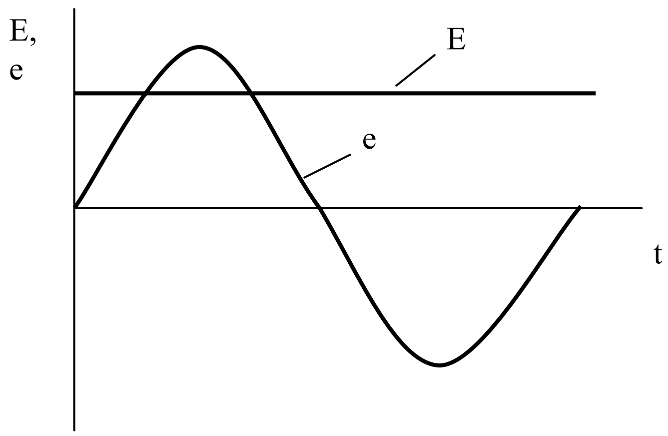


Figure 1.10: Graphics EMF of power supply: constant E and sinusoidal e

The positive direction of the voltage on the external connection terminals of the source is opposite to the positive direction of the current source. Therefore, the positive charges inside the source at this point of time, move in the direction of increasing potential, and their energy is increased by the value $q \cdot E = W \cdot e$ or $q \cdot e = W \cdot e$. At the same time in the receiver the positive charges are moving in the direction of decreasing potential, and their energy decreases by the value of $q \cdot U$ or $q \cdot u$. The power developed by a source $P = E \cdot I$ or $p = e\ i$ and the power of a subcircuit $R = U \cdot I$ or $p = u \cdot i$ will be positive only at coincident positive directions of EMF and current sources, as well as voltage and current of the receiver.

1.5 General rules of implementation of electrical schemes

The schemes are used to study the principle of governing the operation of machines, mechanisms and devices on their adjustment, installation and repair, to clarify the relationship between the parts of the equipment without specification of their design.

Rules for the implementation and execution of schemes regulate the standards of USDD. Types of schemes, general requirements for their implementation must comply with state standard 2.701-84 "USDD. The schemes. Kinds and types. General requirements", the rules of execution of all types of electric schemes – state standard 2.702-75 "USDD. The rules of execution of electric schemes."

Schemes are included in the set of design documentation and together with other documents contain necessary data for the design, manufacture, assembly, adjustment and operation of the equipment.

Graphical and alphanumeric characters are used both for representation of the electric components or devices and abbreviation information about them on the schemes. Conditional graphical symbols and rules for their construction are established by a group of USDD standards.

Alphanumeric characters and the rules of their construction are regulated by state standard 2.710-81. Uppercase letters of the Latin alphabet and Arabic numerals are used to build signs. Signs are written as a sequence of letters and numbers in one line without spaces. In table 1.1 an example of letter symbols of some elements of electric schemes is given.

It should be noted that the standards of USDD are applied to electrical schemes belonging to the design documentation. The academic literature in electrical engineering presents the equivalent scheme signs that are traditionally accepted by many authors but may differ slightly from the requirements of USDD.

For example, in the equivalent schemes the source of EMF is denoted by a lowercase letter E in the Latin alphabet, and in accordance with USDD – it is denoted by G (see tabl. 1.1). Graphical and letter symbols for electric schemes will be given upon further description of physical processes and the principle of work of separate electrical devices, machines and vehicles.

Table 1.1: Letter symbols of the basic elements of electric schemes

The first letter of the code (mandatory)	**Group of elements**	**Examples of the types of items**	**Two-letter code**
C	Capacitors		
G	Generators, power supplies	Battery	GB
K	Relays, contactors, starters	Current relay Voltage relay	KA KV
L	Inductors, chokes	Chokes of luminescent lighting	LL
M	Motors AC and DC		
P	Devices, measuring equipment	Ammeter Voltmeter Wattmeter	PA PV PW
R	Resistors		
S	Devices switching	The switch or jumper Switch automatic	SA SF
T	Transformers, autotransformers	Current transformer Voltage Transformer	TA TV

More detailed rules for the implementation of electrical schemes can be found in the literature [44].

Key findings

1. The main elements of the electric circuit are the source of electrical energy, the consumer (receiver) of electrical energy and transmitting elements.
2. Electric circuit with alternating (sinusoidal) current and EMF are most commonly used.
3. The electric circuit elements are divided into active and passive.
4. Resistance, inductance and capacitance are the main parameters of the passive elements
5. There are no parameters of the passive elements of capacitance and inductance in DC.
6. The equivalent scheme of the electric circuit is a model of the real circuit that allows you to perform calculations of the parameters of the elements and processes in the circuit.

Control questions

1. What is meant by an electric circuit? By the subcircuit?
2. What are the basic elements of an electric circuit and what are their purposes?
3. What is electric current?
4. What are the different types of electric current? In what units is it measured?
5. What is EMF? In what units is it measured?
6. What is difference between the amplitude of the alternating current and the maximum value of AC? What is the difference between the currents themselves in this case?
7. Give an example of an unbranched (branched) electric circuit?
8. What is the difference between nonlinear and linear electrical circuit?
9. Define the parameter of electrical circuit resistance. What units are used to measure resistance?
10. Define the parameter of electrical circuit inductance. What units are used to measure inductance?
11. Define the parameter of the electric circuit of mutual inductance.
12. Define the parameter of electrical circuit capacity. In terms of what is capacity measured?
13. Explain the difference between active and passive elements of an electrical circuit.
14. What is the ideal element of an electric circuit? Give examples.
15. What does the contour of the electric circuit mean?
16. What is node in the electric circuit?
17. What directions of the currents and EMF are considered to be positive?

2 PHYSICAL PROCESSES IN DC CIRCUIT

Key concepts: Ohm's law, the modes of the DC source (nominal, no-load, short circuit, agreed), short circuit current, work of electric current, power of electric current (full, useful, losses), coefficient of performance, sequential (parallel) connection elements, the first Kirchhoff's law, the second Kirchhoff's law, independent contour.

2.1 Ohm's Law

In 1827, a German scientist Ohm experimentally deduced the law establishing the relationship between the three main parameters of electric circuits: current, voltage and resistance

Ohm's law for the entire circuit states the following: ***the current in an electric circuit is directly proportional to the magnitude of the EMF of the source and inversely proportional to the impedance of the circuit***

$$I = \frac{E}{R_F} \ , \tag{2.1}$$

where: $\boldsymbol{I}$ is the current strength in the circuit, A;
$\boldsymbol{E}$ is the EMF of the source, V;
$\boldsymbol{R_F}$ is the full resistance of circuit, Ohm.

The full resistance R_F represents the sum of the resistance load R_{LD} (external circuit resistance), resistance line R_L (resistance of connecting wires) and the internal resistance of the source R_0:

$$R_F = R_{LD} + R_L + R_0 \ . \tag{2.2}$$

For individual subcircuit Ohm's law is:

$$I = \frac{U}{R}, \tag{2.3}$$

where: $\boldsymbol{I}$ is the current strength in the subcircuit, A;
$\boldsymbol{U}$ is the voltage (voltage drop) at the section V;
$\boldsymbol{R}$ is the electrical resistance of the circuit, Ohm.

The physical meaning of this relation is that the larger the potential difference at the boundaries of the section of the circuit (the more the voltage drop across it), the more current at a given resistance value. If we change the resistance value on the constant voltage applied to the circuit, the current will vary inversely proportional to the resistance value.

From Ohm's law it follows that

$$U = I \cdot R, \tag{2.4}$$

i.e., the voltage drop (often referred to as ΔU) in the subcircuit is directly proportional to the current and the resistance value. Therefore, if we know that at some subcircuit with a constant resistance the current strength has increased, it means that the voltage drop in this section has increased too.

From Ohm's law, it also follows that

$$R = \frac{U}{I} \ . \tag{2.5}$$

This ratio allows us to calculate the resistance value of the subcircuit by known values summed to it voltage and amperage. Note that this ratio cannot be considered as the dependence of the electrical resistance from the voltage drop or amperage. From physics [23] it is known that the electrical resistance of a conductor depends on its geometrical dimensions and temperature. However, the expression (2.5) gives the opportunity to establish empirically the value of this resistance on the measured amperage and voltage drop on it.

2.2 Operation modes of the constant current source

Let's consider the nature of changes in the voltage at the connection terminals *a* and *b* of the DC voltage source *E* with internal resistance R_0 depending on the size of the load R_L (see fig. 2.1).

Due to the fact that the EMF source (fig. 2.1) is equal to the sum of the voltage drops on the outer R_L and internal R_0 subcircuit resistance

$$E = U + U_0 \tag{2.6}$$

we can record

$$U = E - U_0 . \tag{2.7}$$

Using Ohm's law, the magnitude of the internal voltage drop on the source can be expressed through the load current I_L and the internal resistance of the source R_0

$$U_0 = I_L \cdot R_0 . \tag{2.8}$$

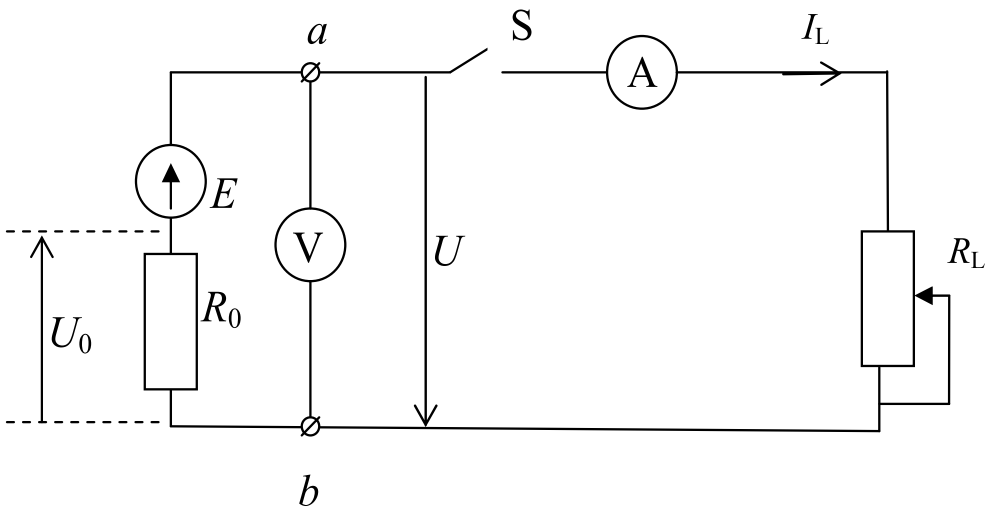


Figure 2.1: A simple DC circuit

After substituting (2.8) into (2.7) we get

$$U = E - I_L \cdot R_0 . \tag{2.9}$$

Expression (2.9) determines the dependence of the connection terminal source from the load, on condition that the magnitude of the EMF and the internal resistance of the source are constant.

Let's consider the possible modes of operation of the electric power source and the nature of changes in the magnitude of its voltage when the load changes.

2.2.1 Idle mode. Idle mode is considered to be the work of the source of electric energy when load is disconnected (fig. 2.1, the key S is open). Then the resistance of the external circuit is endless ($R_L = \infty$) and the current in the circuit is equal to zero:

$$I_L = \frac{E}{R_L + R_0} = \frac{E}{\infty + R_0} = 0\,. \qquad (2.10)$$

The ***idling voltage*** U_{idle} on the connection terminals of the source will be:

$$U_{idle} = E - 0\ R_0 = E\,. \qquad (2.11)$$

i.e. the open-circuit voltage is equal to the emf. source, Therefore at idle, the voltmeter connected to the connection terminals of the source, shows its electromotive force.

2.2.2 The short circuit conditions. The short circuit condition of the source is created when the value of the load resistance is practically zero (fig. 2.1, the resistor R_L is in the upper position). Then the resistance of the electrical circuit is minimum and equal to the resistance of the connecting wires (on the equivalent scheme in fig. 2.1 it is not shown), and the current in the circuit reaches its maximum value, called the ***short-circuit current***:

$$I_{SC} = \frac{E}{0 + R_0} = \frac{E}{R_0}\,. \qquad 2.12)$$

At it the voltage at the connection terminals of the source will be zero:

$$U = I{\cdot}R_L = I{\cdot}0 = 0\,, \qquad (2.13)$$

and the voltage drop inside the source will be equal to its EMF:

$$E = U + U_0 = 0 + U_0 = U_0\,. \qquad (2.14)$$

It must be emphasized that the short circuit condition for the most sources of electrical energy is invalid (malfunction), as the internal resistance sources are typically small and the resulting short-circuit current reaches significant values, leading to its phase out.

The protective devices (such as protectors, circuit breakers), which disconnect the circuit in case if the current exceeds the limit values, are included in the circuit to protect sources of electrical energy from short circuits.

2.2.3 The mode of work on load. Any consumer of electrical energy can be a load of source. Its resistance R_L determines the magnitude of the load current:

$$I_L = \frac{E}{R_L + R_0}\,. \qquad (2.15)$$

The less the load resistance, the greater the load current must be, so the more the load on the source is.

From (2.9) we see that the increase in load current causes the decrease in voltage at the connection terminals of the source. This dependence is linear (because we consider that the magnitude of the EMF and the internal resistance of the source are immutable). Its schedule is presented in figure 2.2.

Point A corresponds to the idling: $I = 0$, $U = E$, $U_0 = 0$.

Point B corresponds to the short circuit conditions: $I = I_{SC}$, $U = 0$, $U_0 = E$.

For any intermediate value of the load current namely I_1 or I_2, the voltage at the connection terminals of the source is less of its EMF on the value of the voltage drop inside the source:

$$U_1 = E - U_{01} = E - I_1 \cdot R_0 \text{ ; } U_2 = E - U_{02} = E - I_2 \cdot R_0 \text{ .}$$

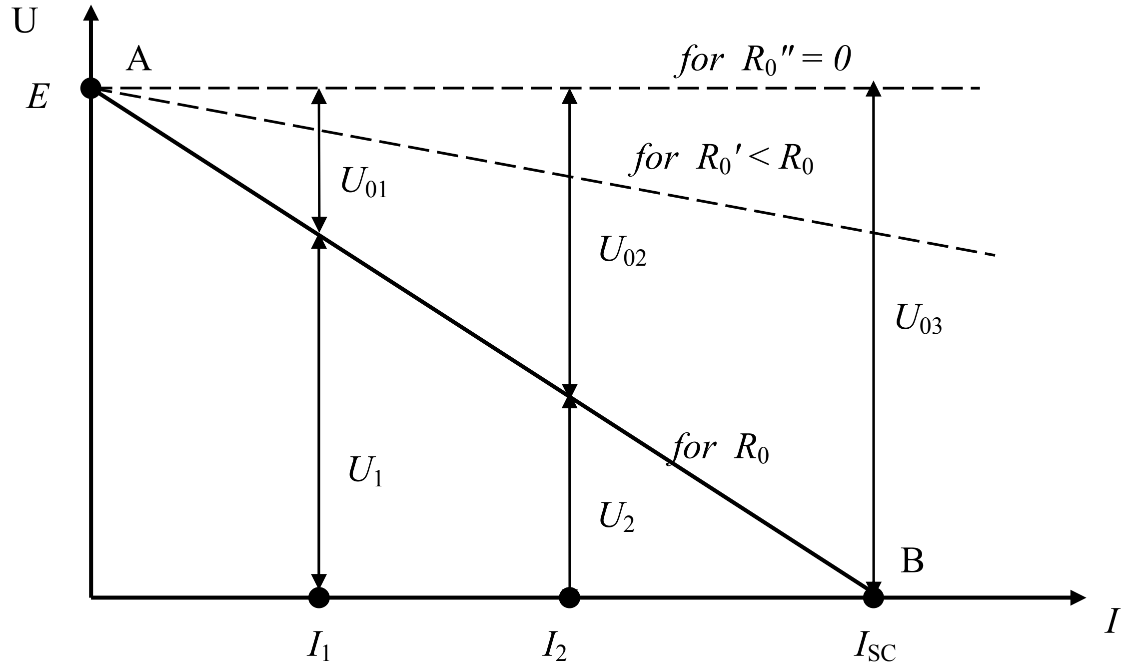


Figure 2.2: The dependence of the voltage source from the load current

Thus, when the load changes, the voltage applied to the consumer, is changing all the time that in most cases is undesirable. To ensure maximum stability of the voltage, it is necessary to achieve the minimum internal resistance of the source, then under the same limits of variation of the load current the changes of the voltage will be lower (fig. 2.2 dotted line $R_0' < R_0$).

2.2.4 The nominal conditions. The nominal source conditions are characterized by the fact that the voltage, current and power of it correspond to the values for which it was intended by the manufacturer. This guarantees the best working conditions (economy, durability, etc).

The values that define the nominal conditions are usually indicated in your passport or on a plate attached to the device. These data are based on the calculations of the electrical schemes.

The main nominal data of electrical devices are their ***rated voltage and current***: U_R and I_R. ***The insulation of the electrical wires is calculated on the nominal voltage, but the conditions of their maximum permissible heat are calculated on the nominal current***.

2.2.5 Agreed mode. The mode in which it gives the greatest power to the external circuit is called agreed mode of the source.

2.3 The generalized Ohm's law for subcircuit

In addition to the cases described in chapters 2.2.1 – 2.2.5, there is another approach to the analysis of the physics of source operation mode. This approach takes into account ***the factor*** if a source gives or consumes electrical energy. The work of a battery can be an example. On charging it works in the mode of consumption of electrical energy, while powering the light bulbs it works in the mode of electric power source.

We will consider this issue on the example of two active subcircuits the schemes of which are shown in figure 2.3, *a* and *b*. To determine the general properties of the mode of the subcircuits let's make the equation of their electrical state.

The first source of EMF works in the mode of an energy source because the directions of the EMF E_1 and current I_1 are the same. The second source of EMF works in the mode of a power receiver, so as the directions of the EMF E_2 and current I_2 are opposite. The directions of the voltage of the resistive elements of the sections coincide with the direction of the current therefore all of these elements operate as receivers. The direction of the common voltage U_{ab} of the first section which is opposite to the direction of the current I_1, allows to conclude that the first section generates electrical energy in the external circuit, which is connected to its connection terminals *a* – *b*.

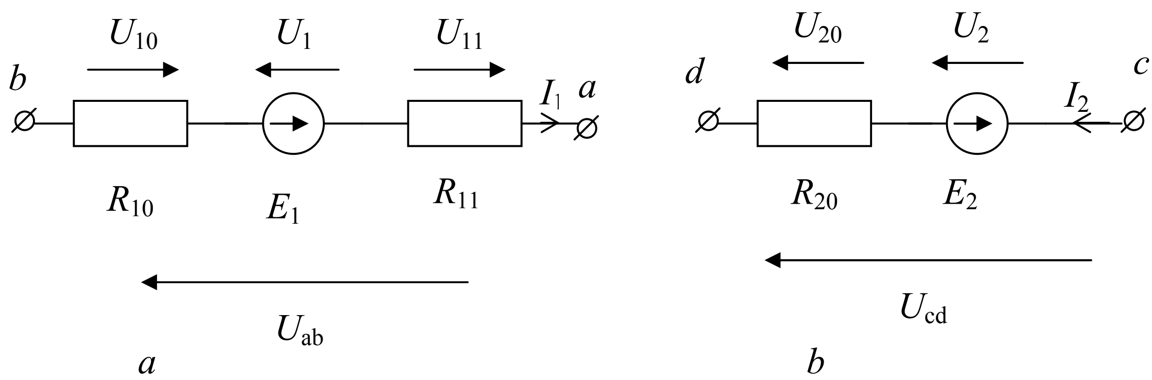


Figure 2.3: The schemes of two active subcircuits:
a – with opposite directions of voltage U_{ab} and current I_1;
b – with the same direction of voltage U_{cd} and current I_2

Coincident directions of the common voltage U_{cd} and current I_2 of the second section point to this section as the active receiver.

The tension of the section is equal to the algebraic sum of the voltages of the elements in it; however, with the sign "+" the equation includes voltage, the directions of which coincide with the assignable voltage, and the sign "-" is a voltage with opposite directions:

$$U_{ab} = - U_{11} + U_1 - U_{10} = U_1 - (U_{11} + U_{10}) ,$$

$$U_{cd} = U_2 + U_{20} .$$

Replacing the voltage of the passive elements by the voltage drops and voltage sources by their EMF, we get

$$U_{ab} = E_1 - (R_{11} + R_{10})\cdot I_1 , \quad (2.16)$$

$$U_{cd} = E_2 + R_{20}\cdot I_2 . \quad (2.17)$$

From the equation (2.16) it follows that the actual direction of the stress section would be the same as the direction which was conventionally selected $U_{ab} > 0$ and marked by the arrow in the scheme, provided that

$$(R_{11} + R_{10})\cdot I_1 < E_1 .$$

This can occur when the load current I_1 of the section is less than the short circuit current $I_{1SC} = E_1/(R_{11} + R_{10})$ of this section.

Therefore, the ***active section gives the energy to the external circuit when the actual direction of the current and voltage is the opposite,*** i.e., when $I_1 < I_{1SC}$.

When $I_1 > C$ voltage $U_{ab} < 0$ and its actual direction coincide with the direction of the current section, i.e. the mode of the receiver takes place.

The equation (2.17) shows that the second section characterizes the mode of the receiver when $R_{20}\cdot I_2 > 0$ or $I_2 > 0$, i.e. when the $U_{cd} > E_2$.

Thus, ***active section with the same positive directions of current and voltage is in the mode of the receiver when connected to it voltage is greater than the EMF of the subcircuit***.

When $U_{cd} < E_2$, the current $I_2 < 0$ and its actual direction are opposite to the indicated one by the arrow on the scheme. In this case this section describes the generating device which gives energy to the external circuit.

To obtain the general form of the equation that links current, EMF, voltage and resistance of the active section, let's determine the currents of the given sections from the equations (2.16) and (2.17):

$$I_1 = \frac{+E_1 - U_{ab}}{R_{11} + R_{10}} ,$$

$$I_2 = \frac{-E_2 + U_{cd}}{R_{20}} .$$

Based on these equations we can write the general equation for the current of the active section:

$$I = \frac{\pm E \mp U}{R} . \quad (2.18)$$

This equation expresses the ***generalized Ohm's law for an active section,*** whereby the ***current of the active section of the circuit is equal to the algebraic sum of the voltage and the EMF divided by the resistance of the section***.

EMF and voltage is taken with the sign "+" if their directions coincide with the direction of the current, and the sign "-" when their direction is opposite to the direction of the current.

2.4 Work and power of electric current

2.4.1 Working electric current. Work on moving charges is performed when electric current passes through the conductor. From physics it is known that the work done by the electric current in the subcircuit is directly proportional to the voltage drop across this subcircuit, amperage and time during which the current passes:

$$A = U \cdot I \cdot t. \tag{2.19}$$

The unit of measurement of electric current is 1 Joule or watt-seconds:

$$1\ \mathrm{J} = \mathrm{V \cdot A \cdot s} = \mathrm{W \cdot s}.$$

From the formula (2.19), using various forms of writing Ohm's law, by simple transformations we can obtain the following relations:

$$A = I^2 \cdot R \cdot t, \tag{2.20}$$

$$A = \frac{U^2 \cdot t}{R} \ . \tag{2.21}$$

Depending on the type of consumer the process of accomplishing the work by the electric current is accompanied by the conversion of electric energy into thermal energy (heating devices), mechanical (electric motors) or light (lighting devices).

2.4.2 The power of the electric current. Power is the quantity that characterizes the rate at which work is done, or energy conversion rate. ***Electrical power (P) is the work done by the electric current per unit time.***

$$P = \frac{A}{t} \ . \tag{2.22}$$

From the expression (2.22) we can write

$$P = U \cdot I, \tag{2.23}$$

i.e., the power developed in the subcircuit by the electric current is directly proportional to the voltage and current strength on this subcircuit.

The unit of power is watt: 1 W = 1V·1A.

From (2.23) the following formulas for calculating the power can be obtained by simple transformations:

$$P = I^2 \cdot R \ , \tag{2.24}$$

$$P = \frac{U^2}{R} \ . \tag{2.25}$$

The power developed by the electric power source in the entire circuit, is called ***apparent power***. Total power (P_{TOT}) is determined by the electromotive force of the source and magnitude of the load current (fig. 2.1):

$$P_{\mathrm{TOT}} = E \cdot I_{\mathrm{L}} \, . \tag{2.26}$$

As the EMF of the source is equal to the sum of voltage drops on the internal and external subcircuits (see fig. 2.1), we can write

$$P_{\mathrm{TOT}} = (U + U_0)\, I_{\mathrm{L}} = U \cdot I_{\mathrm{L}} + U_0 \cdot I_{\mathrm{L}} \, . \tag{2.27}$$

The value of $U \cdot I_{\mathrm{L}}$ expresses the power developed on the external circuit, i.e., the power consumed by the load. It is called the ***useful power*** (load power):

$$P_{\mathrm{L}} = U \cdot I_{\mathrm{L}} \, . \tag{2.28}$$

The value of $U_0\, I$ expresses the power consumed inside the source, and is called the ***power of loss***:

$$P_{LOSS} = U_0 \cdot I_L \ . \tag{2.29}$$

In practical calculations the line resistance R_L is often taken into account in the equivalent schemes. In this case, the power loss is determined by:

$$P_{LOSS} = U_0 \cdot I_L + U_L \cdot I_L = I_L^{\,2} \cdot (R_0 + R_L). \tag{2.30}$$

Thus, the total power is equal to the sum of useful power and power of loss:

$$P_{TOT} = P_L + P_{LOSS} \ . \tag{2.31}$$

2.4.3 The coefficient of performance. Due to the fact that not all of the source power is given to the receiver of the electrical energy, the concept of the ***coefficient of performance*** of the source η as the ratio of useful power to its total power is used:

$$\eta = \frac{P_L}{P_{TOT}} \text{ , or in percent } \eta = \frac{P_L}{P_{TOT}} 100\% \,. \tag{2.32}$$

The coefficient of performance (COP) can be expressed via other quantities characterizing the electrical circuit (see fig. 2.1):

$$\eta = \frac{P_L}{P_{TOT}} = \frac{U \cdot I_L}{E \cdot I_L} = \frac{U}{E}, \tag{2.33}$$

or

$$\eta = \frac{P_L}{P_L + P_0} = \frac{I_L^2 \cdot R_L}{I_L^2 \cdot R_L + I_L^2 \cdot R_0} = \frac{R_L}{R_L + R_0}, \tag{2.34}$$

or

$$\eta = \frac{U}{E} = \frac{E - U_0}{E} = 1 - \frac{U_0}{E} = 1 - \frac{I_L \cdot R_0}{I_{SC} \cdot R_0} = 1 - \frac{I_L}{I_{SC}} \,. \tag{2.35}$$

Figure 2.4 shows a plot of the COP of the source from the load current. When the idle the current is zero and the COP is equal to 1. In case of short circuit the current reaches its maximum value, but the COP turns to zero, as the source does not perform useful work and the whole energy is spent inside the source.

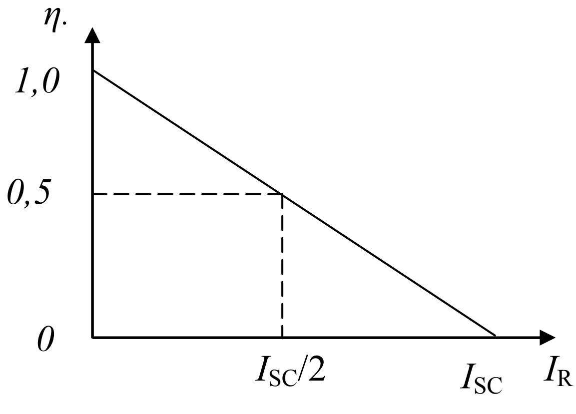


Figure 2.4: Dependence $\eta = f(I_R)$

The equality of the COP to one at idle implies that the mode of the work source approaching to the idle mode has the COP which tends to one. It turns out when the load resistance is many times higher than the internal resistance of the source. In this case loss energy inside the source is much less useful power. From the graph it is seen that under the condition of obtaining the maximum useful power from the source when $I_R = I_{SC}/2$ its COP is only 0.5.

Electric generators of power plants are calculated for work in the modes providing the highest possible COP. The COP of modern generators reaches 98–99%.

2.4.4 Heating effect of current. The electric current passing through the conductor, heats it, because electric energy is converted into thermal energy in the conductor. The physical essence of the phenomenon is that orderly moving charge carriers colliding with atoms of the substance give them some of their energy, due to which the thermal motion of the latter is becoming more intense. In metals, for example, "free" electrons collide with ions of the crystal lattice and give them part of their kinetic energy, causing the increase of oscillatory motion of the ions and, consequently, the temperature of the conductor is being increased too.

When electric current passes through the conductor and there is no additional transformation of electrical energy into chemical, mechanical or light, all the energy spent on moving electric charges, is converted into heat. Then the amount of heat Q that is equivalent to the work of electric current is deposited:

$$Q = A = U \cdot I \cdot t \text{ or } Q = I^2 \cdot R \cdot t\,. \tag{2.36}$$

The heat here is expressed in joules.

In technical calculations heat energy is often measured in calories (1 calorie is the amount of heat required to heat of 1 gram of water by 1 degree of Celsius). One Joule is 0.24 calories, so a value of 0.24 calories/Joule is called ***thermal equivalent of the work***.

The amount of heat, measured in calories, which is highlighted in the conductor when an electric current passes through it, is usually expressed as:

$$Q = 0.24 \cdot I^2 \cdot R \cdot t. \tag{2.37}$$

This ratio was first obtained empirically and independently of one another in 1841–1842 by the English scientist J. P. Joule and Russian scientist H. E. X. Lenz and is called the ***Joule-Lenz's law***.

The amount of heat emitted from the electric current in a conductor is directly proportional to the square of the current, the resistance value of the conductor and the time during which the current flows. In addition to the formulas (2.37) the Joule-Lenz's law can be expressed by the following formulas:

$$Q = 0.24a \cdot U \cdot I \cdot t, \quad \text{or} \quad Q = 0.24\frac{U^2}{R}t\,. \tag{2.38}$$

As in the SM system the thermal energy is measured in joules, the Joule-Lenz's law is written as:

$$Q = I^2 \cdot R \cdot t\,. \tag{2.39}$$

2.5 Conditions of return of source maximum power

When the resistance of resistor of the external circuit (see fig. 2.1) is equal to R_L, the voltage and current in it are related by the equation $U = I_L \cdot R_L$ expressing Ohm's law for the passive subcircuit. Given this, we can write:

$$E = U_0 + U = I_L \cdot R_0 + I_L \cdot R_L \, . \tag{2.40}$$

This equation expresses the electrical state of a simple closed circuit. From (2.40) can be obtained:

$$I_L = E/(R_0 + R_L) \, . \tag{2.41}$$

About it the power of the source power

$$P_{TOT} = I_R^{\,2} \cdot (R_0 + R_L) = E^2/(R_0 + R_L), \tag{2.42}$$

and the power allocated to the load

$$P_L = R_L \cdot I_L^{\,2} = R_L \cdot E^2/(R_0 + R_L)^2 \, . \tag{2.43}$$

The load power P_L at no load ($R_L = \infty$) and at short circuit ($R_L = 0$) is equal to zero. It has a maximum value when the ratio $R_L/(R_0 + R_L)^2$ is maximum. Taking the first derivative of this fraction by R_L and equating it to zero, we get

$$\frac{d}{dR_L} \cdot \frac{R_L}{(R_0 + R_L)^2} = \frac{(R_0 + R_L)^2 - 2R_0(R_0 + R_L)}{(R_0 + R_L)^2} = 0 \, ,$$

or

$$(R_0 + R_L)^2 - 2R_0 \cdot (R_0 + R_L) = 0 \, ,$$

from which

$$R_L = R_0 . \tag{2.44}$$

Therefore, ***the power of the external circuit is maximum when the resistance of the external circuit*** R_L ***is equal to the internal resistance*** R_0 ***of the source,*** i.e., when the external circuit and the source are in the consistent mode.

In the coordinated mode, the power of loss inside the source is equal to half of the power of source:

$$\Delta P_{SOURCE} = R_0 \cdot I^2 = R_L \cdot I^2 = E \cdot I/2 \, . \tag{2.45}$$

Let's examine the changing of the COP of the source, depending on the resistance value R_H. ***The coefficient of performance of the source is equal to the ratio of the power of the external circuit*** P_L ***to the total power*** P_{TOT} ***of the source***:

$$\eta = \frac{P_L}{P_{TOT}} = \frac{R_L \cdot I^2}{(R_0 + R_L) \cdot I^2} = \frac{R_L}{(R_0 + R_L)} = \frac{1}{1 + \frac{R_0}{R_L}} \, . \tag{2.46}$$

The formula (2.46) shows that at idle, when $R_L = \infty$, the COP is $\eta_{idle} = 1$; in case of short circuit when $R_L = 0$, the COP is $\eta_{sc} = 0$; in the coherent mode ($R_L = R_0$) the COP is $\eta_{coord} = 0.5$.

Figure 2.5 shows the graphs of dependences of P_{TOT}, P_L and η from the relative magnitude of the resistance of the external circuit R_L/R_0.

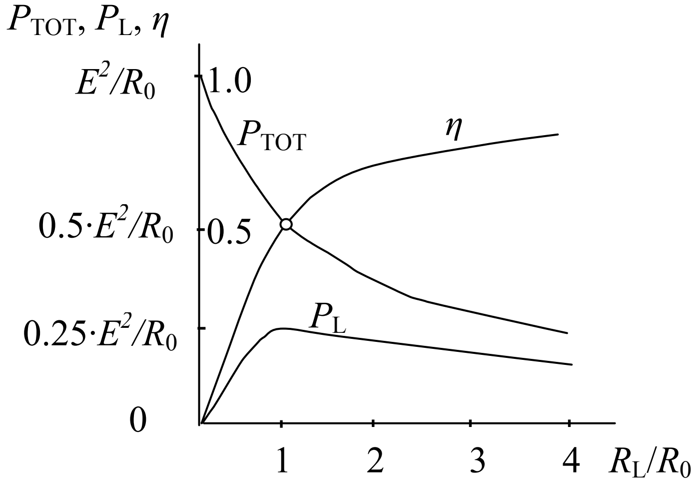


Figure 2.5: Figure of dependency P_{TOT}, P_L and η from the ratio R_L/R_0

Note that in practical terms the nominal mode of the powerful sources rarely coincides with the agreed regime, as the COP is equal to 0.5, and the load current source is significantly larger than the nominal current. The latter circumstance can lead to excessive heat generation inside the source.

We have to deal with the agreed regime when the low COP does not have crucial value because of the low power circuit. When economic issues are critical, the internal resistance R_0 of the source must be small compared with the resistance R_L of the external circuit. In this case, the nominal mode of the source closer to the idle mode and the COP of the source is close to one.

2.6 The connection schemes of the circuit elements

Let's consider the possible ways of connection of the elements of DC circuits using the example of connection of passive collectors.

2.6.1 Serial connection of the circuit elements. ***Serial connection is a connection, in which the same current passes through each of the elements***. At a series connection of n elements, the currents of given (fig. 2.6, *a*) and equivalent (fig. 2.6, *b*) schemes will be the same. Therefore using the second Kirchhoff's law we can write the equation

$$U_1 + U_2 + ... + U_n = U,$$

or

$$R_1 I + R_2 I + \dots + R_n I = R_{EQ}\, I,$$

and determine from it the equivalent resistance

$$R_{EQ} = R_1 + R_2 + \dots + R_n\,. \tag{2.47}$$

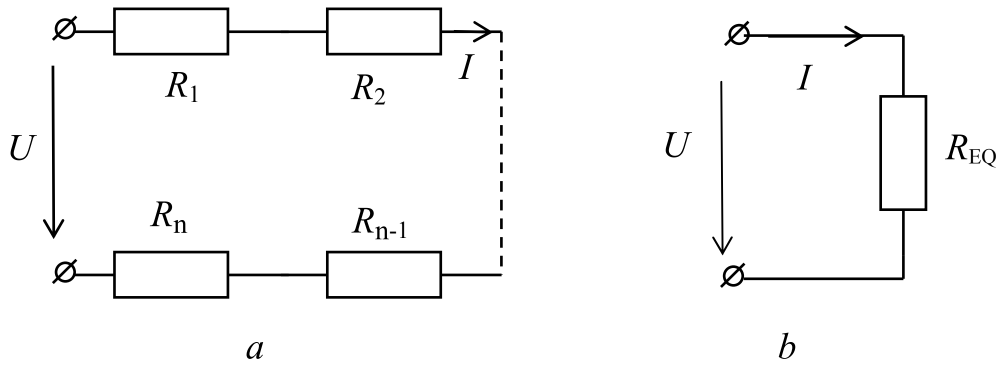


Figure 2.6: Scheme of a circuit with a series connection of active resistance (*a*) and its equivalent resistance (*b*)

The equivalent resistance of the series-connected circuit elements is equal to the sum of the resistance of the individual elements. To analyze the operation modes of the scheme let's record in the general form the following equation of the circuit:

for the current

$$I = U/R_{EQ}\,; \qquad (2.48)$$

for the voltage of the n-th element

$$U_n = R_n \cdot U/R_{EQ}\,; \qquad (2.49)$$

for the power consumption of the n-th element

$$P_n = R_n \cdot I^2 = R_n \cdot U^2/R^2_{EQ}\,.$$

Based on these equations we can obtain some ***general properties of series circuits***:

1. From the equations (2.48) and (2.49) it is shown that there is a linear relationship between the voltage at the input of the scheme, current and voltage of its individual elements. Any change of the voltage U in k times leads to the change of the current and voltage of each element also in k times. The power of the entire circuit and its individual elements varies in it in k^2 times.

2. The current of the entire circuit and the voltage of its individual elements depend on the resistance values of each of the circuit elements. In this case, if the resistance of any element increases, the current in the circuit and voltage elements with the same resistance decreases and the voltage of the element with the increasing resistance grows. In the limit, when the resistance of this element is equal to infinity (idling), the voltage at the connection terminals through which this element was attached to the rest of the circuit will be equal to the circuit voltage.

Full interdependence of work modes of the series-connected elements is the characteristic of this binding.

Serial connection of receivers is used in the case when the nominal voltage is below the voltage of circuit. If, for example, the receivers have a nominal voltage of 110 V, and the voltage is 220 V, these receivers can be connected in series and turn on under voltage. However, *it should be remembered that the resistance of the receiver is inversely proportional to its rated power*: $R = U^2_R/P_R$. Therefore, the receiver of greater rated power will work with underloading, and the receiver of lower rated – power will work with overload. ***Connected in series receivers with the same nominal voltages will have the best working conditions for the same rated power.***

There are multiple applications of serial connections of elements in different areas of technology. For example, when using DC motors sequentially with circuit of anchors the resistors with adjustable resistance are included to limit the starting current (starting resistor, see section 10.9) and for speed control (the regulating rheostat, see sections 10.9, 15.2.2).

In practice of electrical measurements the series-connected resistors are used to form measuring shops of resistance, the consistent inclusion of additional resistors to a voltage meter is applied to expand the limits of measurement of voltage, etc.

2.6.2 Parallel connection of the circuit elements. Figure 2.7, *a* shows a scheme with *n* passive branches attached to the same nodes *a* and *b*, the potential difference between which is equal to the voltage *U* of the source. Therefore, the current in each *n*-th branches is determined by the voltage between the nodes and the resistance R_n or conductance $G_n = 1/R_n$:

$$\begin{cases} I_1 = U/R_1 = G_1 \cdot U\,, \\ \dots\dots\dots\dots\dots \\ I_n = U/R_n = G_n \cdot U\,. \end{cases} \tag{2.50}$$

The fact that the ***parallel connection provides the same voltage to all the receivers and independent from other modes***, is an important advantage of this connection by which it has found wide application. As a rule, all the electric power receivers are included in the circuit in parallel.

To characterize the work of a parallel circuit let's determine its equivalent resistance. Equivalency conditions will be met if the current of I_{eq} (fig. 2.7, *b*) passing through the equivalent circuit is equal to the current *I* in the unbranched part of the circuit or the sum of the currents of the individual parallel branches:

$$I_{eq} = I = I_1 + I_2 + \dots + I_n\,. \tag{2.51}$$

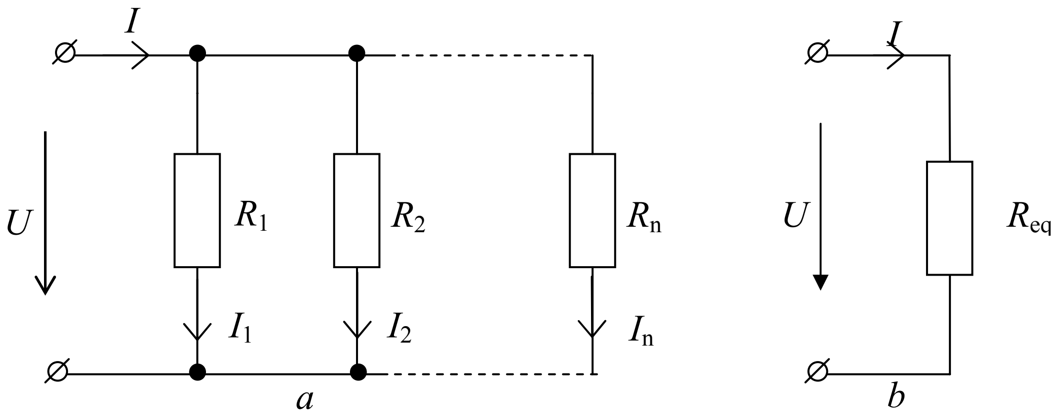


Figure 2.7: Scheme of a circuit with parallel connection of resistive items (a) and its equivalent scheme (b)

Substituting in this equation the values of currents from (2.50), we obtain the expression

$$G_{eq} \cdot U = G_1 \cdot U + G_2 \cdot U + \dots + G_n \cdot U, \tag{2.52}$$

from which you can derive a formula of equivalent conductivity

$$G_{eq} = G_1 + G_2 + \dots + G_n = \sum G_k\,, \tag{2.53}$$

or formula of equivalent resistance

$$1/R_{eq} = 1/R_1 + 1/R_2 + \dots + 1/R_n = \sum 1/R_k\,. \tag{2.54}$$

Therefore, ***with a parallel connection the equivalent conductance of the circuit is equal to the sum of conductance of the individual branches***. Since the branch with the lowest resistance has the largest conductivity, the conductivity of the circuit in parallel connection elements may not be less than the conductivity of the branch with the least resistance. ***The equivalent resistance of the circuit consisting of parallel-connected branches is inversely proportional to its equivalent conductivity***:

$$R_{eq} = 1/G_{eq} \,, \tag{2.55}$$

therefore, it is always less than the smallest resistance in the branch.

You should understand clearly that when you connect the new receiver to the circuit an additional parallel branch is formed, the total conductivity of the circuit increases, and its equivalent resistance decreases. If connected in parallel *n* branches with the same resistance *R*, the equivalent resistance will be *n* times smaller than the resistance of one branch: $R_{eq} = R/n$. The decrease in the total circuit resistance will be accompanied by the increase in current and power:

$$P = U \cdot I = U \cdot (I_1 + I_2 + \ldots + I_n),$$

or

$$P = P_1 + P_2 + \ldots + P_n.$$

The power of circuit consisting of parallel branches is equal to the sum of the powers of its individual branches.

The circuit with two parallel connected resistors with resistance R_1 and R_2 is the matter of practical interest. The equivalent resistance of this circuit is equal to the product of the resistances divided by their sum:

$$R_{eq} = R_1 \cdot R_2/(R_1 + R_2) \,. \tag{2.56}$$

The currents of the branches of this circuit are equal to:

$$\left.\begin{aligned} I_1 = U/R_1 = R_{eq} \cdot I/R_1 = R_2 \cdot I/(R_1 + R_2) \,, \\ I_2 = U/R_2 = R_{eq} \cdot I/R_2 = R_1 \cdot I/(R_1 + R_2) \,. \end{aligned}\right\} \tag{2.57}$$

According to the obtained ratio the current in one of the parallel branches of the circuit is equal to the current of the unbranched subcircuit, multiplied by the ratio of the resistance of the opposite branch and the sum of the resistances of the branches.

2.6.3 ***Mixed connection of the circuit elements***. A combination of serial and parallel connections is called mixed connection of elements. The simplest and most common in the practice of mixed binding is a circuit of normal parallel connection of the receivers to the distribution panel which is connected to a power source by wires.

The schemes of mixed connection of various electrical devices are very diverse. As an example, let's consider the scheme of figure 2.8, *a*.

Let all the resistances of the resistive elements of the branches and the voltage at the input of this scheme are set and you want to determine the currents of its sections. For the calculation we use the ***method of equivalent transformations, by which certain sections of the scheme with parallel or series-connected elements are replaced with one equivalent element***. The gradual transformation of sections simplifies the scheme and result in the simplest scheme, consisting of one equivalent element.

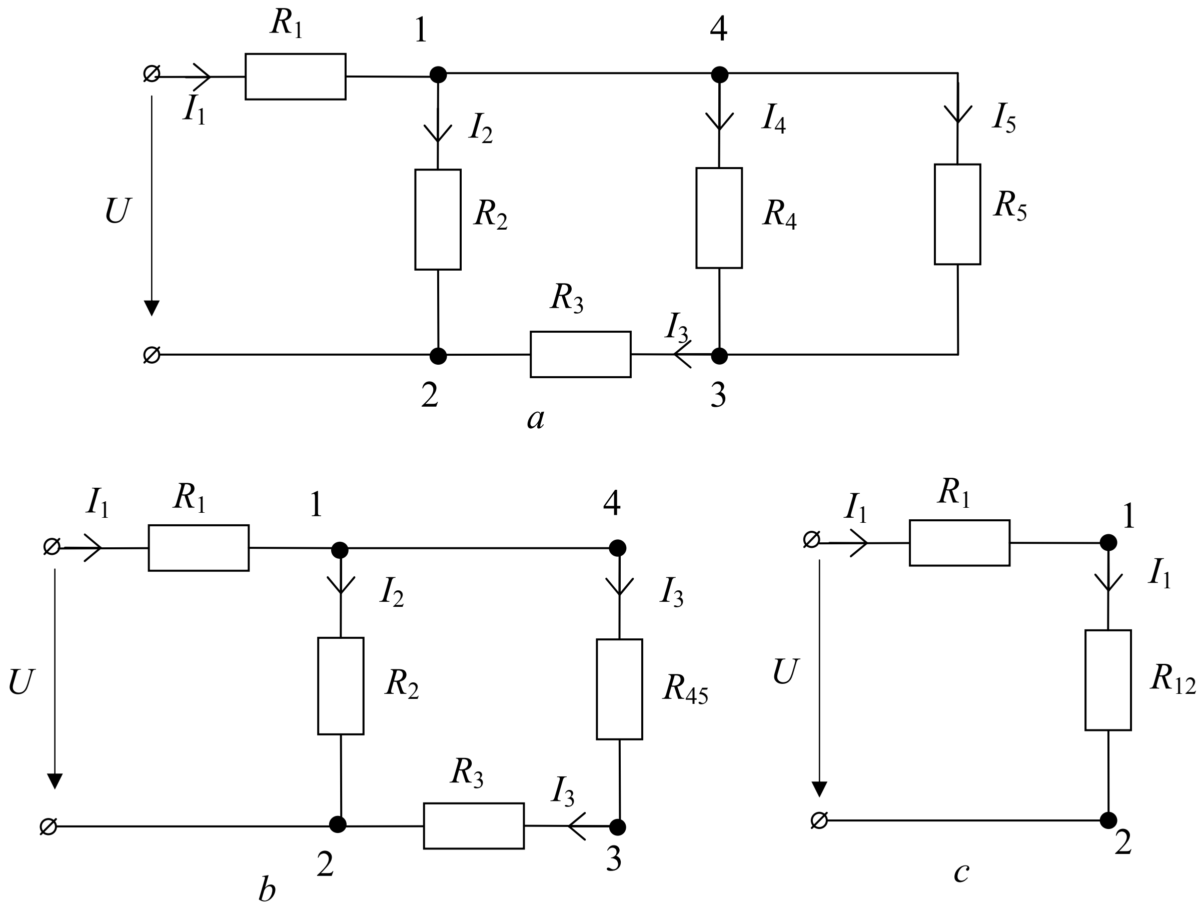


Figure 2.8: Scheme of a circuit with mixed connection of resistive elements (*a*) and equivalent schemes to it (*b*) and (*c*)

Thus, the branches with resistances R_4 and R_5, the schemes figure 2.8, *a* are connected in parallel and can be replaced with one equivalent branch with resistance

$$R_{43} = \frac{R_4 \cdot R_5}{R_4 + R_5}. \tag{2.58}$$

After that, the scheme is simplified and takes the form of the scheme figure 2.8, *b*, the elements of which with the resistances R_3 and R_{43} are connected serially. In turn, the branch 1–4–3–2 is connected in parallel with the branch R_2, so the equivalent resistance of the two branches of the section 1-2

$$R_{12} = \frac{R_2(R_3 + R_{43})}{R_2 + R_3 + R_{43}} . \tag{2.59}$$

The resistive element with a resistance R_{12} is connected serially with a resistive element with a resistance R_1, as indicated in the scheme figure 2.8, *c*. General or the input resistance of this scheme $R_{in} = R_{12} + R_1$ gives the opportunity to define a common current source of the initial scheme figure 2.8, *a*:

$$I_1 = U/R_{in} . \tag{2.60}$$

Then, returning to the scheme figure 2.8, *c*, you can find the voltage on the section 1–2:

$$U_{12} = R_{12} \cdot I_1,$$

and the currents in the resistors R_2 and R_3 of the scheme figure 2.8,*b*

$$I_2 = U_{12}/R_2 \text{ and } I_3 = U_{12}/(R_{43} + R_3). \quad (2.61)$$

The current I_3 passes also the equivalent element with the resistor R_{43}, the voltage drop of which is equal to the voltage on the section 4-3 of the source scheme figure 2.8, *a*:

$$I_3 = \frac{U_{12}}{R_{43} + R_3} \text{ and } U_{43} = I_3 \cdot R_{43}.$$

Knowing the voltage U_{43}, you can find the currents of the other branches:

$$I_4 = U_{43}/R_4 \text{ and } I_5 = U_{43}/R_5. \quad (2.62)$$

The examined above calculation procedure of the scheme with mixed connection of passive elements has been called the ***method of equivalent transformations***. The essence of this method, as it can be seen from the above example, is reduced to the successive simplification of the scheme of connection of the passive elements by replacing sections with serial or parallel connection of the elements by their equivalent schemes derived from the equations (2.45) for the sections with serial connection of elements or the equations (2.53 and 2.54) for the sections with parallel connection of elements.

2.6.4 Equivalent conversion of the bindings of the passive elements by star and triangle. There are schemes with complex connection of elements, which can be attributed neither to parallel nor serial connection. Let's consider one of these schemes, when part of it forms a triangle, the vertices of which are the three nodes, and the parties are the three passive branches connected between these nodes. In many cases to simplify the calculation of similar schemes it is convenient to replace the triangle by the equivalent triactinal star.

Let's consider a scheme with two triangles of resistors 1–2–3 and 4–5–6 (fig. 2.9) and its transformation into the scheme with equivalent stars (fig. 2.10) with the resistances of resistor rays R_1, R_2, R_3, and R_4, R_5, R_6. This conversion of the scheme of connection of elements in the triangle to the scheme of connection of the elements by the star gives you the opportunity to simplify the calculation of the scheme.

Let's determine the equations linking the equivalent resistance of the triangle and the star (fig. 2.11, *a* and *b*).

To solve the problem, let's use a common condition of equivalence, according to which the currents in the unconverted branches of the scheme, will remain unchanged. This means that the currents which are directed to the nodes 1, 2 and 3 on the wires of the schemes of the triangle (fig. 2.9) and stars (fig. 2.10) should be the same. The condition of equivalence must be observed in all modes, including the precipice of one of the wires that are attached to the nodes 1, 2 and 3.

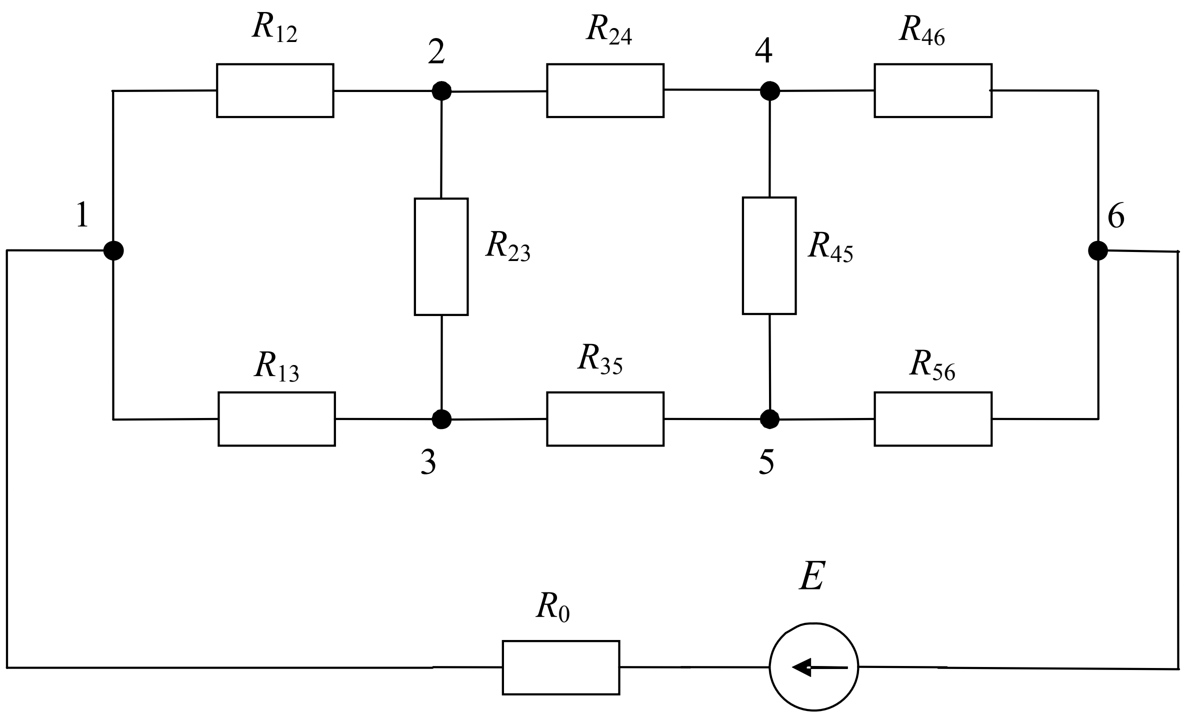


Figure 2.9: Circuit scheme with two sections of the connection of resistances by triangle

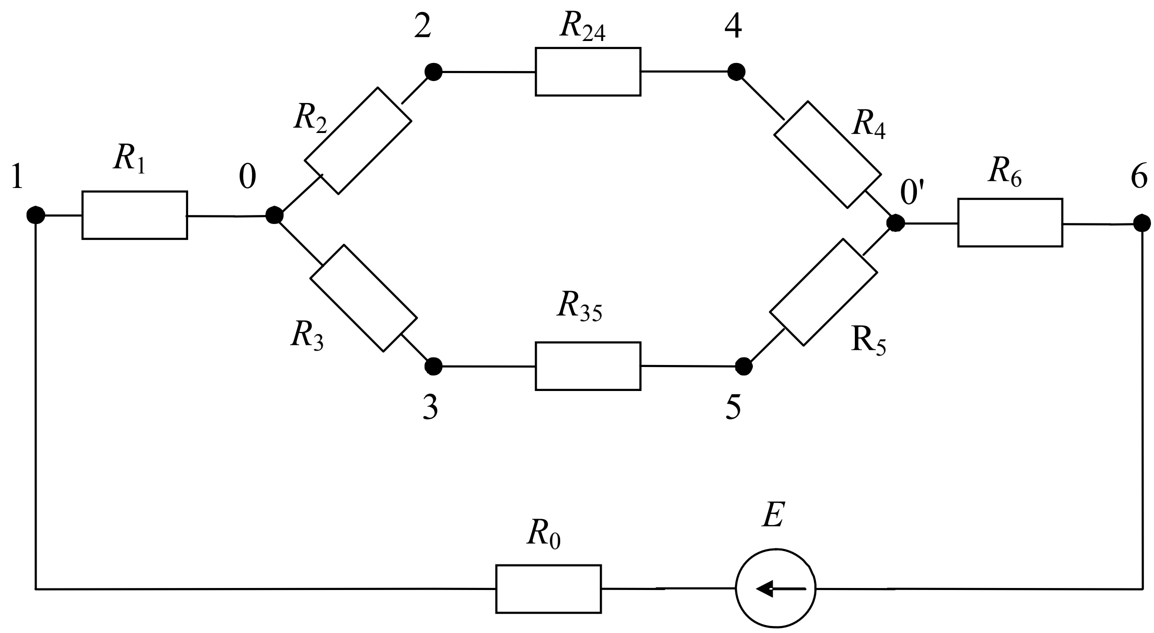


Figure 2.10: The equivalent scheme with two sections of the connection of resistances of star

If the broken wire is connected to the node 1, the voltage between the nodes 2 and 3, and the currents of the wires attached to these nodes must be identical to the schemes of the triangle and star. Therefore, the resistance between the nodes 2 and 3 of the schemes of the triangle and star must be equal between each other. In the star scheme (fig. 2.10) the current through resistor R_1 fails. So the section, consisting of series-connected two rays of the star, the total resistance of which is equal to $R_2 + R_3$, will be included between the nodes 2 and 3.

In the scheme of the triangle there are two parallel branches between the nodes 2 and 3, to one of which a resistor with a resistance R_{23} is included and two series-connected resistors with resistances R_{31} and R_{12} are included to the other. The total resistance of this circuit

$$\frac{R_{23}(R_{31}+R_{12})}{R_{23}+R_{31}+R_{12}} .$$

On the condition of equivalence

$$R_2 + R_3 = \frac{R_{23}(R_{31}+R_{12})}{R_{23}+R_{31}+R_{12}} . \tag{2.63}$$

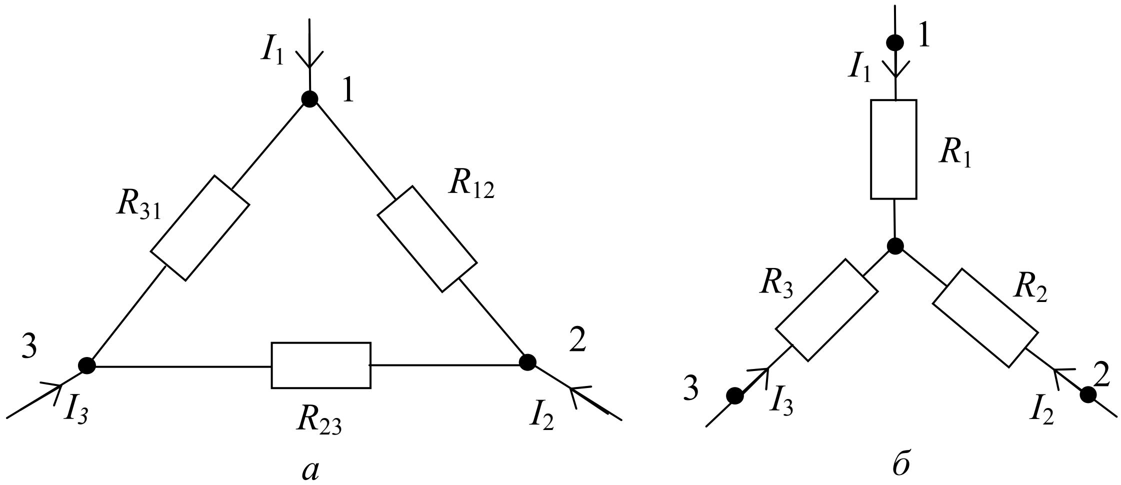


Figure 2.11: Equivalent connections of resistors of triangle (*a*) and star (*b*)

Repeating the above arguments for the case when the ends of the wire attached to the node 2, and then to the node 3 are broken, we get two more equations:

$$R_3 + R_1 = \frac{R_{31}(R_{12}+R_{23})}{R_{23}+R_{31}+R_{23}} , \tag{2.64}$$

$$R_1 + R_2 = \frac{R_{12}(R_{23}+R_{31})}{R_{23}+R_{31}+R_{12}} . \tag{2.65}$$

Solving the resulting system of three equations (2.63) – (2.65) relative to the resistance of the star, we find:

$$R_1 = \frac{R_{12}\cdot R_{31}}{R_{12}+R_{23}+R_{31}} , \tag{2.66}$$

$$R_2 = \frac{R_{23}\cdot R_{12}}{R_{12}+R_{23}+R_{31}} , \tag{2.67}$$

$$R_3 = \frac{R_{31}\cdot R_{23}}{R_{12}+R_{23}+R_{32}} . \tag{2.68}$$

Therefore***, the resistance of any beam of an equivalent star is equal to the product of resistances of the sides of a triangle adjacent to the beam, divided by the sum of resistances of all sides of a triangle***.

If the resistances of resistors of the sides of a triangle $R_{12} = R_{23} = R_{31} = R_{\Delta}$ are equal, the resistance of resistors of the rays of an equivalent star $R_1 = R_2 = R_3 = R_Y$ will be three times less than the resistances of resistors of the sides of a triangle: $R_Y = R_{\Delta}/3$.

Replacing a three-beam star by an equivalent triangle the resistors of the triangle R_{12}, R_{23}, R_{31} can be determined by known resistances of resistors of the star R_1, R_2, R_3, if we solve the system of equations (2.66), (2.67) and (2.68) relative to R_{12}, R_{23}, R_{31}. To do this, we multiply pairs (2.66) on (2.67), (2.67) on (2.68), (2.68) on (2.66), sum up these products and perform the appropriate conversion. The result will be

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} \cdot R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \ .$$

Dividing this equation in turn to (2.68), (2.67) and (2.66), we will find the transition formulas in final form:

$$\begin{cases} R_{12} = R_1 + R_2 + \dfrac{R_1 \cdot R_2}{R_3} \ , \\ R_{23} = R_2 + R_3 + \dfrac{R_2 \cdot R}{R_1} \ , \\ R_{31} = R_3 + R_1 + \dfrac{R_3 \cdot R_1}{R_2} \ . \end{cases} \qquad (2.69)$$

From the obtained formulas ***it*** can be seen that ***the resistance of resistor of any side of an equivalent triangle is equal to the sum of the resistances of resistors of the star beam adjacent to that side of the triangle, and a fraction, the numerator of which is equal to the product of the resistances of resistors of these beams, and the denominator of which is equal to the resistance of resistor of the third star beam***.

2.7 Kirchhoff's Laws

In the theory of electrical circuits the laws which were experimentally established by the German physicist R. Kirchhoff in 1847 are of great importance. They are called the 1st and 2nd laws of Kirchhoff.

2.7.1 The first Kirchhoff's Law (Kirchhoff Current Law). This law applies to the nodes of the circuit and for the case of constant currents is as follows: ***the algebraic sum of the currents meeting at a node is zero***

$$\sum_{k=1}^{n} I_k = 0 \, . \qquad (2.70)$$

When writing equations on the first Kirchhoff's law the summation of the currents is made algebraically: the currents directed to the node have a single character, for example positive and the currents directed from the node have another character, for example negative. So there is often another formulation of the first Kirchhoff's law: ***the sum of the currents flowing to the node is equal to the sum of the currents coming from it***.

If several generators of current are connected to the node, the amount of the generated currents $\sum I_G$ should be presented with the appropriate signs in the left part of the equation (2.70).

The first Kirchhoff's law is a consequence of ***the conservation law of quantity of electricity*** [23] according to which at none point charges can't accumulate infinitely: ***the amount of electricity flowing to a given point for a certain period of time must be equal to the amount of electricity flowing from it at the same time.***

Let's accept for the scheme in figure 2.12 the currents incoming to the node d as positive (I_G and I_1), and the currents outgoing from the node as negative (I_2 and I_3). Then you can write the following equation using the first Kirchhoff's law

$$I_G + I_1 - I_2 - I_3 = 0, \qquad (2.71)$$

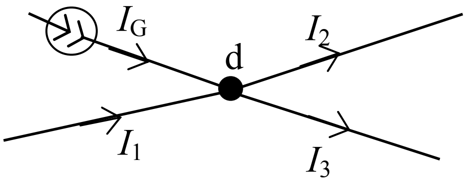


Figure 2.12: Scheme of a subcircuit

that will be correspond to the 1-st formulation of the first Kirchhoff's law, or in the form

$$I_G + I_1 = I_2 + I_3\ , \qquad (2.72)$$

that corresponds to the 2nd formulation of the first Kirchhoff's law, or obtained easily by converting ratio (2.71).

2.7.2 The second Kirchhoff's law (Kirchhoff Voltage Law). This law is a consequence of the energy conservation law, according to which the change of potential in a closed contour is zero. The change of potential between a pair of nodes of the section is characterized by a potential difference or equal to it voltage.

During traversal of a closed contour on separate sections the potential target of the end node m of this section increases relatively to its initial potential of the starting node n on the quantity of voltage, if the direction of traversal is opposite to the direction of the arrow of voltage, and decreases when the traversal path and the direction of the arrow of voltage are the same. Therefore, potential changes in the closed contour can be defined as the summation of voltages with a glance of their characters. According ***to the second Kirchhoff's law the algebraic sum of voltages of the sections of a closed contour is equal to zero*** (first formulation):

$$\Sigma U_{mn} = 0 \quad . \qquad (2.73)$$

In this case voltages, positive directions of which coincide with the direction of a contour traversal are taken with positive signs, and voltages, positive directions of which are opposite to the direction of traversal are taken with negative signs.

Applied to the equivalent schemes with sources of EMF the ***second Kirchhoff's law*** is formulated as follows: ***the algebraic sum of voltages on the***

resistive elements of a closed contour is equal to the algebraic sum of the EMF included in this contour (second formulation)

$$\Sigma I \cdot R = \Sigma E . \qquad (2.74)$$

2.7.3 The order of analysis of circuits. The application of the first and second Kirchhoff's laws for analysis of electric circuits makes it possible to calculate almost any circuit. As a rule, EMF and resistance of all elements of the scheme are known in the study and it is necessary to determine the values of currents and powers in the branches of the scheme. Let's consider the order of calculation on the example of the scheme of circuit shown in figure 2.13.

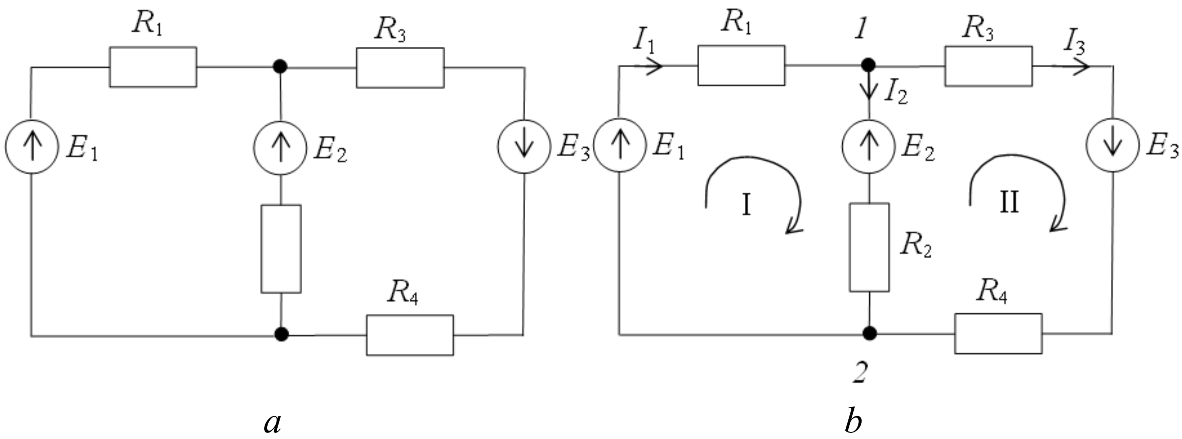


Figure 2.13: The scheme of an electric circuit:
a – initial, *b* – with applied signs nodes, currents and directions of bypass of contour

The algorithm of calculation:

1. Apply the arbitrary directions of the currents in the branches (in fig. 2.13, *b* currents I_1, I_2 and I_3) in the original scheme. If the branch has the EMF, the direction of the current is better to set coincident with the direction of this EMF. Let's apply the signs of the nodes (nodes 1 and 2 in fig. 2.13, *b*).

2. Determine the number of nodes *n*, branches *m* and independent contours *k*.

Contours (path) ***is considered to be independent if it includes at least one new branch***. The number of independent paths is equal to k = m - (n - 1).

3. For independent contours we set an arbitrary direction of bypass (in fig. 2.13, *b* contour I and II, the direction of bypass on the clockwise direction).

To determine unknown currents in the branches it is necessary to make a system of linear algebraic equations, the number of which is equal to the number of unknown currents. According to the first Kirchhoff's law we can make *n-1* independent equations. It is impossible to use all *n* equations, as one of them is sure to be dependent. This is due to the fact that the currents of the branches will be included twice in the equation designed for all *n* nodes, with different signs, because the same current is directed from one node (the minus sign in the equation) to another node (the plus sign). When adding all the equations the left and right sides will be equal to zero, and this means that one of the equations can be obtained by summing the n-1 equations and replacing the signs of all the currents on the opposite. Thus, the n-s equation will always be dependent and that's why it can't be used to determine the currents.

4. Set the number of independent equations according to the first Kirchhoff's law and write these equations.

5. Form the missing equations using the second Kirchhoff's law. The number of equations formed by the second Kirchhoff's law must be equal to the number of independent contours k.

6. Solve the resulting system of linear algebraic equations as for the unknown currents in the branches.

7. To verify the correctness of the calculation of the received current values make the balance equation of power sources and receivers of electric energy

$$\Sigma E \cdot I = \Sigma\, I^2 \cdot R, \tag{2.75}$$

in which the right part describes the power of passive receivers of electric energy, and the left part characterizes the power of active elements of the circuit. When writing the power balance equation we should bear in mind that the components for which the directions of the EMF and current are the same are written with a plus sign in the left part of it. If the direction of the EMF and current are opposite (the work of a source in the mode of energy consumption), the corresponding terms are written with a minus sign in the left side of the equation or with a plus sign in the right part of it, which corresponds to the power of active receiver of electrical energy.

The scheme of the electrical circuits of figure 2.13, *a* contains two nodes (n = 2), three branches (m = 3) and two independent contours

$$\mathrm{k = m - (n - 1) = 3 - (2 - 1) = 2.}$$

According to the first Kirchhoff's law it can be only one independent equation for the scheme, for example, for the node 1

$$I_1 - I_2 - I_3 = 0. \tag{2.76}$$

Using the second Kirchhoff's law we should form two equations for two independent contours (fig. 2.13, *b* of the contour I and II). Taking into account the adopted bypass directions of the contours, these equations have the form:

for contour I

$$R_1 \cdot I_1 + R_2 \cdot I_2 = E_1 - E_2, \tag{2.77}$$

for contour II

$$-R_2 \cdot I_2 + (R_3 + R_4\,) \cdot I_3 = E_2 + E_3. \tag{2.78}$$

When writing equations (2.77), (2.78) the components in which the current and EMF have a direction coincident with the bypass direction of a contour are recorded with a plus sign.

Solving the system of equations (2.76), (2.77) and (2.78), we can determine the unknown currents (three equations, three unknowns I_1 , I_2 , I_3). If the solution of these equations has negative values of the currents, it will mean that the true directions of the currents in the branches of the circuit are opposite to the directions, which we have agreed in figure 2.13, *b*.

Let's make the equation of power balance and perform the test solution. For our case (fig. 2.13, *b*) the equation of power balance has the form

$$E_1 \cdot I_1 + E_2 \cdot I_2 + E_3 \cdot I_3 = I_1^2 \cdot R_1 + I_2^2 \cdot R_2 + I_3^2 \cdot (R_2 + R_4) .$$

For engineering calculations relative error of the obtained solution $\delta = 2 \div 5$ % for most cases is considered to be satisfactory.

Key findings

1. The greater the difference of potentials at the subcircuit boundaries, the greater is the current force at a given quantity of circuit resistance.

2. To improve voltage stability at the consumer it is necessary to lower the internal resistance of the source.

3. To obtain a high COP of the consumer its internal resistance should be many times higher than the internal resistance of the source.

4. The power of the external circuit is maximum if the resistance of the external circuit is equal to the internal resistance of the source.

5. Connected in series receivers with the same nominal voltages have the best working conditions for the same rated power.

6. Power of circuit consisting of parallel branches is equal to the sum of powers of its individual branches.

7. The resistance of any beam of an equivalent star is equal to the product of the resistance of the triangle sides adjacent to the shoulder, divided by the sum of resistances of all sides of the triangle.

8. The resistance of resistor of any part of the equivalent triangle is equal to the sum of resistances of the star beams adjacent to that side of the triangle, and the fraction, the numerator of which is equal to the product of the resistances of resistors of these beams, and the denominator is equal to the resistance of resistor of the third star beam.

9. The sum of currents flowing to the node is equal to the sum of the currents coming from it (the first Kirchhoff's law).

10. The algebraic sum of voltages of sections of a closed contour is equal to zero (the second Kirchhoff's law).

11. The number of independent equations composed by the first Kirchhoff's law for an arbitrary circuit is equal to the number of circuit nodes minus 1.

12. The number of independent equations composed by the second Kirchhoff's law for an arbitrary circuit is equal to the number of independent contours of the circuit.

Control questions

1. Give the definition of Ohm's law for the entire circuit.
2. In what modes can the source operate?
3. What is meant by the nominal data of the collector?
4. How do you calculate the power (work) of electric current?
5. Give the definition of the power of loss, the complete and useful power.
6. Write down the basic equations to define the COP of an electric circuit.
7. Explain the Joule-Lenz’s law.
8. Under what condition does the source give the maximum power to the external circuit?
9. Explain the general properties of the series connection of the circuit elements.
10. Explain the general properties of the parallel connection of the circuit elements.
11. What is the essence of the equivalent conversion method?
12. Write the formula of the equivalent transformation of a star into a triangle and vice versa.
13. Explain the physical meaning of the first (second) Kirchhoff's law.
14. How many independent equations can we compose for the scheme of arbitrary configuration using the first Kirchhoff's law?
15. How many independent equations can we compose for the scheme of arbitrary configuration using the second Kirchhoff's law?
16. Explain the algorithm for finding the unknown currents for the scheme of arbitrary configuration according to the first and second Kirchhoff’s laws.

3 CALCULATION METHODS OF COMPLEX DC CIRCUITS

Key concepts: contour current, contour current method, an intrinsic resistance of contour, the superposition principle, the method of superposition, the principle of reciprocity, the equivalent generator (active two-terminal circuit), the method of nodal voltages.

A direct application of Ohm's law and Kirchhoff's laws is a classic calculation technique of complex circuits. However, in the case of strongly branched circuits it is necessary to solve a system with a large number of equations, so the desire to find less time-consuming methods of calculation of circuits is natural. Different methods are applied to simplify calculations. Among them are the methods of nodal potentials, contour currents, overlay, equivalent generator, etc. All these methods are based on the laws of Ohm and Kirchhoff.

The choice of a calculation method of complex schemes depends on the source of data, structures of the observable circuit and the objectives of its research. This topic discusses the basic calculation methods of complex DC circuits.

3.1 Application of Kirchhoff's laws in analyzing complex circuits

Let's consider the use of Kirchhoff's laws to determine the currents of the branches of the circuit schemes (fig. 3.1) if the EMF and resistance of its elements are set.

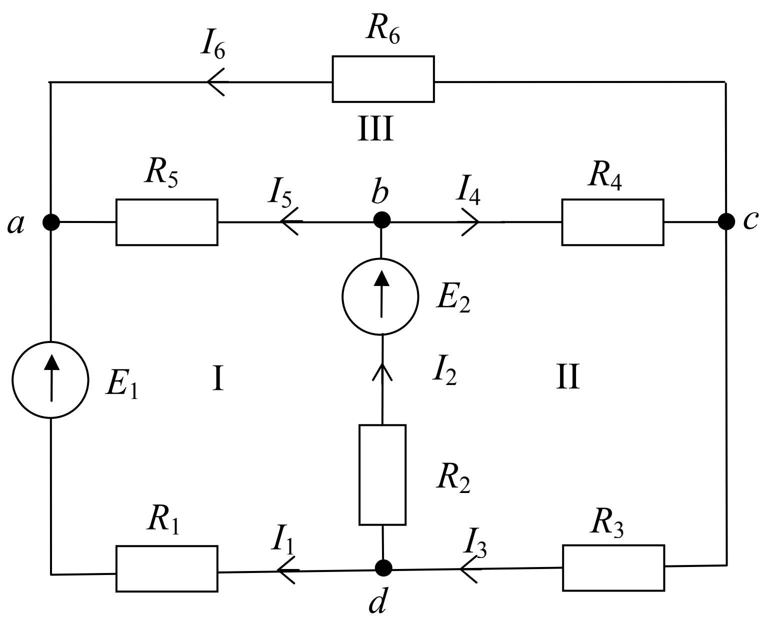


Figure 3.1: The scheme of a complex circuit to determine the currents of branches according to Kirchhoff's laws

The number of unknown currents of the scheme is equal to the number m of its branches. Therefore, to solve the tasks it is necessary to create a system consisting of $m = 6$ independent equations.

Let's assume that a scheme has n nodes. In the specified scheme (fig. 3.1) $n = 4$. Let us take an arbitrary direction of the currents in the separate branches of the scheme. Let's agree that the currents which are directed to the nodes have the plus sign, and the currents directed from the nodes have the sign "minus". According to the first Kirchhoff's law it is possible to compose $n\text{-}1=3$ independent equations. Selecting the nodes b, c and d as independent, you can make the following system of equations:

$$\begin{array}{ll} \text{for node } b: & \\ \text{for node } c: & \\ \text{for node } d: & \end{array} \quad \begin{cases} I_2 - I_4 - I_5 = 0, \\ I_4 - I_6 - I_3 = 0, \\ I_3 - I_1 - I_2 = 0. \end{cases} \qquad (3.1)$$

Since the number of branches m is always greater than the number of nodes n, then ***using the second Kirchhoff's law*** the ***missing number of equations*** m - (n - 1) ***is possible to compose***. Each of the equations will be independent from the previous one if the whole scheme is broken into independent contours. The breakdown should start with choosing the easiest contour (with the least number of branches). Each next contour should be independent from the previous one and contain at least one branch which was not included in the examined before contours.

Let's choose three independent contours, as indicated in the scheme figure. 3.1, and take the bypass of them in a clockwise direction. Then according to the second Kirchhoff's law, we obtain:

$$\begin{array}{ll} \text{for contour I:} & \\ \text{for contour II:} & \\ \text{for contour III:} & \end{array} \quad \begin{cases} R_1{\cdot}I_1 - R_5{\cdot}I_5 - R_2{\cdot}I_2 = E_1 - E_2; \\ R_2{\cdot}I_2 + R_4{\cdot}I_4 + R_3{\cdot}I_3 = E_2\ ; \\ R_5{\cdot}I_5 - R_6{\cdot}I_6 - R_4{\cdot}I_4 = 0. \end{cases} \qquad (3.2)$$

The equations (3.1) and (3.2) give a system of linear algebraic equations

$$\begin{cases} \begin{array}{cccccccc} 0 & +I_2 & +0 & -I_4 & -I_5 & +0 & = & 0 \\ 0 & +0 & -I_3 & +I_4 & +0 & -I_6 & = & 0 \\ -I_1 & -I_2 & +I_3 & +0 & +0 & +0 & = & 0 \\ R_1I_1 & -R_2I_2 & +0 & +0 & -R_5I_5 & +0 & = & E_1-E_2 \\ 0 & +R_2I_2 & +R_3I_3 & +R_4I_4 & +0 & +0 & = & E_2 \\ 0 & +0 & +0 & -R_4I_4 & +R_5I_5 & -R_6I_6 & = & 0 \end{array} \end{cases} \qquad (3.3)$$

the solution of which gives the values of the currents in the branches of the scheme.

As noted in section 2.7.3, if the solution of the equations of currents of individual subcircuits is negative, it means that their actual direction is opposite to that which has been set before the calculation of the scheme.

Let's consider the solution of the problem of the circuit mode calculation in the general case, when the equivalent scheme of the circuit has n nodes and m

branches, of which m_j branches contain current sources. At given EMF and resistances of the branches, the calculation is to find the currents in the m branches.

First, let's consider the calculation of the circuit mode for the scheme without current sources. As it has been already noted, for solving the problem n - 1 independent equations must be composed according to the first Kirchhoff's law and $k = m - (n - 1)$ independent equations are formed according to the second Kirchhoff's law. The resulting system of linear algebraic equations in matrix notation has the form

$$\mathbf{A\,I} = \mathbf{C}\,, \tag{3.4}$$

where: **A** is the matrix of coefficients of the system;
I is the matrix-column of unknown currents of the system;
C is the matrix-column-right part of the system.

For system (3.3) the matrix **A, I** and **C** have the form

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ R_1 & -R_2 & 0 & 0 & R_5 & 0 \\ 0 & R_2 & R_3 & R_4 & 0 & 0 \\ 0 & 0 & 0 & -R_4 & R_5 & -R_6 \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}. \quad \mathbf{C} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E_1 - E_2 \\ E_2 \\ 0 \end{bmatrix}.$$

The system of algebraic equations (3.4) for complex circuits is usually solved by numerical methods on the PC with the use of modern software packages, such as MATLAB or MATCAD.

When calculating schemes with current sources the order for solving the system decreases. Since the currents of m_j branches are known, the number of independent contours (without current sources) for which you want to make the equations using the second Kirchhoff's law, is equal to $k = m - m_j - (n - 1)$.

3.2 The method of nodal potentials

The method of nodal potentials allows to reduce the number of jointly solving equations to n - 1, where n is the number of nodes in the equivalent scheme of circuit. The method is based on the application of the first Kirchhoff's law and is as follows.

1. One node of the equivalent scheme is set as basic with zero potential. This assumption does not change the values of the currents in the branches, as the current in each branch depends only on the difference of potentials of the nodes, and not on the actual values of these potentials.

2. For the remaining n - 1 nodes a system of equations is composed according to the first Kirchhoff's law, expressing the currents in the branches through the potentials of the nodes.

3. The potentials of n - 1 nodes relative to the base are determined by the solution of the resulting system, and then the currents of the branches are determined by the generalized Ohm's law (2.18).

Let's consider the use of the method of nodal potentials on the example of the scheme of a circuit (see fig. 3.2), containing $n = 3$ nodes. Let's accept the node 3 as the basic, i.e., $\varphi_3 = 0$. For the nodes 1 and 2 let's make up the equation using the first Kirchhoff's law.

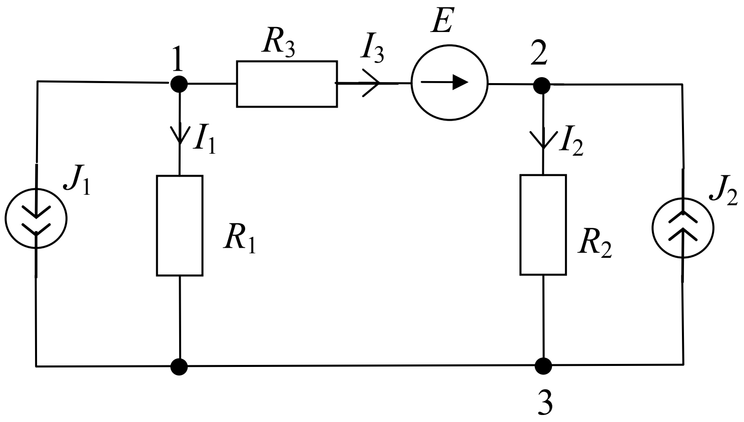


Figure 3.2: Calculation scheme

For node 1

$$I_1 + I_3 + J_1 = 0\ ,$$

for node 2

$$I_2 - I_3 - J_2 = 0\ ,$$

where

$$I_1 = (\varphi_1 - \varphi_3)/R_1 = \varphi_1/R_1\ ;$$
$$I_2 = (\varphi_2 - \varphi_3)/R_2 = \varphi_2/R_2\ ;$$
$$I_3 = (\varphi_1 - \varphi_2 + E)/R_3.$$

After substitution of the obtained values of the currents in the equations we get the system of equations for 1 and 2 nodes

$$\begin{cases} \left(\dfrac{1}{R_1} + \dfrac{1}{R_3}\right)\varphi_1 - \dfrac{1}{R_3}\varphi_2 = -J_1 - \dfrac{E}{R_3}, \\ -\dfrac{1}{R_3} + \left(\dfrac{1}{R_2} + \dfrac{1}{R_3}\right)\varphi_2 = J_2 + \dfrac{E}{R_3}. \end{cases} \tag{3.5}$$

Matrix notation of the system (3.5) has the form

$$\begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_3} & -\dfrac{1}{R_3} \\ -\dfrac{1}{R_3} & \dfrac{1}{R_2} + \dfrac{1}{R_3} \end{bmatrix} \times \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -J_1 & -\dfrac{E}{R_3} \\ J_2 & \dfrac{E}{R_3} \end{bmatrix}. \tag{3.6}$$

or in a more convenient form of writing

$$\begin{bmatrix} G_{11} & -G_{12} \\ -G_{21} & G_{22} \end{bmatrix} \times \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} -J_1 & -G_3E \\ J_2 & G_3E \end{bmatrix}, \tag{3.7}$$

where: $\boldsymbol{G_{11}}$ and $\boldsymbol{G_{22}}$ are intrinsic conductivity of nodes 1 and 2, which are defined as the sum of conductances of the branches connected respectively to the nodes 1 and 2;
$\boldsymbol{G_{12}}$ and $\boldsymbol{G_{21}}$ are mutual conductances of nodes 1 and 2 (conductivity branches connecting nodes 1 and 2);
$\boldsymbol{G_3}$ is the conductivity of the branch with EMF E. In our case, $G_{12} = G_3$.

Let's note that the 1-st equation of the system (3.7) is recorded in relation to the node 1, and the second is relative to the node 2. The right part of the system contains the nodal currents, defined as the algebraic sum of currents of the branches of the source currents and short circuit currents of the branches with the sources of EMF, converging to the node. In this case the summands are taken with a plus sign (minus), if the current source and the EMF are directed to the node (from the node).

In the general case, the system (3.7) has the form

$$\mathbf{G}_{ik}\,\boldsymbol{\varphi}_k = \mathbf{I}_{yi}\ , \ \text{if } i,k = \overline{1, m - m_j - 1}\ , \tag{3.8}$$

where: $\mathbf{G}_{ik}$ is the matrix of own and mutual conductance of the nodes;
$\boldsymbol{\varphi}_k$ is the matrix-column of the desired potentials of the nodes;
$\mathbf{I}_{yi}$ is the matrix-column of the nodal currents.

The method of nodal potentials is more efficient than contour current method, if the number of nodes in the scheme is less than or equal to the number of independent contours. It is particularly effective when we calculate the electric circuits with two nodes and a large number of parallel branches. In this case if we take the potential of one of the nodes equal to zero, for example, $\varphi_2 = 0$, then the voltage between the nodes will be equal to the potential of another node

$$U_{12} = \varphi_1 = \mathbf{I}_{y1}/\mathbf{G}_{i1} = \frac{\sum_{k=1}^{m} E_k G_k}{\sum_{k=1}^{n} G_k}\ , \tag{3.9}$$

where: ***n*** is the number of parallel branches of a circuit;
m is the number of branches containing sources of EMF.

In some cases, the method of nodal potentials is called as the method of node voltages, and its special case for two nodes is called as nodal voltage method.

3.3 The method of contour currents

The method of contour currents is considered to be one of the widely used methods. It allows you to reduce the total number m of jointly solving equations in $(n - 1)$ and to take down the system to the number $k = m - (n - 1)$ of equations composed by the second Kirchhoff's law.

This method is based on the concept of ***contour currents***, which are the ***estimated (conditional) currents, connected only by their contours***.

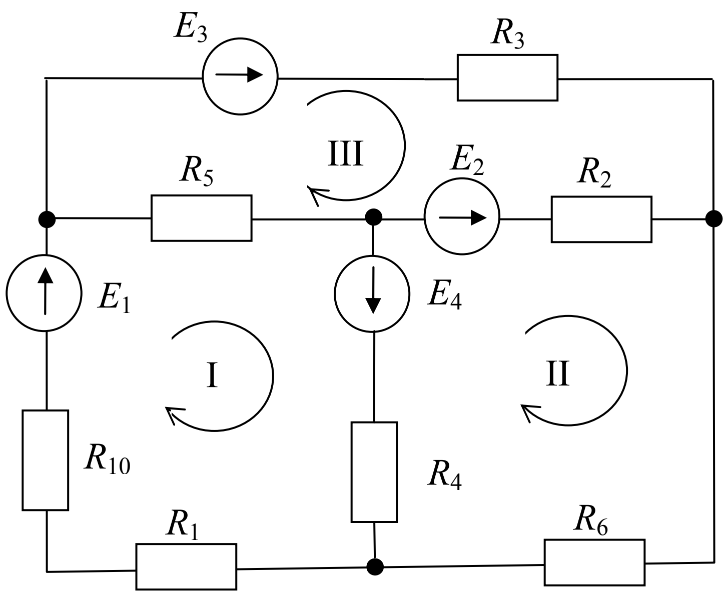


Figure 3.3: Scheme of a complex circuit to determine the currents by the method of contour currents

For example, let's consider a circuit scheme figure 3.3. Let's divide it into three contiguous contours and agree that the contour current passes through each of them I_{I} , I_{II} , I_{III}. Let's select the same - clockwise direction of these currents in all contours, as shown in the scheme. Comparing contour currents with the currents of the branches, the direction of which is also marked on the scheme, you can set that the values of the contour currents coincide with the values of the actual currents only in the external branches:

$$I_{\text{I}} = I_1 , \qquad I_{\text{II}} = -I_6 , \qquad I_{\text{III}} = I_3 . \tag{3.10}$$

The currents of adjacent branches are equal to the difference of contour currents of the neighboring contours:

$$\begin{cases} I_2 = I_{\text{II}} - I_{\text{III}} , \\ I_4 = I_{\text{I}} - I_{\text{II}}, \\ I_5 = I_{\text{III}} - I_{\text{I}} . \end{cases} \tag{3.11}$$

Therefore, using the known contour currents of the scheme we can easily determine the actual currents of its branches.

To determine the contour current of this scheme it is sufficient to make only three equations for each of the contours:

$$\begin{array}{l} \text{for contour I:} \\ \text{for contour II:} \\ \text{for contour III:} \end{array} \begin{cases} (R_1 + R_{10} + R_5 + R_4)\cdot I_{\text{I}} - R_4\cdot I_{\text{II}} - R_5\cdot I_{\text{III}} = E_1 + E_4 ; \\ (R_2 + R_6 + R_4)\cdot I_{\text{II}} - R_4\cdot I_{\text{I}} - R_2\cdot I_{\text{III}} = E_2 - E_4 ; \\ (R_2 + R_5 + R_3)\cdot I_{\text{III}} - R_5\cdot I_{\text{I}} - R_2\cdot I_{\text{II}} = E_3 - E_2 . \end{cases} \tag{3.12}$$

Solving the resulting system of the equations, we define the contour currents, and then we can define the actual currents of the branches.

The method of contour currents is often used to prove other possible methods of calculation and analyze circuits in general. In this case, the equations made for contour currents are recorded in a generalized form. For this purpose, the ***total resistance of this circuit is denoted by two subscripts indicating the number of the path***, and is ***called its intrinsic resistance of contour***.

So, the intrinsic resistance of three contours of the scheme is equal to:

$$\begin{cases} R_{11} = R_1 + R_{10} + R_5 + R_4 ; \\ R_{22} = R_2 + R_6 + R_4 ; \\ R_{33} = R_2 + R_5 + R_3 . \end{cases} \tag{3.13}$$

The ***total resistances of adjacent contours*** are considered as the coefficients of the currents and denoted by two subscripts indicating where (between which neighboring contours) this resistance is included. For example, for the considered scheme

$$R_{12} = R_4 , \qquad R_{13} = R_5 , \qquad R_{23} = R_2 . \tag{3.14}$$

Taking into account these notations, the equation (3.12) can be rewritten in a more general way:

$$\begin{cases} R_{11}\cdot I_{\text{I}} - R_{12}\cdot I_{\text{II}} - R_{13}\cdot I_{\text{III}} = E_{\text{I}} , \\ - R_{21}\cdot I_{\text{I}} + R_{22}\cdot I_{\text{II}} - R_{23}\cdot I_{\text{III}} = E_{\text{II}} , \\ - R_{31}\cdot I_{\text{I}} - R_{32}\cdot I_{\text{II}} + R_{33}\cdot I_{\text{III}} = E_{\text{III}} . \end{cases} \tag{3.15}$$

The EMF in these equations

$$E_{\text{I}} = E_1 + E_4 , \qquad E_{\text{II}} = E_2 - E_4 \quad \text{and } E_{\text{III}} = E_3 - E_2 , \tag{3.16}$$

is the contour EMF, the magnitude of which is determined by the algebraic summation of the individual EMF branches of this contour. Thus the electromotive force coinciding with the direction of the contour current is summed with the sign "plus".

Matrix notation of the system (3.15) has the form

$$\begin{bmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{bmatrix} \times \begin{bmatrix} I_I \\ I_{II} \\ I_{III} \end{bmatrix} = \begin{bmatrix} E_I \\ E_{II} \\ E_{III} \end{bmatrix}, \qquad (3.17)$$

in which the coefficients of the matrix column of the contour EMF is determined by the relations (3.16). In general, for the arbitrary scheme configuration the system of equations of contour current has the form

$$\mathbf{R} \cdot \mathbf{I} = \mathbf{E}, \qquad (3.18)$$

where: **R** is a square matrix of coefficients under the unknown contour currents;
I is a matrix-column of the unknown contour currents;
E is a matrix-column of outline EMF.

The solution of the system of the equations (3.18) has the form

$$\mathbf{I} = \mathbf{R}^{-1} \cdot \mathbf{E}, \qquad (3.19)$$

where $\mathbf{R}^{-1}$ is the matrix, inverse to the matrix of coefficients **R**.

3.4 The superposition principle, application method

The principle of superposition (overlay) is one of the most important physical principles, which is used to consider phenomena that occur under the influence of several reasons. Complex phenomena based on this principle are divided into simpler phenomena, in which each reason functions separately and independently of the others, and the results of these actions (responses), colliding one upon the other, form the total response. In electrostatics, for example, the field strength at any point from several point charges is determined on the basis of the superposition principle as the geometric sum of the field strengths of point charges, operating independently from each other, and the potential points of this field are defined as a result of overlap (the algebraic sum) of the potentials of each of the charges separately. In mechanics, the superposition principle is considered as the principle of independent action of forces.

As applied to electrical circuits, ***the superposition principle*** implies that the ***impact of several sources on any circuit element can be considered as the sum of impacts of each of the EMF source on this element separately and independently of the others***.

The superposition principle is used to replace the result of the influence of one EMF of a complex shape with the influence of the components of the EMF of simpler forms.

The method of circuit calculation using the superposition principle is called the overlay method. Using this method, the calculation of a complex circuit with multiple EMF is taken down to the calculation of several circuits with single power supply. The current in any branch is considered as the result of the superposition currents, resulting from separate EMF which acts independently from each other.

Using the superposition method let us consider the order of calculation on the example of the scheme figure 3.1. To determine the currents initially we believe that it is the only EMF E_1. Thus the resistance of all resistors, including the internal resistance of the source, is considered to be immutable. We take down the definition of partial currents I'_1, I'_2,..., I'_6 of separate branches from EMF to the calculation of the circuit scheme figure 3.4, *a*. Then we repeat the calculation in turn for all other EMF. In our case for EMF E_2, using the circuit scheme figure 3.4, *b* we define partial currents I''_1, I''_2 , ... , I''_6.

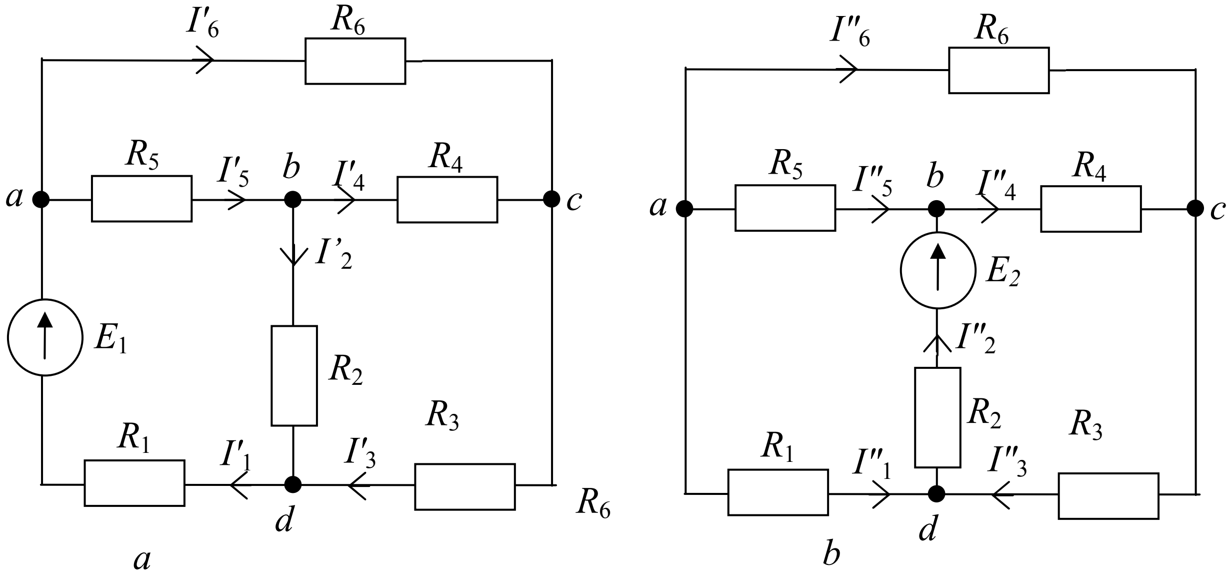


Figure 3.4: The scheme of a circuit to determine the currents according to the method of overlap from EMF E1 (*a*) and E2 (*b*)

The algebraic summation of partial currents gives the value of actual currents of the branches, the positive directions of which must be preliminarily brought to the original scheme (see fig. 3.1).

Taking into account the directions of the partial and resulting currents we will receive:

$$\begin{cases} I_1 = I'_1 - I''_1\,, \\ I_2 = -I'_2 + I''_2\,, \\ \;\; . \; . \; . \; . \; . \; . \; . \; . \; . \;, \\ I_6 = -I'_6 + I''_6\,. \end{cases} \tag{3.20}$$

The number of terms in the system equations (3.20) is equal to the number of EMF of the scheme.

You should pay attention to the fact that the method of superposition is not applicable to the calculation of the power, since the values of the latter are proportional to the squares of the currents.

3.5 The principle of reciprocity

Linear electrical DC circuits with a single power supply possess the property of reciprocity (reversibility). It lies in the fact that if the EMF E of branch n of a circuit causes current I_k in branch k, then the same EMF acting in branch k, will call current I_k of the same size $I_n = I_k$ in branch n.

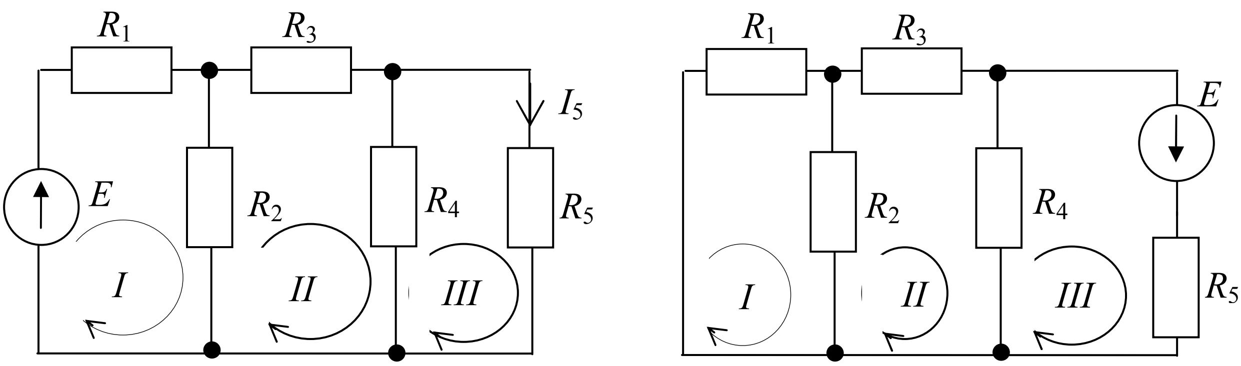
Let's consider the principle of reciprocity using the example of the schemes of the circuits figure 3.5. The EMF E of the first branch of the scheme figure 3.5, a causes in the resistor R_5 the current I_5 which is equal to the third contour current $I_{III} = I_5$. To find the current I_{III} using determinants, we write EMF, intrinsic and general resistance for each of the contours.

For contour I:

$$E_I = E, \qquad R_{11} = R_1 + R_2, \qquad R_{12} = R_2, \qquad R_{13} = 0 .$$

For contour II:

$$E_{II} = 0, \qquad R_{22} = R_2 + R_3 + R_4, \qquad R_{21} = R_2, \qquad R_{23} = R_4 .$$



a *b*

Figure 3.5: Schemes of circuits for the application of the principle of reciprocity for the currents I_5 (a) and I_1 (b)

For contour III:

$$E_{III} = 0, \qquad R_{33} = R_4 + R_5, \qquad R_{32} = R_4, \qquad R_{31} = 0 .$$

The third current path ($k = 3$) from the EMF of the primary contour (n = 1)

$$I_3 = \sum_{n=1}^{3} \frac{A_{nk}}{\Delta} E_n = \frac{A_{13}}{\Delta} E_I + \frac{A_{23}}{\Delta} E_{II} + \frac{A_{33}}{\Delta} E_{III} ,$$

or

$$I_{III} = I_5 = \frac{A_{13}}{\Delta} E_1 = \frac{A_{13}}{\Delta} E \quad . \tag{3.21}$$

The determinant of the system Δ, its minor M_{13} and algebraic addition A_{13} are respectively equal to:

$$\Delta = \begin{vmatrix} r_{11} & -r_{12} & -r_{13} \\ -r_{21} & r_{22} & -r_{23} \\ -r_{31} & -r_{32} & r_{33} \end{vmatrix} , \quad M_{13} = \begin{vmatrix} -r_{21} & r_{22} \\ -r_{31} & -r_{23} \end{vmatrix} = r_{21} r_{23} + r_{22} r_{31} ,$$

$$A_{13} = (-1)^{1+3} M_{13} = M_{13} .$$

For the circuit scheme figure 3.5, *b* the intrinsic and total resistances, and therefore the determinant Δ will remain unchanged. Contour EMF of this scheme: $E_{\mathrm{I}} = 0$, $E_{\mathrm{II}} = 0$ and $E_{\mathrm{III}} = E$. Therefore, the current of the first contour ($k = 1$) from the EMF of the third contour ($k = 3$)

$$I_{k=1} = \sum_{n=1}^{3} \frac{A_{nk}}{\Delta} E_n = \frac{A_{11}}{\Delta} E_1 + \frac{A_{21}}{\Delta} E_{II} + \frac{A_{31}}{\Delta} E_{III} ,$$

or

$$I_I = \frac{A_{31}}{\Delta} E_{III} = \frac{A_{31}}{\Delta} E \quad . \tag{3.22}$$

Minor M_{31} is obtained from the determinant Δ by deletion in it the third row and the first column:

$$M_{31} = \begin{vmatrix} -r_{12} & -r_{13} \\ r_{22} & -r_{23} \end{vmatrix} = r_{12} r_{23} + r_{22} r_{13} \ .$$

Algebraic addition

$$A_{31} = (-1)^{3+1} M_{13} = M_{31}$$

Values $A_{13} = A_{31}$, therefore, according to (3.21) and (3.22) $I_{\mathrm{I}} = I_{\mathrm{III}} = I_5$.

3.6 The method of equivalent generator

In some cases, it is necessary to investigate the operation modes of one of the branches of a complex electric circuit when the resistance of the same branch is changing. Thus there is no need to carry out bulky calculation of the entire circuit using any of the above mentioned methods, and it is better to use the method of equivalent generator. According to this method, ***the impact of all sources of a complex electric circuit on the analyzed branch can be replaced by the influence of the equivalent generator*** which is ***sequentially connected with a branch and*** has an EMF E_{eqv} and internal resistance R_{eqv}.

We will show the possibility of such substitution to determine the current in the branch of the resistor with a variable resistance *R* of the circuit scheme figure 3.6, *a*. Let EMFs E_1, E_2, E_3 and resistances R_1, R_2, R_3 of the resistors in this scheme are set. To establish the dependence of the current from the resistance *R* let's assign this branch and enclose the rest of the scheme in the dotted rectangle, showing the connection terminals *a* and *b*, by which it is joined with an observable branch (fig. 3.6, *b*). The selected part of the scheme that has two connection terminals is an active two-terminal circuit *A*. The letter *A* in the rectangle of the scheme figure 3.6, *b* shows, therefore, that the final effects of EMF E_1, E_2 and E_3 on the analyzed branch are not equal to zero.

Let's include to the study branch the two EMF E' and E'' which are equal in magnitude and opposite in direction (fig. 3.6, *c*). The current *I* of the branch will not change and will be equal to the current of the source scheme figure 3.6, *b*.

We consider the current *I* of the scheme figure 3.6, *c* as the result of the superposition of the currents I_{a} of the scheme figure 3.6, *d* from the action of EMF E', E_1, E_2, E_3 and the current I_{b} of the scheme figure 3.6, *d* from the action of EMF E''. The branches of the circuit figure 3.13, *e* enclosed in the rectangle, are passive, so they are marked with the letter Π.

The current I of the study branch will be equal to the current I_b, scheme figure 3.6, *e*, if we select in the scheme figure 3.6, *d* the EMF E' of such a value at which the current I_a is equal to zero. This condition can be met if the EMF E' is equal to the open circuit voltage between connection terminals *a* and *b* of the scheme: $E' = U_{ab.idle}$.

The value of the EMF E' for these conditions can be defined analytically too. In this case, for the scheme figure 3.6, d let's make an equation for the current using Ohm's law for a section with EMF E' and the voltage U_{ab}:

$$I_a = \frac{U_{ab} - E'}{R} \quad .$$

From this equation it is seen that when the current I_a is equal to zero the EMF E' is equal to the open circuit voltage $U_{ab.idle}$ between connection terminals *a* and *b* of the scheme.

Thus, to determine the current I in the source scheme it is sufficient to consider only the scheme figure 3.6, *e* with an EMF $E'' = E' = U_{ab.idle}$ in it. This scheme consists of series-connected resistive element with a resistance R of the studied branch and a resistive element with a resistance of the rest of the circuit relative to the connection terminals *a* and *b* (fig. 3.6, *f*).

To determine the input resistance relative to the connection terminals of the left part of the scheme figure 3.6, *e*, it is necessary to convert the resistance of the passive two-terminal circuit into equivalent resistance equal to the input resistance R_{ab}. The sequence of transformations is shown in figure 3.7. Let's define the equivalent resistance between points 1–2.

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} \quad .$$

The equivalent resistance between points 2-3

$$R_{23} = \frac{R_3(R_{12} + R_5)}{R_3 + R_{12} + R_5} \quad .$$

The input resistance of the scheme:

$$R_{IN} = R_{43} = \frac{R_6(R_4 + R_{23})}{R_6 + R_4 + R_{23}} \quad . \tag{3.23}$$

The impact of EMF $E'' = U_{ab.idle}$ on the resistance R of the scheme figure 3.6, *f* can be represented as the impact of the equivalent generator with EMF E_{equ} equal to the open circuit voltage between connection terminals *ab*, to which the under study branch is connected:

$$E_{eq} = E'' = U_{ab.idle} \; . \tag{3.24}$$

The internal resistance of the equivalent generator is equal to the input resistance of the rest of the passive part of the circuit relative to the connection terminals *a* and *b*, which are attached to the under study branch:

$$R_{eq} = R_{in} \quad . \tag{3.25}$$

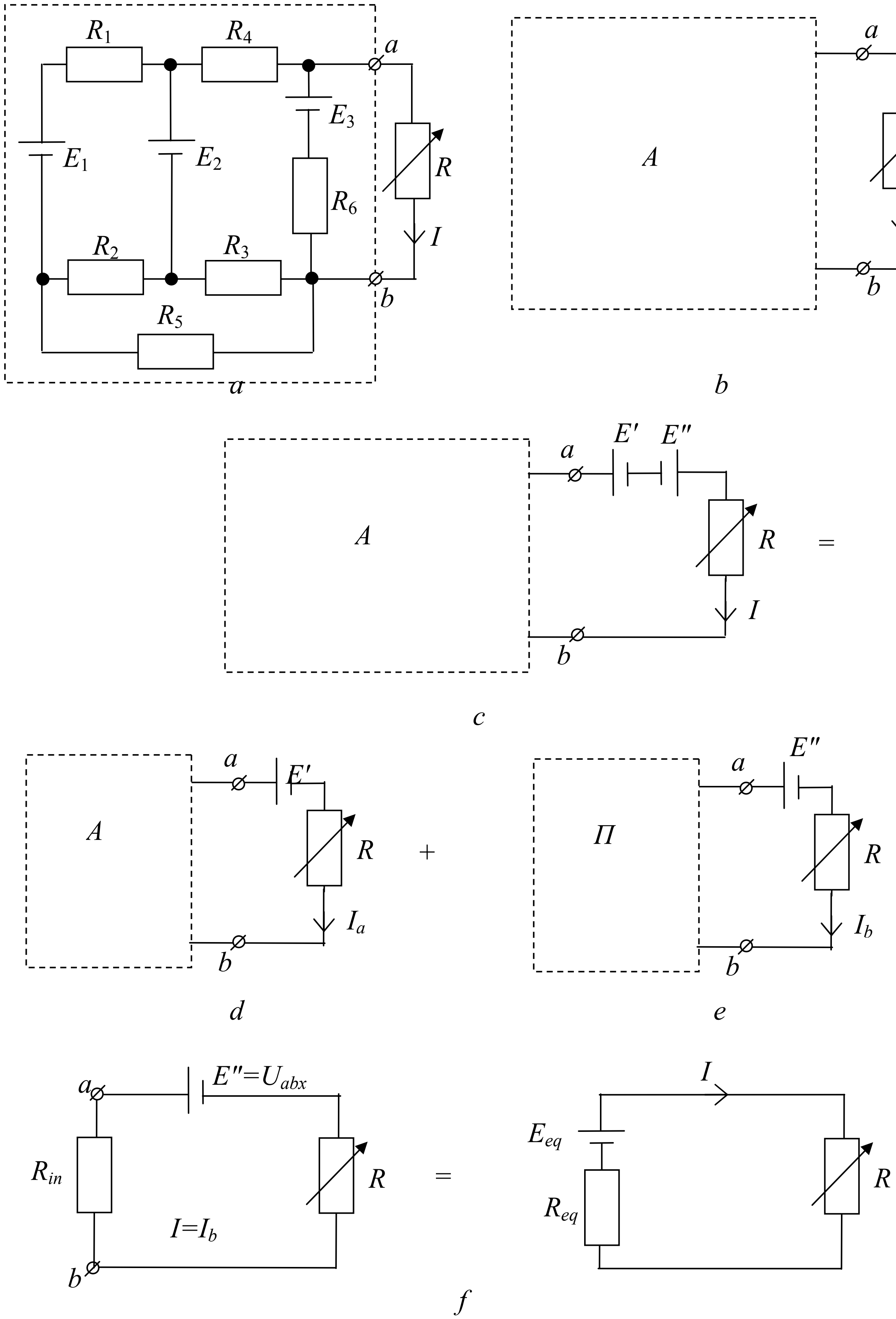


Figure 3.6: Scheme of a circuit for calculation by the method of equivalent generator

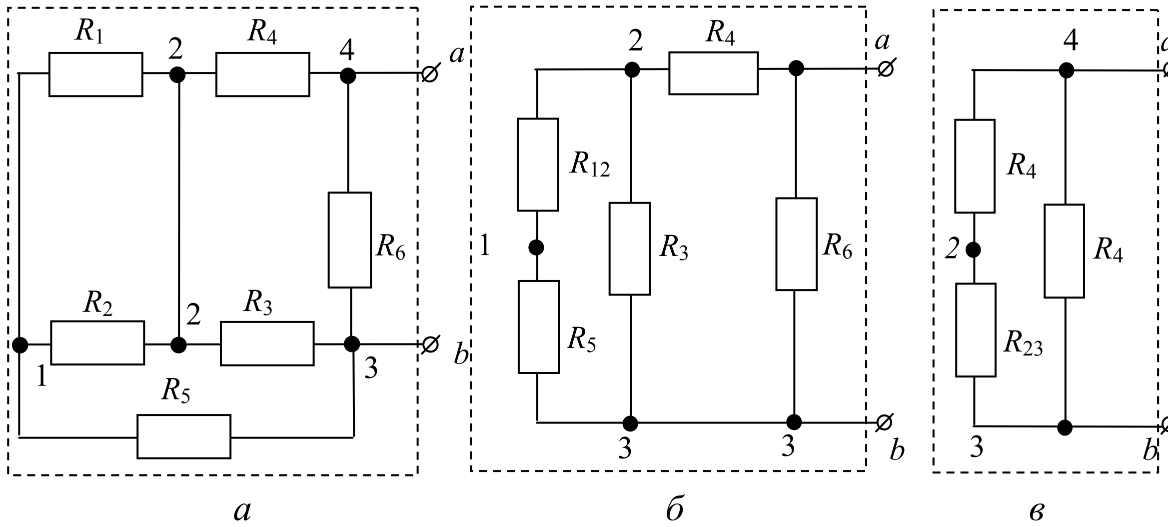


Figure 3.7: Sequence of resistance conversion
of a passive two-terminal circuit

Knowing EMF E_{eq} and internal resistance R_{eq} of the equivalent generator (see fig. 3.6, *f*) you can determine the current of the under study branches

$$I = \frac{E_{eq}}{R + R_{eq}} \quad . \tag{3.26}$$

The described method for determination of current in one branch of a complex electric circuit is called as the ***method of equivalent generator or method of active two-terminal circuit***. The method of equivalent generator is called so because the impact of the rest of the circuit on the analyzed branch is replaced by the influence of the equivalent generator. The method of active two-terminal circuit is called so, because in relation to the under studied branch the rest of the circuit attached to the under study branch with two output connection terminals *a* and *b*, is called two-terminal circuit. The total resistance of the branches, forming a passive two-terminal circuit (see fig. 3.6, *e*) with respect to connection terminals *a* and *b*, is called the ***input resistance of two-terminal circuit***. It is equal to the internal resistance of the equivalent generator.

Key findings

1. There are several calculation methods of complex circuits. The choice of the method depends on the configuration of the scheme, available source data and objectives of the study of circuits.

2. Using the laws of Ohm and Kirchhoff you can calculate any scheme. For complex structure schemes the result is a system with a large number of equations.

3. The method of nodal potentials allows us to take down the order of the solved system to n - 1 equations composed by the first Kirchhoff's law.

4. The method of contour currents allows to take down the order of the system to m-n+1 equations composed by the second Kirchhoff's law. The method is based on the estimated (conditional) contour currents locked on the

adjacent contours of the branched electrical circuits. The true values of the currents in the branches of the circuit are determined by the values of contour currents.

5. The method of nodal potentials is more efficient than contour current method, if the number of nodes in the scheme is less than or equal to the number of independent contours.

6. The impact of several power sources (EMF and voltage) on any circuit element can be considered as the total result of influence on this item of each source individually, independently from other sources (application method).

7. Using the method of equivalent generator it is relatively easy to determine the current in any one branch of the complex electric circuit and to investigate the behavior of this branch when its resistance is changing. Thus in relation to the under studied branches a complex circuit with multiple power sources is replaced with an equivalent active two-terminal circuit with a single power supply (equivalent generator) with EMF $E_{экв}$ and internal resistance $R_{экв}$.

8. Two-terminal circuit is considered to be active if there is at least one source of EMF or current inside of it. In the absence of sources inside two-terminal circuit it is considered to be passive.

Control questions

1. What is the number of independent nodes (contours) of the scheme of arbitrary configuration?
2. How to choose the base node in the method of nodal potentials?
3. How many equations can be made according to the method of nodal potentials?
4. Explain the calculation procedure of a scheme using the method of nodal potentials?
5. Explain the matrix form of these equations using the method of nodal potentials?
6. Explain the advantages of the method of nodal potentials?
7. What is meant under contour current?
8. How to select the direction of contours in the method of contour currents?
9. Explain the matrix form of equations using the method of contour currents?
10. Explain the essence of the superposition principle?
11. Explain the essence of the reciprocity principle?
12. Explain the essence of the method of equivalent generator?
13. Are there any cases when it is preferable to apply the method of equivalent generator?

SECTION II.
AC ELECTRICAL CIRCUITS

4 PHYSICAL PROCESSES IN AC CIRCUITS

Key concepts: alternating current, periodic alternating current, active value of voltage (EMF, current), the average value of the voltage (EMF, current), phase, initial phase, angular frequency, phase shift angle, vector diagram, the integrated value, a vector of complex value.

4.1 Basic information about alternating currents

Circuits with changeable alternating currents compared to DC circuits have a number of features. These features are defined so that the alternating currents and voltages of the individual elements of the electrical devices generate electric and magnetic fields. These changes in an electrical circuit lead to phenomena of self-induction, mutual induction and displacement currents, which have a significant impact on flowing in the circuit processes. Analysis of processes in circuits is more complicated.

Currently alternating current is widely applied in engineering, because it is easily transformed and transmitted over large distances at high voltage and low loss. The economic effect is huge. In addition, electrical machinery and other electrical devices intended for use in alternating current circuits are relatively simple and reliable in operation.

Alternating current is used in various fields of electrical engineering (electric drive, electrothermics, telecommunications, radio engineering and so on). Its use in the construction industry and on construction sites allows you to implement many of the technological processes of the industry effectively.

The electric current changing over time is called the alternating current. If instantaneous values and current direction at equal intervals of time (periodically) repeat, such current is called ***periodically changing***.

The electric circuit of the periodic AC is classified depending on the curve shape of current and its frequency, nature settings, the complexity of the electrical equivalent schemes, destination.

The following types of electrical AC circuits are distinguished as: single-phase and polyphase; linear and nonlinear; with lumped and distributed parameters; with mutual inductance and without mutual inductance; simple and complex.

From all possible forms of periodic currents the most widely spread are sinusoidal currents. Compared with other currents sinusoidal currents have the following advantages: it allows the most economical carrying out, production, transmission, distribution and utilization of electrical energy. Using only sinusoidal currents can keep curve shapes of voltages and currents in all subcircuits of complex linear electric circuits.

In Ukraine, as in most countries, the production and transmission of electric energy is carried out using a three-phase sinusoidal current with frequency of 50 Hz (in the US, Canada, Japan and some countries in Central and South America 60 Hz).

Different techniques use a wide range of sinusoidal current depending on the technical needs. In aviation, for example, sinusoidal current with frequency of 400 Hz is successfully applied, because overall dimensions and weight of aircraft equipment are reduced at this frequency. In electrothermal installations the frequency range from 500 Hz to 50 MHz is used. Frequency of several hundred megahertz to milliard Hertz is used in radio engineering.

This topic will review some of the issues related to circuits with currents, changing on arbitrary law. In the analysis of such circuits electric equivalent schemes consisting of ideal elements is composed - an ideal source of EMF, a resistive element, an inductive element, a capacitive element, and the element of mutual induction. Each of these elements represents a particular phenomenon and is introduced into the equivalent scheme, when this phenomenon of replaced circuit want is considered.

The ideal elements R, L, C (fig. 4.1, *a*, *b*, *c*) are passive, so the positive directions of the currents and voltages in them coincide. In the power supply (fig. 4.1) the positive direction of the current and EMF coincide. The positive direction of the voltage of source opposite to the positive direction of its EMF.

On these directions positive values of the instantaneous powers of the receiver $p = u{\cdot}i$ and source $p = e{\cdot}i$ mean that the first one is the receiver, and the second - source. At negative values of the instantaneous powers the first is in source mode and the second mode is in the mode of the receiver.

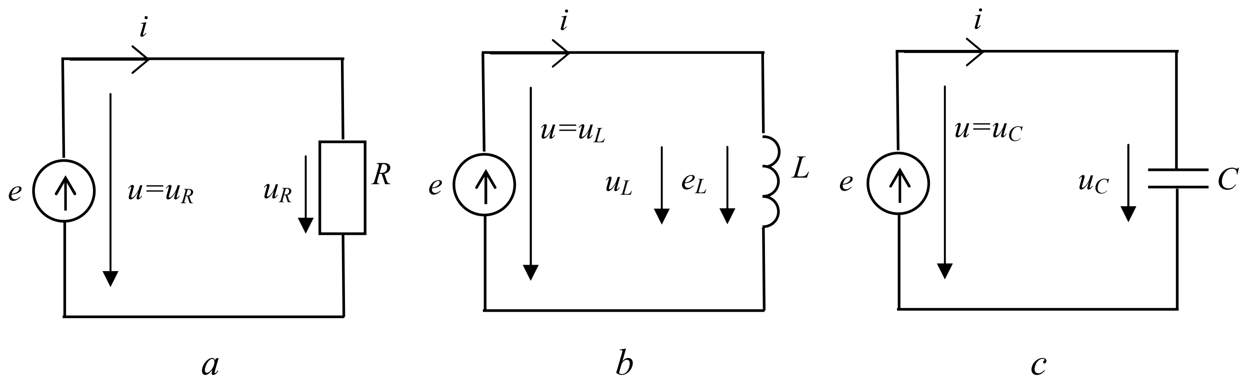


Figure 4.1: Positive direction of EMF, current and voltage ideal resistive (a) inductive (b) and capacitive (c) elements

Let us consider each of the elements of the circuit in detail, the equivalent scheme with the changing currents.

4.2 The elements of the equivalent schemes with changing currents

4.2.1 Resistive element of equivalent schemes. The resistive element is a passive element of equivalent scheme, characterizing the presence of the substituted element of irreversible processes of transformation of electric energy into other forms of energy. Parameter of resistive element is its active resistance r, in which electrical energy equal to the energy consumed by replaced real element of the electric circuit is absorbed.

Resistive element, for example, may characterize the real resistance of the conductor passing through it current. It dissipates the energy equal to the thermal energy generated in the conductor. It should be borne in mind that the resistance of the conductor greater than its DC resistance. This is because the density of the alternating current is unevenly distributed over the section of the conductor has a surface effect (displacement of current to the conductor surface), resulting in loss of energy to heat increase. Conductor resistance that characterize these losses increases.

Resistive element in the equivalent scheme also takes into account the presence of energy losses in the magnetic core of the coil included in the circuit with variable current.

In the future you will see other examples, when the resistive element in the equivalent scheme is characterized by presence of irreversible processes of transformation of electric energy into other forms of energy in the real circuit.

The voltage u and current i of a resistive element connected between each other by an equation, based on the Ohm's law for instantaneous values:

$$u_R = R \cdot i \, . \tag{4.1}$$

Equation (4.1) indicates a very important property of a resistive element: curve u_R repeats the curve shape of the current, i.e., curves of the voltage and current of a resistive element are similar.

Instantaneous power of resistive element is determined by the formula

$$p_R = u_R \cdot i = R \cdot i^2 \, , \tag{4.2}$$

it does not depend on the sign of the current and is always positive. Positive power indicates that the resistive element regardless of the direction of current energy always comes from a source and converts it into heat energy.

4.2.2 Inductive element of equivalent schemes. The inductive element of equivalent scheme of a real circuit with variable current is characterized by the presence of a changing magnetic field created by this current.

In electrical circuits of DC magnetic field created by the current is not changed and, therefore, ***does not affect the mode of operation of the circuit***. In circuits with changing current any change of current I in the element circuit, causing a change in its own flux linkage is ψ_L in accordance with the law of electromagnetic induction is accompanied by induction of EMF e_L in this element. This phenomenon is called ***self-induction*** and induced at this EMF – EMF of self-induction.

The law of electromagnetic induction (Faraday's law) is formulated as follows: the magnitude of the EMF e induced in a closed conductor is proportional to the rate of change of the magnetic flux Φ, penetrating this contour

$$e = \frac{-d\Phi}{dt} \, .$$

The minus sign reflects the Lenz's law [36], on which induced current always seeks to prevent change of the magnetic flux of contour.

According to the law of electromagnetic induction EMF of self-induction is determined by the rate of change of own flux linkage

$$e_L = -\frac{d\Psi_L}{dt} \ . \tag{4.3}$$

The quantity of flux self-linkage of inductive element ψ_L is proportional to the current i in it: $\psi_L = L \cdot i$.

Therefore, the formula for the EMF of self-induction can be written in more general form:

$$e_L = -L \cdot \frac{di}{dt} \ . \tag{4.4}$$

From (4.3) and (4.4) it is shown that the ***inductance L*** of any element of the circuit can be considered as ***the coefficient of proportionality between flux linkage ψ_L and the current i of the element***, or as the coefficient of proportionality between the rate of change of current of element of circuit *di/dt* and self-induction EMF e_L induced by this element.

In the study of circuits with self-induction EMF it is agreed that positive direction of the EMF of self-induction coincide with the positive direction of the current, which induces this EMF. Therefore, the arrow EMF e_L and arrow of current i in the scheme (see fig. 4.1, *b*) have the same direction.

In accordance with this, the actual direction of the EMF coincides with the direction indicated in the scheme by the arrow when the decrease of current in the circuit, when $di/dt < 0$, and $e_L > 0$; increasing in the circuit the current induces the EMF e_L, the actual direction of which is opposite to the direction indicated by the arrow.

For the inductor alternating current was lossless, between its contacts there must be a voltage equal in absolute value and in every moment in the opposite direction of the EMF of self-induction

$$u_L = -e_L = L\frac{di}{dt} = \frac{d\Psi}{dt} \ . \tag{4.5}$$

The basic unit of flux linkage and magnetic flux in the system SM – Weber (WB), 1 WB = 1 V·s; inductance is Henry (H), 1 H = WB/A = 1 V·A/s.

Let’s consider change of the current in the inductive element of scheme (fig. 4.1, *b*). If within a certain time interval the instantaneous value of the current is positive ($i > 0$) and is formed by a growing segment of the curve ($di/dt > 0$), then the voltage on the inductive element will be also positive ($u_L > 0$). This means that at the specified time interval, the direction of the voltage u_L coincides with the positive direction of the voltage indicated on the scheme by the arrow.

Instantaneous power of inductive element $p_L = u_L \cdot i$ will be positive ($p_L > 0$). Therefore, the energy in this interval of time comes from a source in the circuit and is converted to the energy of the magnetic field $L \cdot i^2/2$.

When positive ($i > 0$), but descending current in the coil ($di/dt < 0$) values of voltage and instantaneous power are negative ($u_L < 0$, $p < 0$). Energy from the magnetic field return back into the source. Thus, in the process of increase and decrease of current in the inductive element it is an exchange of energy between the source and the magnetic field.

4.2.3 Capacitive element of equivalent schemes. Between different parts of the electrical devices there is an electric field of charges being on these parts of the devices. A capacitive element (capacitor) is introduced into the equivalent scheme of a real circuit with variable current, when you want to take into account the influence of the changing electric field of the circuit elements.

If between the capacitor plates attached to the changing voltage u_C (see fig. 4.1, *c*), charge accumulates on its plates

$$q = C \cdot u_c\,, \tag{4.6}$$

where the coefficient of proportionality ***C*** is called the capacitance of the capacitor.

The voltage and current of the capacitive element are connected by the equation

$$i = \frac{dq}{dt} = C\frac{du_c}{dt}\ . \tag{4.7}$$

If the voltage u_c increases, then the current is positive ($i > 0$). This means that at this point in time, the current has a direction coinciding with conditional positive direction of voltage u_c (see fig. 4.1, *c*). The charge and energy of the electric field $W_E = C \cdot u_c^2/2 = q \cdot u_c/2$ increase. The energy from the source is transmitted to the electric field.

When the voltage u_C decreases, energy from field desreases and returns back into the source. Therefore, there is the exchange of energy between the source and the electric field in the capacitive element. If the law of change of the current is set in the capacitive element, the voltage on it can be determined from the equation

$$u_c = \frac{1}{C}\int idt + const\ . \tag{4.8}$$

When processes in circuits with changing currents constant *const* in equation (4.8) it is usually considered equal to zero, as the voltage u_C doesn't have the constant component (u_C = const = 0).

4.3 Effective and average values of the periodic voltages and currents

4.3.1 Effective values of the periodic voltages and currents. To assess the effectiveness of action of periodic current heat or electrodynamic action is used and it is compared to similar effect of DC for the same interval of time equal to one period.

The value of the periodic current, equal to such value of the DC current, which during one period produces the same heat or electrodynamic effect as the periodic current is called ***the operating value of the periodic current***. Operating values of current, voltage and EMF denote by capital letters without indices: *I, U, E.*

When evaluating periodic current i, using a thermal effect, accept that the constant current I and a periodic current i of the same resistive element with active resistance R produce the same amount of heat for a time T equal to one period

$$R \cdot I^2 \cdot T = \int_0^T Ri^2 dt \ .$$

From this equation, let's get the current value of the current:

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \ , \tag{4.9}$$

equal to the meansquare value of the periodic current.

Similarly, the effective values of voltage and EMF are the meansquare values of periodic voltage and EMF

$$U = \sqrt{\frac{1}{T}\int_0^T u^2 dt} \quad \text{and} \quad E = \sqrt{\frac{1}{T}\int_0^T e^2 dt} \ . \tag{4.10}$$

Electrical measuring instruments of thermal, electromagnetic, electrodynamic and electrostatic systems have moving parts, deflection of which are proportional to the meansquare values measured by them values. Therefore, these devices measure the effective values of periodic currents and voltages (see chapter 7).

4.3.2 Average values of the periodic voltages, currents and power. In General, under the average value of periodic functions it is understood their arithmetic average values for the period.

The average value of power for the period is determined by the equation

$$P_{cp} = \frac{1}{T}\int_0^T pdt = \frac{1}{T}\int_0^T uidt \ . \tag{4.11}$$

If the positive and negative half-wave of curve of change of power is not equal, then the average value is determined by the difference of the areas bounded by curves of the half-wave and x-axis.

As a rule, negative half-wave of periodic currents, voltages and EMF repeat their positive half-waves. Therefore, under the average values of periodic currents of voltages and EMF it is understood the average maximum value for their half-waves:

$$\begin{cases} I_{cp} = \dfrac{2}{T}\displaystyle\int_0^{T/2} idt, \\ U_{cp} = \dfrac{2}{T}\displaystyle\int_0^{T/2} udt, \\ E_{cp} = \dfrac{2}{T}\displaystyle\int_0^{T/2} edt \ . \end{cases} \tag{4.12}$$

4.4 Representation of sinusoidal voltages and currents on a plane of Cartesian coordinates

Sinusoidal currents and voltages can be represenedt graphically, using equations with trigonometric functions and they can be presenedt in the form of a rotating vectors in the Cartesian or the complex plane.

Shown in figure 4.2, *a* and *b* two graphs of sinusoidal EMF e_1 and e_2 are corresponded to the equations:

$$e_1 = E_{1m} \, sin(\omega t + \psi_{e1}) \, ,$$
$$e_2 = E_{2m} \, sin(\omega t - \psi_{e2}) \, .$$

The arguments of the sine functions of the $\omega t + \psi_{e1}$, and the $\omega t - \psi_{e2}$ are called the ***phases of the sinusoids***, and the value of the phase at the initial moment of time the ψ_{e1} and the ψ_{e1} – ***initial phase***.

If the first coming from the origin of the transition point of the sine waves from the instant negative to positive values of the instantaneous values (points a and b of curves in figure 4.2) consider the start of the first period of the sinusoid, then the initial phase to the left of the ordinate axis, is counted with a plus sign, and initial phase, located to the right of the ordinate axis, - with the sign "minus".

The value ω in the phases of the sinusoids, which characterizes the rate of change of the phase angle, is called the ***angular frequency***. As the phase angle of the sine wave during one period *T* is changed to 2π, the angular frequency

$$\omega = 2\pi/T = 2\pi f \, . \qquad (4.13)$$

On the joint consideration of two sinusoidally varying quantities with the same frequency, the difference of their phase angles equals to the difference between the initial phases, called ***phase shift angle***. The phase shift angle of the same name sinusoidal functions (EMF, voltages, currents) is denoted by α. The phase shift angle between the sine waves of voltage and current of element of the circuit is denoted by φ.

For sinusoids EMF e_1 and e_2 graphs of which are shown in figure 4.2, the phase shift angle

$$\alpha = \omega t + \psi_{e1} - (\omega t - \psi_{e2}) = \psi_{e1} + \psi_{e2} \, . \qquad (4.14)$$

For sine waves of voltage and current

$$u = U_m \, sin(\omega t + \psi_u) \, ,$$
$$i = I_m \, sin(\omega t + \psi_i),$$

graphs of which are shown in figure 4.3, the phase shift angle

$$\varphi = \psi_u - \psi_i \, .$$

Using the phase angle, the equations of voltage and current can be rewritten as:

$$u = U_m \sin(\omega t + \psi_u) \, ,$$
$$i = I_m \sin(\omega t + \psi_u - \varphi) \, ,$$

or

$$i = I_m \sin(\omega t + \psi_i) \, ,$$
$$u = U_m \sin(\omega t + \psi_i + \varphi) \, .$$

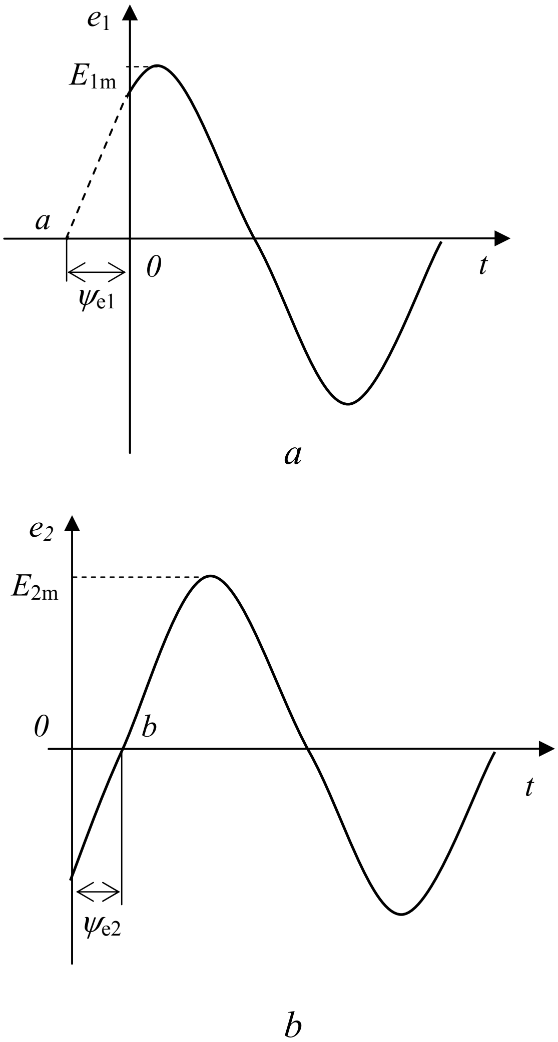


Figure 4.2: Graphs of sinusoidal EMF with different initial phases

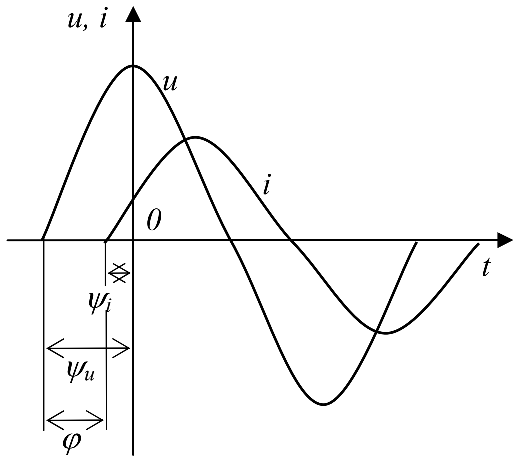


Figure 4.3: denotations of the initial phases and phase shift of sinusoidal voltage and current

These expressions show that sinusoidal current lags on the phase from the sinusoidal voltage on angle φ, or sinusoidal voltage is ahead of the phase the sinusoidal current on the angle φ.

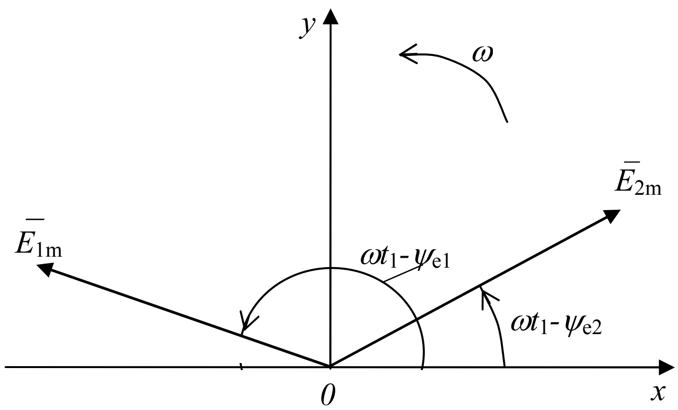


Figure 4.4: Image of sinusoidal EMF by rotating vectors at $t = t_1$

At the representation of sinusoidal EMF, voltages and currents by rotating vectors on the Cartesian plane from the origin take vectors equals to the amplitude values of the sinusoidal quantities and rotate these vectors counterclockwise with an angular velocity equal to the angular frequency ω. The phase angle when the rotation is counted from the positive x-axis, as shown in figure 4.4 for $t > 0$. The projection of the rotating vector on the y-axis is equal to the instantaneous values of EMF e_1 and e_2.

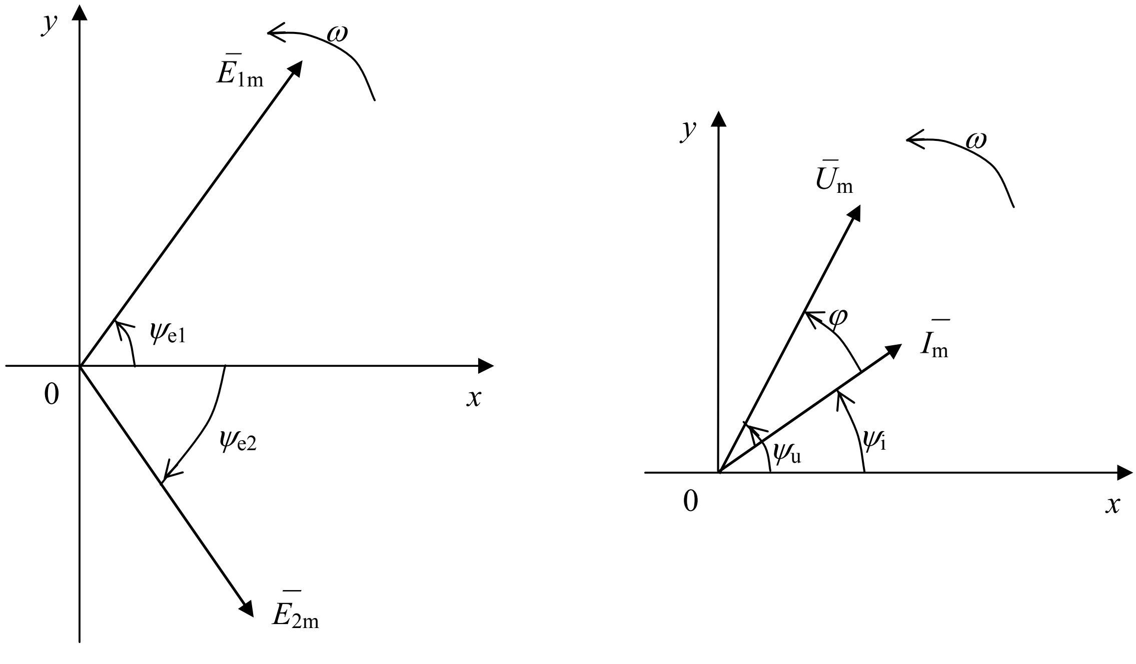


Figure 4.5: The location of the vectors representing the sine waves of EMF, voltage and current for the initial moment of time

The set of vectors representing a sinusoidal EMF, voltage and current of one frequency, called the ***vector diagrams***.

At construction the vector diagrams of vectors it is convenient to arrange for the initial time (t = 0). In this case, the vectors of sinusoids of EMF E1 and E2 (fig. 4.2) are placed, as shown in figure 4.5, *a*, while the vectors of sine waves of voltage and current *i* (fig. 4.3) – as in figure 4.5, *b*.

Vector diagrams are widely used in the analysis of the modes of operation of the circuits of sinusoidal current. Their use makes circuit calculation more intuitive and simple.

For example, at the point of branching circuit the total current equals to the sum of the currents i_1 and i_2 of the two branches:

Each of these currents are sinusoidal and can be represented by the equation

$$i_1 = I_{1m} \cdot \sin(\omega t + \psi_1) ,$$
$$i_2 = I_{2m} \cdot \sin(\omega t + \psi_2) .$$

The resulting current will also be sinusoidal:

$$i_3 = I_{1m} \cdot \sin(\omega t + \psi_1) + I_{2m} \cdot \sin(\omega t + \psi_2) = I_{3m} \cdot \sin(\omega t + \psi_3) .$$

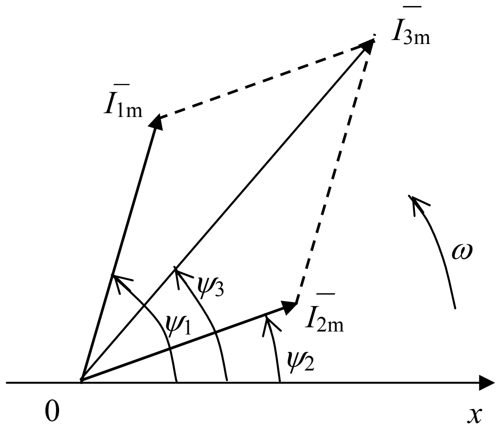


Figure 4.6: Replacement of addition of the instantaneous values of currents by addition of depicting their vectors

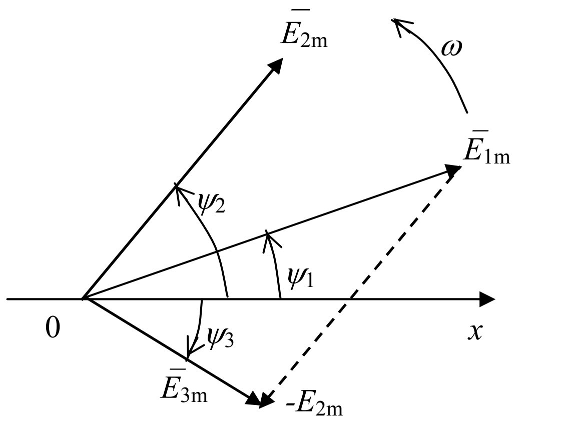


Figure 4.7 – Replacement of subtraction of the instantaneous values of EMF by subtraction depicting their vectors

Determination of the amplitude I_{3m} and initial phase ψ_3 of this current by the corresponding transformation of received equations are cumbersome and a little clearly. Using vector diagrams is easier. Figure 4.6 shows the initial position of vectors of the currents, the projection of which on the y-axis gives the instantaneous values of the currents for time $t = 0$. During the rotation of these vectors with the same angular speed ω their mutual arrangement will not change and the phase shift angle between them will remain equal to $\alpha = \psi_1 - \psi_2$.

As the algebraic sum of the projections of the vectors on the y-axis is equal to the instantaneous value of the total current, the vector of the total current is equal to the geometric sum of the vectors of currents:

$$\overline{I}_{3m} = \overline{I}_{1m} + \overline{I}_{2m} \ .$$

Building of a vector diagram in scale allows you to set the values I_{3m} and the ψ_3 from the chart.

Subtraction of the instantaneous values of, for example, EMF $e_3 = e_1 - e_2$ where $e_1 = E_{1m} \cdot \sin(\omega t + \psi_1)$ and $e_2 = E_{2m} \cdot \sin(\omega t + \psi_2)$ can be replaced by subtraction depicting their vectors $\overline{E}_{3m} = \overline{E}_{1m} - \overline{E}_{2m}$ as shown in figure 4.7. The chart to determine the amplitude $\overline{E}_{3m}$ of the result vector EMF to vector $\overline{E}_{1m}$ it is added reverse vector $-\overline{E}_{2m}$. In accordance with the diagram resulting EMF is determined by the equation

$$e_3 = E_{3m} \cdot sin(\omega t - \psi_3) \ .$$

4.5 Complex form of presentation of sinusoidal voltages and currents

Currently calculations of electrical circuits with sinusoidal EMF, voltages and currents are effective. It is a complex method of analysis. On the representation of rotating vectors of sine values on the complex plane the x-axis of plane of Cartesian coordinates combine with the axis of the valid or real values (axis +1) of complex plane. Then the instantaneous sinusoidal quantities get on the axis of imaginary parameters (axis +j).

In order to represent sinusoidal EMF

$$e = E_m \cdot \sin(\omega t + \psi), \quad (4.15)$$

with an initial phase ψ let's draw on the complex plane (fig. 4.8) from the origin at an angle ψ to the axis of valid values the vector whose length in scale of the building is equal to the amplitude of the EMF E_m. The end of this vector is at a point which corresponds to a certain complex number – the ***complex amplitude*** of the EMF:

$$\dot{E}_m = E_m e^{j\psi} = E_m \angle \psi . \quad (4.16)$$

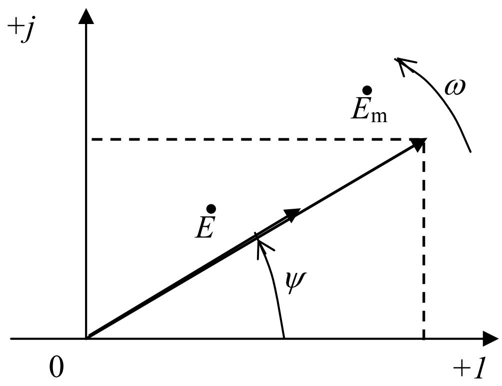


Figure 4.8: Respresention of sinusoidal EMF by rotating vector on the complex plane

With increase in time the phase of EMF $\omega t + \psi$ the angle between the vector and the axis of valid values increases, i.e. it is turned out the rotating vector

$$E_m e^{j(\omega t+\psi)} = E_m \cos(\omega t + \psi) + jE_m \sin(\omega t + \psi) .$$

As you can see, the imaginary part of the rotating vector is equal to a given sinusoidal EMF.

The vector on the complex plane, whose length in scale of the building is equal to the operative value of sinusoidal EMF, is called the ***complex operative value of sinusoidal EMF***

$$\dot{E} = \frac{\dot{E}_m}{\sqrt{2}} = Ee^{j\psi} = E\angle\psi . \quad (4.17)$$

As indicated by the itself vector on the complex plane (fig. 4.8).

There are three forms of complex value of sinusoidal EMF, current and power. Let's consider them on the case of sinusoidal EMF.

Algebraic notation $\dot{E} = \mathrm{Re}\,\dot{E} + j\,\mathrm{Im}\,\dot{E}$, or other designation $\dot{E} = E' + jE''$, where $E' = \mathrm{Re}\,\dot{E} = E\cos\psi$ and $E'' = \mathrm{Im}\,\dot{E} = E\sin\psi$ – real and imaginary components of the complex values of sinusoidal EMF,

$$E = \sqrt{(\mathrm{Re}\,\dot{E})^2 + (\mathrm{Im}\,\dot{E})^2} \ ; \quad \psi = arctg\frac{\mathrm{Im}\,\dot{E}}{\mathrm{Re}\,\dot{E}} .$$

Algebraic notation is more convenient with the addition and subtraction of complex numbers.

Trigonometric notation is derived from algebraic and comfortable during the transition from exponential to algebraic notation. Considering the fact that $\cos\psi = E'/E$, $\sin\psi = E''/E$ trigonometric notation has the form $\dot{E} = E\cos\psi + jE\sin\psi$.

Exponential notation is derived from the trigonometric and looks $\dot{E} = Ee^{j\psi} = E\angle\psi$. This notation is more convenient when multiplication, division, extraction of roots of complex numbers.

The transition from exponential form of writing of the sine values for trigonometric performed is made using the Euler formula

$$e^{j\psi} = \cos\psi + j\sin\psi \ .$$

So it is, if the instantaneous value of voltage (current, and so on) in the form of a sine wave $e = E_m \cdot \sin(\omega t + \psi)$, then the complex amplitude is recorded at first record in exponential form, and then by the formula of Euler is converted to algebraic form.

When analyzing circuits of sinusoidal current, mainly complex operative values of sine values is applied, abbreviated ***complex values*** and the corresponding vectors on the complex plane – ***vectors of complex values***.

Using a vector diagram, addition and subtraction of complex values can be replaced by addition and by subtraction of the corresponding vectors. This simplifies the calculations and makes them clear.

The direction of sinusoidal quantities (current, voltage) in the circuit periodically changes, but one of the two directions is taken positive. This direction is chosen arbitrarily and is shown by the arrow in the diagram of corresponding subcircuit. On selected positive direction sinusoidal amount is represented by the instantaneous value (for example, for a voltage $e = E_m \cdot \sin(\omega t + \psi)$) and the corresponding complex value ($\dot{E} = E\angle\psi$ – see fig. 4.9). Therefore, one-to-one representation of sinusoidal currents, voltages and other quantities in the form of instantaneous and integrated values is corresponded their the same positive directions (fig. 4.9).

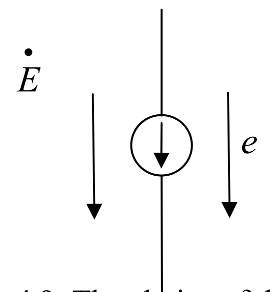


Figure 4.9: The choice of the positive direction of sinusoidal EMF

Usage of complex numbers allows from the geometric addition or subtraction of vectors on a vector diagram move to an algebraic action on complex numbers of these vectors. For example, to determine the complex amplitude of the resulting current (see fig. 4.6) it is enough to sum up two complex numbers, the corresponding by complex amplitudes of the currents which are summed up:

$$\dot{I}_{3m} = \dot{I}_{1m} + \dot{I}_{2m} = \dot{I}_{3m}\, e^{j\psi_3} \ .$$

To determine the complex amplitude of the resulting EMF (see fig. 4.7) it is enough to determine the difference of complex numbers corresponding to complex amplitudes of EMF $\dot{E}_{1m}$ and $\dot{E}_{2m}$

$$\dot{E}_{3m} = \dot{E}_{1m} - \dot{E}_{2m} = \dot{E}_{3m}\, e^{-j\psi_3} \ .$$

4.6 Operative and average values of sinusoidal voltages and currents

To determine the operating value of sinusoidal current let's use the formula (4.9), substituting it instead of instantaneous values of sinusoidal current its expression

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t dt} \ .$$

As

$$\int_0^T I_m^2 \sin^2 \omega t dt = I_m^2 \int_0^T \frac{1-\cos 2\omega t}{2} dt = \frac{I_m^2}{2} T \ ,$$

Then the effective value of sinusoidal current is less than its amplitude in $\sqrt{2}$ time:

$$I = I_\text{m} / \sqrt{2} = 0{,}707\ I_\text{m} \ . \qquad (4.18)$$

Similarly it is determined by the effective value of sinusoidal voltage and EMF:

$$U = U_\text{m} / \sqrt{2} \ \text{and} \ \ E = E_\text{m} / \sqrt{2} \ . \qquad (4.19)$$

Scales of electrical measuring instruments are used for measurement of sinusoidal currents and voltages, graduated to the current values, and to determine the amplitudes of the sinusoidal value of their indication it is sufficient to increase in $\sqrt{2}$ time.

Under the average value of sinusoidal currents it is realized their average values over the half-period. If the current $i = I_\text{m} \sin\omega t$, then its average value

$$I_{cp} = \frac{\int_0^T i dt}{T/2} = \frac{2}{T}\int_0^T I_m \sin \omega t dt = \frac{2}{\pi} I_m \ , \qquad (4.20)$$

Therefore, the average value of the sinusoidal current is $2/\pi$ of its peak value. Similarly, it is determined by the average value of sinusoidal voltage and EMF:

$$U_{cp} = \frac{2}{\pi} U_m \ , \qquad E_{cp} = \frac{2}{\pi} E_m \ .$$

The average value of current is measured by the devices of electromagnetic system (see section 7.4), a measuring circuit which includes a rectifier of current.

4.7 Complex form of Kirchhoff's laws

In the general case on the first Kirchhoff's law in complex form the algebraic sum of the integrated values of currents of branches converging at a node is equal to zero

$$\sum_{k=1}^n \dot{I}_k = \sum_{k=1}^n I_k' + j\sum_{k=1}^n I_k'' = 0 \ , \qquad (4.21)$$

where: $\dot{\boldsymbol{I}}_\text{k}$ is the complex value of the current in the *k*-th branch has two form of record

$$\dot{I}_k = I_k' + jI_k'' = I_{ak} + jI_{pk} \ ,$$

where: $I'_k = I_{ak}$ is the active component of the current of k-th branch (in the theory of complex numbers is a valid component);
$I''_k = I_{pk}$ – reactive component of the current of k-th branch (in the theory of complex numbers – imaginary component);
n is the number of branches converging at the node.

According to the second Kirchhoff law for any closed contour, the algebraic sum of the integrated values of the voltage drops on its sections is equal to the algebraic sum of the integrated values of EMF, operating in the contour:

$$\sum_{k=1}^{n} \dot{U}_k = \sum_{k=1}^{m} \dot{E}_k \, , \tag{4.22}$$

where: $\dot{U}_k$ is the complex value of the voltage drop on the k-m section of contour;
$\dot{E}_k$ – integrated value of the EMF of the k-th section of the contour;
n is the number of subcircuits with passive elements, m - is the number of sections with EMF.

For voltage and EMF, as in the case of complex currents, there are two forms of record

$$\dot{U}_k = U'_k + jU''_k = U_{ak} + jU_{pk} \, , \tag{4.23}$$

$$\dot{E}_k = E'_k + jE''_k = E_{ak} + jE_{pk} \, , \tag{4.24}$$

where $U'_k = U_{ak}$ and $E'_k = E_{ak}$ is the active component of the voltage drop and EMF in the k-th branch (in the theory of complex numbers is a valid component);
$U''_k = U_{pk}$, $E''_k = E_{pk}$ – reactive component of the voltage drop and EMF in the k-th branch (in the theory of complex numbers imaginary component).

The rule of signs in the complex form of these equations under the laws of Kirchhoff remains the same as in DC circuits.

Consider the record of Kirchhoff's laws on the example of scheme of electrical circuit (fig. 4.10, *a*), in which they are sinusoidal EMF, instantaneous value of which is given by the equations

$$e_1 = E_{1m}\sin(\omega t + \psi_1),$$
$$e_2 = E_{2m}\sin(\omega t + \psi_2) \, .$$

Under the action of these EMF in all branches of the circuit sinusoidal currents appear and in some sections a sinusoidal voltage drops will arise.

To determine the currents of this scheme on given values of EMF e_1 and e_2 and resistance (resistive $R_1 – R_5$, inductive L_4 and capacitor C_5) according to the laws of Kirchhoff it is necessary to make a system of five equations. The procedure of the composition of equations is the same as in the case of DC current (see section 2.7).

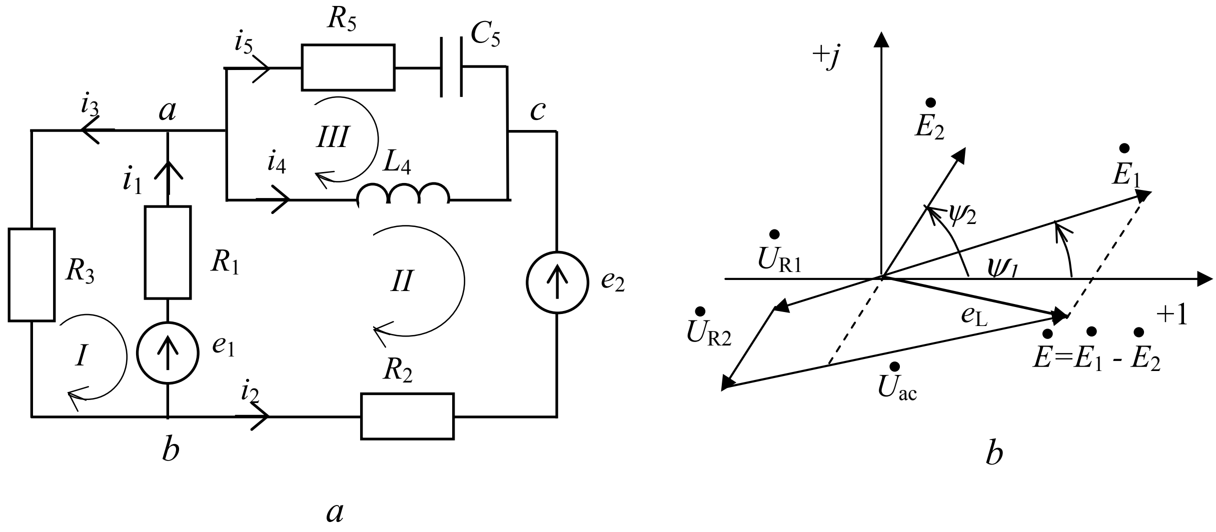


Figure 4.10: Scheme of electrical circuit of AC current (a) and vector diagram of the EMF and the voltage of the second contour (b)

The equations on the first law of Kirchhoff, composed for nodes *a* and *b* will be:

for node a: $\dot{I}_1 - \dot{I}_3 - \dot{I}_4 - \dot{I}_5 = 0$, (4.25)

for node b: $-\dot{I}_1 - \dot{I}_2 + \dot{I}_3 = 0$. (4.26)

Equations composed for contours (I), (II) and (III) the second Kirchhoff's law, when indication of the bypass of contours in a clockwise direction, will be:

for contour I: $-R_1\dot{I}_1 + R_3\dot{I}_3 = -\dot{E}_1$; (4.27)

for contour II: $R_1\dot{I}_1 + jX_4\dot{I}_4 - R_2\dot{I}_2 = \dot{E}_1 - \dot{E}_2$; (4.28)

for contour III: $(R_5 - jX_5)\dot{I}_5 - jX_4\dot{I}_4 = 0$. (4.29)

Thus, the received equations (4.25) - (4.29), are Kirchhoff's laws, written in complex form for the scheme of electrical circuit in figure 4.10, *a*.

In figure 4.10, *b* as the example it is shown a vector diagram of EMF and voltage of contour II, which illustrates the second Kirchhoff's law in complex form.

Key findings

1. Due to a number of advantages in technical-economic indicators AC (sinusoidal) current was more prevalent in comparison with DC current.

2. In electrical circuits of AC current, unlike DC current inductive and

capacitive elements influence to the mode of operation of the circuit

3. To estimate the effective value of alternating current to produce a comparison of the effect from its passage through the scheme element with the same effect when passing through this element of DC.

4. The average value of an alternating (sinusoidal) current is determined by half of its period.

5. Sinusoidal currents and voltages can be represented in the form of trigonometric functions, graphs of changes over time, in the form of a rotating vectors and complex numbers.

6. Integrated values of sinusoidal currents and voltages can be represented in exponential form, trigonometric form and algebraic form.

7. Application of complex numbers to analyzing electrical circuits of AC allows to simplify the calculations and to make them more visible.

Control questions

1. What is alternating current?
2. What is periodic current?
3. Give a description of resistive (inductive, capacitive) element of an electric circuit.
4. What is the inductance?
5. What is the difference of the physical processes in the inductive element during the passage through it direct and alternating current?
6. What is meant by capacity?
7. What is the difference between physical processes in the capacitive element in his work in DC and AC?
8. What is the operating value of the periodic current?
9. What is the operating value of the periodic voltage?
10. What is the average value of a periodic sinusoidal current?
11. Explain the parameters of sinusoidal current.
12. Explain the parameters of sinusoidal current voltage.
13. What is meant by vector diagram? What are its advantages in the analysis of electrical circuit?
14. What form of writing complex values of sinusoidal currents are applied in practice?
15. Write Kirchhoff's laws in complex notation and give explanations to them.

5 THE ELECTRIC CIRCUIT OF SINGLE-PHASE AC

Key concepts: resistance (resistive, inductive, capacitive), impedance (full, active, reactive, inductive, capacitive), real power (reactive, apparent), triangle of resistance, the resonance of voltages, the active conductivity (reactive, full), active and reactive components of current, the resonance of currents, power coefficient.

5.1 Electrical circuit with a resistive element

Let's assume that at the input to the circuit with a resistive element having a ***active resistance*** *R* (fig. 5.1, *a*) is given as sinusoidal voltage

$$u = U_m \cdot \sin(\omega t + \psi_u) . \qquad (5.1)$$

You need to define, how the current and power of this circuit will change.

The current in the circuit can be determined using Ohm's law for instantaneous values:

$$i = u/R = U_m \cdot \sin(\omega t + \psi_u)/R,$$

or

$$i = I_m \cdot \sin(\omega t + \psi_i), \qquad \psi_i = \psi_u . \qquad (5.2)$$

From a comparison of equations (5.2) from (5.1) we see that the sine wave of current has the same frequency as the sine wave of voltage and coincides with it in phase.

The amplitude of the current is connected with the voltage amplitude with ratio

$$I_m = U_m/R . \qquad (5.3)$$

If the left and right side of the expression (5.3) is divided by $\sqrt{2}$, you will get a new formula for the effective values of current and voltage

$$I = U/R , \qquad (5.4)$$

expressing Ohm's law for circuit with a resistive element. According to this law, the effective value of current in circuit with the resistive element is equal to the effective value of the voltage divided by the active resistance of the element.

The instantaneous value of the power of this circuit is equal to the product of the instantaneous values of voltage and current:

$$p = ui = U_m I_m \sin^2(\omega t + \psi_u) = U_m I_m \frac{1 - \cos 2(\omega t + \psi_u)}{2},$$

or

$$P = I \cdot U\cos(\omega t + \psi_u) . \qquad (5.5)$$

The average for the period the value of power

$$P = \frac{1}{T}\int_0^T p dt = \frac{1}{T}UI\int_0^T dt - \frac{1}{T}UI\int_0^T \cos 2(\omega t + \psi_u)dt ,$$

or

$$P = I \cdot U .$$

If in the expression for the average power voltage is substituted by the value $U = R{\cdot}I$ from (5.4), we find that ***the average value of power in the circuit is equal to its real (active) power***.

$$P = I{\cdot}U = R{\cdot}I^2\ . \tag{5.6}$$

To illustrate the changes of voltage, current and power in the resistor in figures 5.1, *b* and 5.1, *c* on the equations (5.1), (5.2) and (5.5) the graphs *p, u* and *i* were built for the case when the initial phase is $\psi_u = 0$.

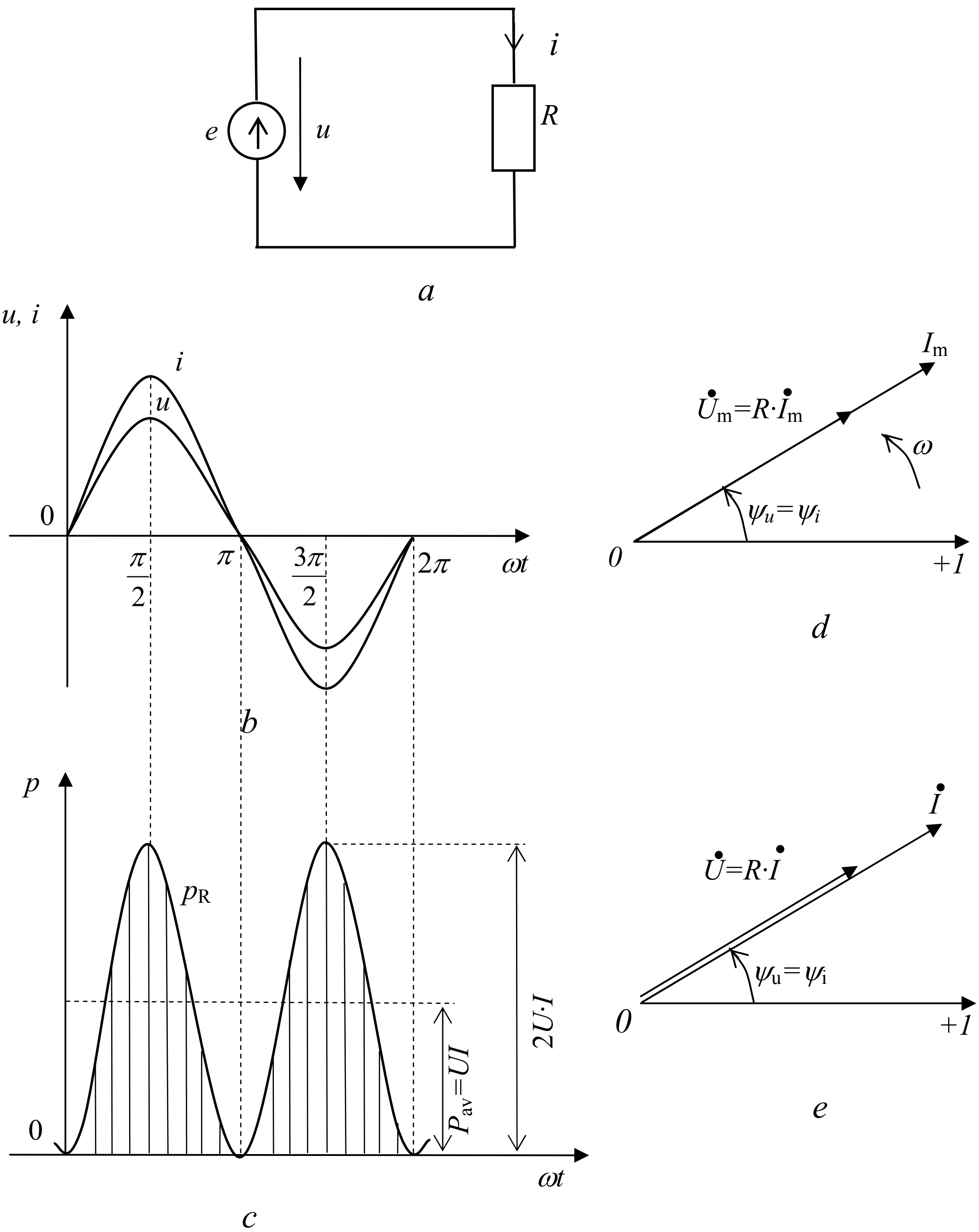


Figure 5.1: Circuit with a resistive element: a – scheme; b – graphs of instantaneous values of voltage and current; c – graphics of instantaneous values of power; d – vector diagrams of complex amplitudes; e - vector diagrams of complex values of current and voltage

It is seen from the graphs that the instantaneous power in the resistor pulses from zero to $2P = 2U \cdot I$, while remaining is always positive. This means that ***in any direction of the current in the resistive element energy comes from a source in the circuit and is converted into heat energy***. The magnitude of the converted energy for the period can be characterized by the shaded area, limited power curve and the x-axis.

To construct vector diagrams of the voltage and current of circuit on the complex plane let's write their complex amplitude in accordance with equations (5.1) and (5.2):

$$\dot{U}_m = U_m e^{j\psi_u} ; \qquad \dot{I}_m = I_m e^{j\psi_i} = I_m e^{j\psi_u} . \tag{5.7}$$

But the amplitude of the voltage of the formula (5.3) can be expressed through the amplitude of the current, therefore, the complex amplitude of the voltage can be written differently:

$$\dot{U}_m = U_m e^{j\psi_u} = R \cdot I_m e^{j\psi_u} = R \cdot \dot{I}_m . \tag{5.8}$$

From the expression (5.8) it follows that the vector representing the sine wave of voltage on the resistor coincides in direction with the vector representing the sine wave of current. Vector diagram of the complex amplitudes of the voltage and current built on figure 5.1, *d*.

However, for calculation circuits of sinusoidal current instead of a vector of complex amplitudes it is accepted to construct the vectors of complex operating values of voltage $\dot{U}$ and current $\dot{I}$. These vectors respectively coincide in direction with vectors $\dot{U}_m$ and $\dot{I}$ differ from them only by its size:

$$\dot{I} = \frac{\dot{I}_m}{\sqrt{2}} = Ie^{j\psi_u} , \qquad \dot{U} = \frac{\dot{U}_m}{\sqrt{2}} = R \cdot Ie^{j\psi_u} = R \cdot \dot{I} . \tag{5.9}$$

In figure 5.1, *e* vector diagram is built of the complex values of the voltage and current of a resistive element. From the diagram it follows that ***the vector of the voltage on the resistor coincides in direction with the vector of current and is equal to the complex value of the current multiplied by the active resistance of resistor***.

From (5.9) we can obtain the expression

$$\dot{I} = \frac{\dot{U}}{R} \tag{5.10}$$

which specify the Ohm's law in complex form for the circuit with a resistive element. According to this law, ***the complex value of current in a circuit with a resistor equal to the complex value of the voltage divided by the active resistance of the resistor***.

5.2 Electrical circuit with an ideal inductive coil

Let's suppose that in a coil with inductance L, the resistance of which is very small (R is 0), there is a sinusoidal current (fig. 5.2, *a*)

$$i = I_m \sin(\omega t + \psi_i) , \tag{5.11}$$

which brings to it EMF of self-induction

$$e_L = -L\frac{di}{dt} = E_{Lm}\sin(\omega t + \psi_i - \frac{\pi}{2}) , \tag{5.12}$$

where $\boldsymbol{E_{Lm}} = \omega \cdot L \cdot I_m$ – the amplitude of the sinusoidal EMF.

From (5.11) and (5.12) it follows that the sine wave of EMF of self-induction lags in phase from the sine wave of current at the phase shift angle $\pi/2$.

External voltage of source $u = u_L$ balanced by EMF of self-induction e_L. Sine wave of this voltage

$$u = U_{Lm}\sin(\omega t + \psi_i + \frac{\pi}{2}) . \tag{5.13}$$

From (5.13) we see that the sine wave of voltage of an ideal coil is ahead on the phase of the sine wave of current at the phase shift angle $\pi/2$.

The amplitude of the sine wave of voltage on the coil

$$U_{Lm} = \omega \cdot L \cdot I_m . \tag{5.14}$$

The effective value of this voltage

$$U_L = \omega \cdot L \cdot I . \tag{5.15}$$

The complex amplitude of current and voltage

$$\dot{I}_m = I_m e^{j\psi_i} ,$$

$$\dot{U}_{Lm} = U_{Lm} e^{j(\psi_i + \pi/2)} = \omega L I_m e^{j\psi_i} e^{j\pi/2} = \omega L \dot{I}_m ,$$

or

$$\dot{U}_{Lm} = j\omega L \dot{I} .$$

Complex values of current and voltage of coil:

$$\dot{I} = I e^{j\psi_i} , \qquad \dot{U}_L = j\omega L \dot{I} . \tag{5.16}$$

In figure 5.2, *b* the graphs of the sine of voltage u_L current i, and the self-induced EMF e_L, are shown and figure 5.2, *d* – corresponding to these sinusoids vectors of their complex values $\dot{U}_L$, $\dot{I}$, and $\dot{E}_L$ for the case $\psi_i = 0$.

The product ω·L has the dimension of resistance

$$[\omega \cdot L] = 1/\text{s} \cdot \text{H} = 1/\text{s} \cdot \text{ohm} \cdot \text{s} = \text{ohm}.$$

***It is denoted by X_L and called inductive resistance of coil*:**

$$X_L = \omega \cdot L = 2\pi f \cdot L . \tag{5.17}$$

The value $j\omega \cdot L = jX_L$ is called ***complex inductive impedance of an ideal coil or complex inductive resistance***.

The inductive resistance is directly proportional to the inductance of the coil and the frequency of the current in it.

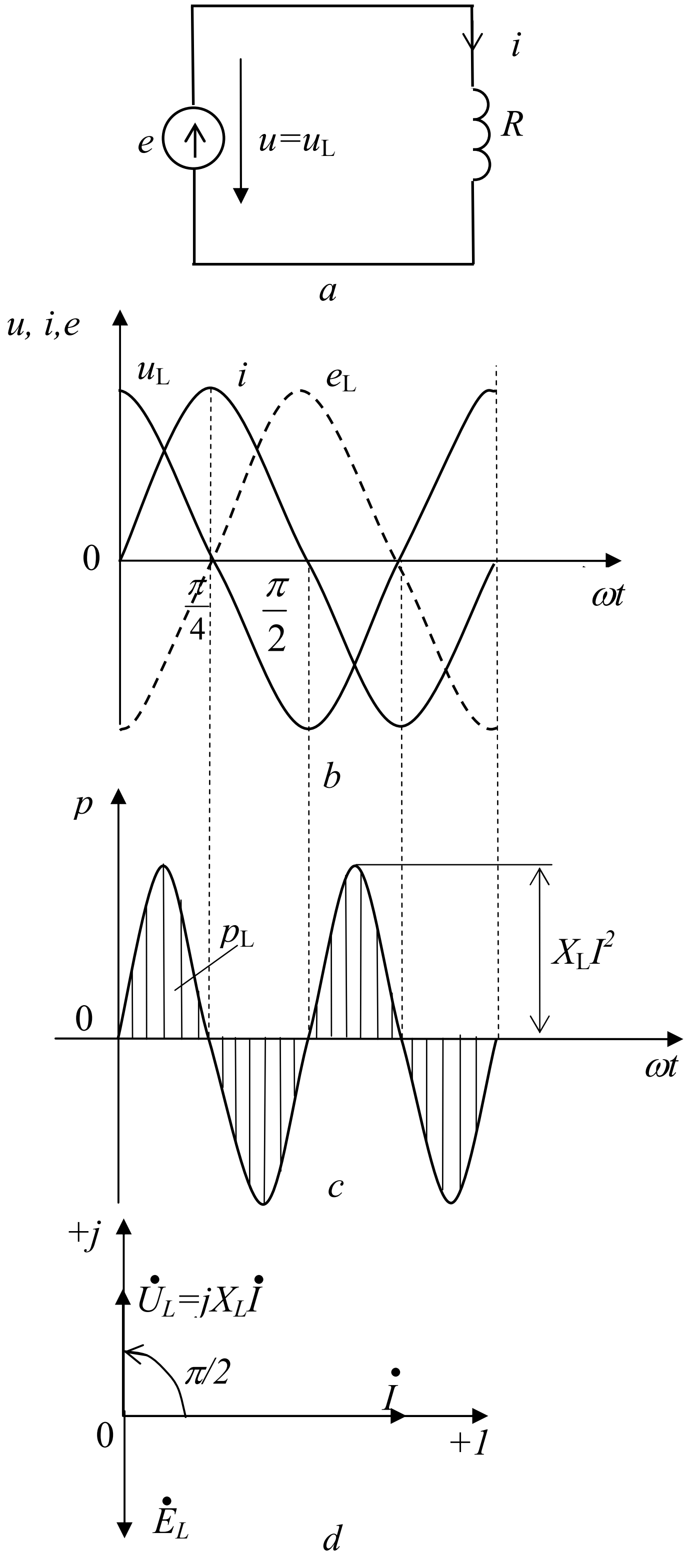


Figure 5.2: Scheme (*a*), graphs of instantaneous values (*b*, *c*) and vector diagram (*d*) of circuit with an ideal coil

According to the expression (5.15) the effective value of induced voltage U_L of coil is equal to the effective value of the current I multiplied by the inductive resistance of coil.

Equations (5.16) show that the ***voltage vector on an ideal coil is ahead of the phase of the vector of current on the phase shift angle π/2***.

From equations (5.16), we can obtain the formula for the complex value of current

$$\dot{I} = \frac{\dot{U}_L}{j\omega L} = \frac{\dot{U}_L}{jX_L}, \tag{5.18}$$

which expresses Ohm's law in complex form for the circuit with ideal inductive coil. That is, in accordance with Ohm's law ***complex value of current*** $\dot{I}$ ***in a circuit with an ideal coil is equal to the complex value of the voltage*** $\dot{U}_L$ ***on the coil divided by the complex value of the inductive reactance of the coil*** jX_L.

The instantaneous value of the power in a circuit with an ideal coil

$$p_L = u_L i = U_{Lm} I_m \sin(\omega t + \psi_i + \pi/2)\sin(\omega t + \psi_i) =$$

$$= U_{Lm} I_m \frac{1}{2}[\cos(+\pi/2) - \cos(2\omega t + 2\psi_i + \pi/2) =$$

$$= -U_L I \cos(2\omega t + 2\psi_i + \pi/2),$$

or

$$p_L = U_L I \sin(2\omega t + 2\psi_i) \ . \tag{5.19}$$

The graph of this power for the case of the $\psi_i = 0$ it is shown in figure 5.2, *c*.

In the first quarter of the period when the current and voltage are positive, power is also positive. Energy $W_L = L \cdot i^2/2$ from the source passes in the circuit and is spent on the creation of a magnetic field. By the end of the first quarter of the period field has a maximum energy $L \cdot I_m^2/2$, it is proportional to the shaded area bounded by the x-axis and the first half-wave of sine wave of power.

In the second quarter the current *i* decreases, but remains positive. Voltage u_L and the power p_L are negative. The magnetic field energy is returned back to the source. By the end of the second quarter of the period the entire stock of energy $L \cdot I_m^2/2$ will be returned to the source. Therefore, the average for the period value of the power of circuit with an ideal coil is equal to zero:

$$P_L = \frac{1}{T}\int_0^T p_L dt = 0 \, .$$

Thus, in circuit ***with an ideal coil there is a continuous fluctuation (change) of energy between the source and the magnetic field of the coil without expenditure of energy of source***.

By analogy with the active power in the circuit with an ideal resistor, in circuit with an ideal coil the concept of ***inductive reactive power*** is introduced

$$Q_L = U_L \cdot I = X_L \cdot I^2 \, . \tag{5.20}$$

Reactive inductive power has the same dimension as the active power. But with the purpose of better understanding the units of measurement of reactive power another name – volt-ampere reactive (VAr) is introduced.

5.3 Electrical circuit with an ideal capacitor

Let the capacitor (fig. 5.3, *a*), the dielectric of which is perfect and has no energy losses, sinusoidal voltage to be brought

$$u_C = U_{Cm} sin(\omega t + \psi_u). \qquad (5.21)$$

The current in the capacitor (4.7)

$$i = C\frac{du_C}{dt} = \omega \cdot C \cdot U_{Cm} cos(\omega t + \psi_u),$$

or

$$i = I_m \sin(\omega t + \psi_u + \pi/2), \qquad 5.22)$$

where $\boldsymbol{I_m}$ – the amplitude of the current

$$I_m = \omega \cdot C \cdot U_{Cm} . \qquad (5.23)$$

The effective value of current

$$I = \omega \cdot C \cdot U_C = \frac{U_C}{\frac{1}{\omega \cdot C}} = \frac{U_C}{X_C}. \qquad (5.24)$$

Value

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot f \cdot C} \qquad (5.25)$$

has the dimension of resistance

$$[X_C] = \frac{c \cdot B}{coulomb} = \frac{B \cdot c}{A \cdot c} = ohm,$$

and is called the ***reactance of a capacitor*** or a ***simple capacitive resistance***.

Capacitive reactance is inversely proportional to the frequency of the power supply and the capacitor capacitance.

Comparing equations (5.21) and (5.22), we see that the ***sine wave of capacitive current is ahead of the phase of the sine wave of voltage on the capacitor on the phase shift angle π/2***.

On equations (5.21) and (5.22) in figure 5.3, *b* the graphs i, u_c were built and on figure 5.3, *d* – vectors of the effective values of current and voltage on the capacitor for the case when the initial phase is $\psi_u = 0$.

The complex amplitudes of the voltage and current corresponding to the equations (5.21) and (5.22), are equal to:

$$\dot{U}_{Cm} = U_{Cm} e^{j\psi_u} \quad ,$$

$$\dot{I}_m = I_m e^{j(\psi_u + 90^o)} = \frac{U_{Cm} e^{j\psi_u}}{X_C} e^{j90^o} = j\frac{U_{Cm} e^{i\psi_u}}{X_C} = j\frac{\dot{U}_{Cm}}{X_C} = \frac{\dot{U}_{Cm}}{-jX_C}.$$

Dividing the right and left side of the last expression on $\sqrt{2}$ we get the equation linking the complex values of current and voltage:

$$\dot{I} = \frac{\dot{U}_C}{-jX_C}, \qquad (5.26)$$

where: $\boldsymbol{jX_C}$ – complex of capacitance.

Equation (5.26) expresses Ohm's law in complex form for a subcircuit with an ideal capacitor: ***the complex of current of the capacitor is equal to the complex of voltage divided by the complex of capacitance of the capacitor.***

The capacitor voltage is determined from the formula (5.26), is equal to the product of its current and complex of capacitance:

$$\dot{U}_C = -jX_C \cdot \dot{I}. \qquad (5.27)$$

From equation (5.27) or from the vector diagrams of figure 5.3, *d*, it follows that ***the voltage vector on an ideal capacitor lags on the phase of the vector current in the phase shift angle π/2***.

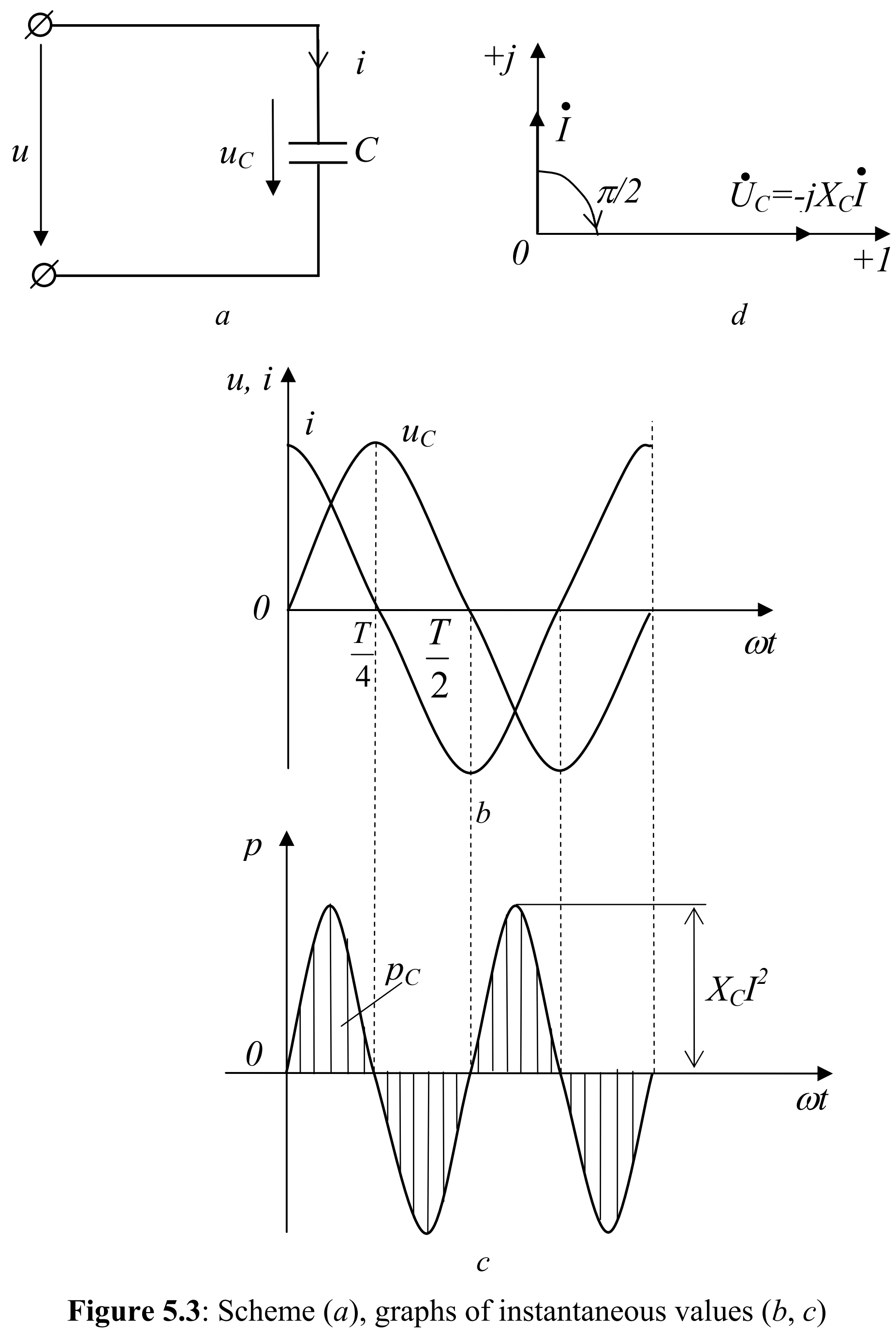


Figure 5.3: Scheme (*a*), graphs of instantaneous values (*b*, *c*) and vector diagram (*d*) of circuit with an ideal capacitor

The instantaneous power value

$$p_C = u_C i = U_{Cm} I_m \sin(\omega t + \psi_u)\sin(\omega t + \psi_u + \pi/2),$$

or $$p_c = U_c I \sin 2(\omega t + \psi_u). \quad (5.28)$$

The average for the period value of the power of circuit with an ideal capacitor is zero:

$$P_C = \frac{1}{T}\int_0^T p_C dt = 0. \quad (5.29)$$

As in circuit with an ideal coil, the processes of fluctuation of energy $W_c = C \cdot u_C^2/2$ take place and the alternation of periods of time during which energy from the source is stored in the electric field of the capacitor, with periods of time when the energy of the circuit is returned back to the source. To illustrate these processes in figure 5.3, *c* we built the diagram of changes of the power of circuit for the case $\psi_c = 0$. Comparing it with the graphs of variations of voltage and current in the circuit, we see that in the first quarter of period the value of u_c, i and p_c are positive, the capacitor is charging. At this time there is an accumulation of energy in the electric field of the capacitor due to the energy received from the power source. By the end of the first quarter of the period field stores the maximum energy $C \cdot U_{Cm}^2/2$. During the second quarter of the period the voltage u_c decreases, the capacitor is discharged. The current i, and power p_c are negative. The energy of the field is returned back to the source.

The amplitude of the power oscillations in a circuit with a capacitor is called ***capacitive reactive power*** and refer to Q_c. According to equation (5.28) value of this power

$$Q_C = U_C \cdot I = X_C \cdot I^2. \quad (5.29)$$

As inductive reactive power, capacitive reactive power is measured in volt-amperes reactive (VAr).

5.4 Electrical circuit with real inductive coil

Let the real inductive coil with inductance L and resistance R (the equivalent scheme presented in figure 5.4.a) current flows

$$i = I_m \sin(\omega t + \psi_i). \quad (5.30)$$

Determine the law of variation of the voltage u at its connection terminals.

Instantaneous value of the voltage u let's record from the second Kirchhoff's law

$$u = u_R + u_L = R \cdot i + L \cdot di/dt, \quad (5.31)$$

where: $\boldsymbol{u_R}$, $\boldsymbol{u_L}$, respectively, the voltage on the resistive and inductive elements of the coil (fig.5.4, *a*).

In chapters 5.1 and 5.2, it was shown that each of the voltages u_R and u_L is sinusoidal and has a frequency equal to the frequency of the current i. Therefore, the voltage u is also sinusoidal. Suppose that it can be recorded by the equation

$$u = U_m \sin(\omega t \pm \psi_u). \quad (5.32)$$

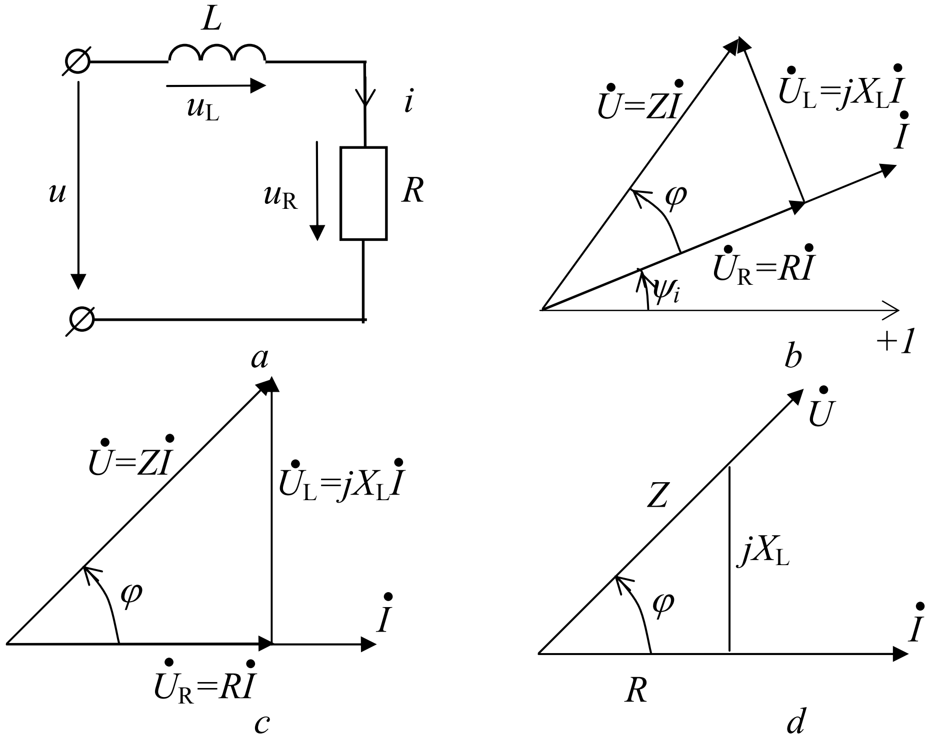


Figure 5.4: Scheme (*a*), triangles voltages and resistances of coil with active resistance and inductance (b, c, d)

The amplitude U_m and the initial phase ψ_u of the voltage u let's determine, using a complex method. Let us write the equation (5.31) in complex form

$$\dot{U} = \dot{U}_R + \dot{U}_L . \tag{5.33}$$

According to equation (5.31) complex value of current

$$\dot{I} = I \cdot e^{j\psi_i} . \tag{5.34}$$

Then the complex values of voltages:

$$\dot{U}_R = R \cdot \dot{I} \text{ and } \dot{U}_L = jX_L \cdot \dot{I} .$$

Substituting the values $\dot{U}_R$ and $\dot{U}_L$ in (5.33), we obtain the equation for the complex voltage at the input of scheme:

$$\dot{U} = R \cdot \dot{I} + jX_L \cdot \dot{I} = (R + jX_L) \cdot \dot{I} = \underline{Z} \cdot \dot{I} . \tag{5.35}$$

A complex quantity $\underline{Z}$ has the dimension of resistance and is the proportionality coefficient between the complex values of the voltage and current of circuit. So $\underline{Z} = R + jX_L$ it is called ***the complex of impedance of the inductive coil***. Real part is the active resistance R and the imaginary part of the complex inductive resistance of coil – jX_L.

In formulas, in which the value $\underline{Z}$ is or as a multiplier or divider, convenient to use not algebraic, but exponential form of its writing:

$$\underline{Z} = R + jX_L = Z \cdot e^{j\varphi_L} , \tag{5.36}$$

where: $Z = \sqrt{R^2 + X_L^2}$ the modulus of the complex impedance of the inductive coil, and the $\varphi_L = \text{arctg}X_L/R$ – its argument.

You should pay attention to the fact that the designation of the complex impedance differs from the notation of complex of current and voltage - instead of a dot above the letter symbol of the complex impedance has a bottom line. This difference is explained by the fact that the complex $\underline{Z}$ is not the representation of a sinusoidal function, and is a complex number, with help of which are compared complex representations of current and voltage.

Substituting in (5.35) the value of Z from (5.36), and the value $\dot{I}$ of (5.34) we get

$$\dot{U} = Ze^{j\varphi_L} \cdot Ie^{j\psi_i} = Z \cdot Ie^{j(\psi_i + \varphi_L)} = Ue^{j\psi_u} \tag{5.37}$$

where $$U = Z{\cdot}I, \qquad \psi_u = \psi_i + \varphi_L \,. \tag{5.38}$$

The initial phase ψ_u is positive, so in the equation (5.32), it must be taken with the sign "plus".

Knowing the values of U and ψ_u , equation (5.32) can be written in its final form:

$$u = Z{\cdot}I_m \, sin(\omega t + \psi_i + \varphi_L) \,. \tag{5.39}$$

Comparing equations (5.39) and (5.30), we see that the sine wave of voltage at the input of coil is ahead of the phase of the sine wave of current at the phase shift angle $\varphi = \varphi_L$.

From the expression (5.35) we can obtain the formula of Ohm's law for the inductive coil in complex form:

$$\dot{I} = \frac{\dot{U}}{\underline{Z}} \,. \tag{5.40}$$

According to this formula the ***complex of current in the inductive coil is equal to the complex of voltage divided by the complex of impedance of the coil.***

In figure 5.4.b it is shown a vector diagram of the scheme of figure 5.4.a.

When construction of this diagram in the original taken the vector of current $\dot{I}$, located under the corner ψ_i to axis +1.

The vector of the voltage on the resistor $\dot{U} = R \cdot \dot{I}$ coincides in phase with the vector of current, and the voltage vector on the inductive element $\dot{U}_L = jX_L \cdot \dot{I}$ is ahead of the phase of the vector of current in the phase shift angle π/2. The voltage vector $\dot{U}$ is equal to the geometric sum of vectors: $\dot{U} = \dot{U}_R + \dot{U}_L$. It is ahead of the phase the vector of current on the angle of shift $\varphi = \varphi_L$.

The vector diagram of figure 5.4, *b* is called the ***triangle of voltages***. To simplify the diagram the initial phase of the current is believed to be equal to zero, then the vector of the current coincides with the axis of the +1 and triangle of voltage is located on a plane as shown in figure 5.4, *c*.

If each of the sides of a triangle of v,oltage (see fig. 5.4, *c*) is divided by $\dot{I}$, you will get a ***triangle of complexes of resistance*** (fig. 5.4, *d*). From this figure it is seen that the module Z of the complex of impedance $\underline{Z}$ is the hypotenuse of a square triangle of complex resistances, the sides of which are active R and inductive jX_L resistance. From it is possible to determine the phase shift angle:

$$\cos\varphi_L = R/Z . \quad (5.41)$$

5.5 Power of inductive coil

To simplify the consideration, we assume that the initial phase of the current in the coil (fig. 5.4, *a*) $\psi_i = 0$. Then the instantaneous value of the current (5.30) can be written $i = I_m \sin\omega t$, and the voltage on the inductive coil $u = U_m \cdot \sin(\omega t + \varphi_L)$. In figure 5.5, a the graphs of instantaneous values of voltage and current in this coil are presented.

The instantaneous value of power equals to the product of instantaneous values of voltage and current:

$$p = u \cdot i = U_m I_m \sin(\omega t + \varphi_L) \sin\omega t =$$
$$U_m \cdot I_m [\cos\varphi_L - \cos(2\omega t + \varphi_L)]/2. \quad (5.45)$$

In figure 5.5, *b* a graph of instantaneous power based on equation is presented (5.45).

The average value of power for the period

$$P_{av} = P = \frac{1}{T}\int_0^T p dt = \frac{UI}{T}\int_0^T [\cos\varphi_L - \cos(2\omega t + \varphi_L)]dt \ ,$$

or

$$P_{av} = U \cdot I\cos\varphi_L .$$

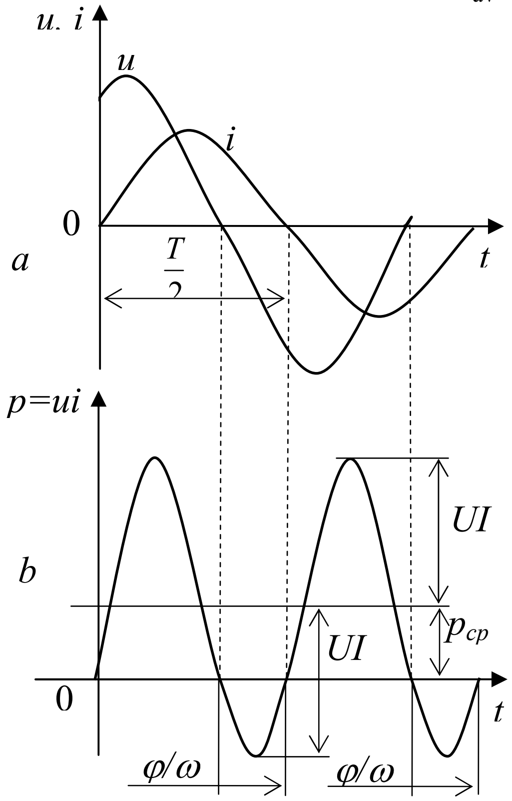


Figure 5.5: Charts of instantaneous values of voltage, current and power for the scheme of figure 5.4, *a*

As $U = Z \cdot I$, and $\cos\varphi_L = R/Z$, the average value of power can be defined differently:

$$P = U \cdot I\cos\varphi_L = Z \cdot I \cdot I \cdot R/Z = R \cdot I^2 . (5.46)$$

From the obtained relations it can be seen that ***the average value of the power of circuit is equal to its active power***. Therefore, the average power of harmonic current circuit is usually called the active power. ***The active power is equal to the product of the effective values of voltage and current on the cosine of the phase shift angle between them***.

The greatest value of active power that can be obtained with these values of voltage and current, referred to ***apparent power*** and mark S. From equation (5.46) implies that at $\cos\varphi = 1$ apparent power

$$P_{max} = S = U \cdot I. \quad (5.47)$$

The value

$$X_L \cdot I^2 = X_L \cdot I \cdot I = U_L \cdot I = U \cdot I \sin\varphi_L,$$

is ***reactive inductive power*** of circuit:

$$Q_L = X_L \cdot I^2 = U \cdot I \sin\varphi_L \,. \qquad (5.48)$$

Active, reactive and apparent power are bound by the formula:

$$P^2 + Q^2_L = (U \cdot I \cos\varphi_L)^2 + (U \cdot I \sin\varphi_L)^2 =$$
$$(U \cdot I)^2(\cos^2\varphi_L + \sin^2\varphi_L) = (U \cdot I)^2 = S\,,$$

or

$$S = \sqrt{P^2 + Q^2}\,. \qquad (5.49)$$

Although all three power of circuit (active, reactive and apparent) have the same dimension, for their differences units of different names are introduced: for active power – watts (W), reactive power is the volt-amperes reactive (VAR), for apparent power – volt-amperes (VA).

In order to determine the values of each power let's return to the consideration of the graphs of the instantaneous values of voltage, current and power based on figure 5.5.

For a more detailed analysis of the energy process let's provide voltage u by active and reactive components $u = u_R + u_L$ and insert it into the formula for instantaneous power:

$$p = u \cdot i = (u_R + u_L) \cdot i = p_R + p_L \,.$$

The first term represents the instantaneous value of the active power, the diagram of which was built on figure 5.1, *b*. The second term is the inductive reactive power diagram of which was built earlier in figure 5.2, *c*. For building the diagrams of these powers in figure 5.6.a it was built the sine wave of current transferred from figure 5.5, *a* graphics of powers the p_R and p_L, as in figures 5.1, *c* and 5.2, *c*. By summing the ordinates of the curves p_R and p_L curve of resulting power p is obtained, a similar curve to this power is presented on figure 5.5, *b*.

From the comparison of graphs (fig. 5.6) it is shown that in the first quarter of period the current is positive and it increases. All power p_R, p_L and p are also positive. This means that at this time, the energy source consumed for thermal energy and the energy stored in the magnetic field of the coil. By the end of the first quarter of the period the magnetic field stores the maximum energy $L \cdot I_m^2/2$.

In the second quarter of period the current decreases. Magnetic field and energy coil also decrease. Part of the energy of the magnetic field is returned back to the source ($p_L < 0$).

During the time from $t = T/4$ to $t = T/2 - \varphi_L/\omega$ a power source p is less active than power p_R. At this time, part of the energy, which is dissipated as heat in the resistor R, is partially comes from a source and partly from the magnetic field. At time $t = T/2 - \varphi_L/\omega$ all thermal energy is covered by arriving in the circuit energy from the magnetic field ($p = 0$, $p_R = p_L$).

During the time from $t = T/2 - \varphi_L/\omega$ up to $t = T/2$ power $p_L < 0$, the power $p_R > 0$, but the ordinate of the curve p_L numerically equal to the sum of the ordinates of the curves p and p_R. This means that the energy coming from the magnetic field back to the source, partly spent on thermal energy, and returns back to the source. For time $\Delta t = \varphi_L/\omega$ to the source will return the energy equal to the shaded figure the area bounded by the curve p at this point in time and the abscissa axis.

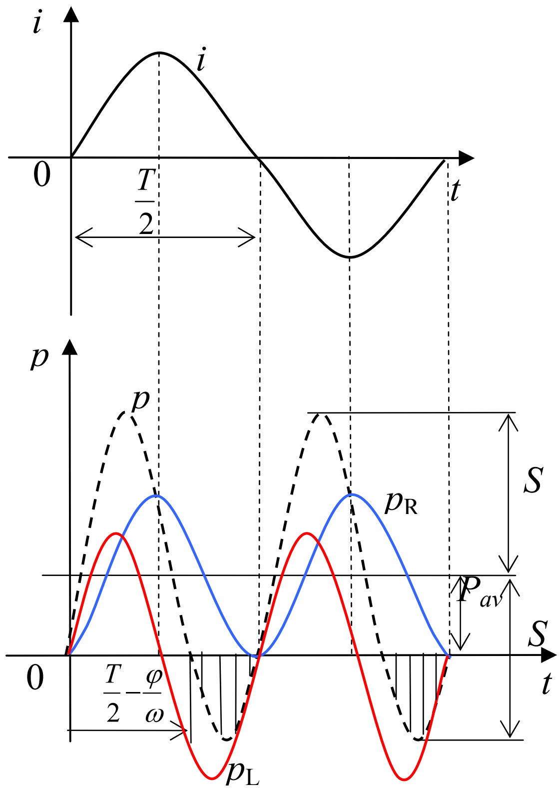


Figure 5.6: Graphs of instantaneous values of current, power sections p_R, p_L and apparent power p of the scheme of figure 5.4, *a*

In the following two quarters of the period of energy the process will repeat in the other direction of the current.

From all mentioned above it follows that graphically ***apparent power S characterizes the amplitude of the power oscillations S = U·I about average value of power.*** Power P varies from a positive value $U{\cdot}I + P$ to negative values of $U{\cdot}I - P$.

The active power P is the average power of conversion of electrical energy into other forms. The value of R depends not only on the current and voltage, but also from $cos\varphi$, which is usually referred to as power coefficient:

$$cos\varphi = P/S. \qquad (5.50)$$

The power coefficient depends on the ratio between active and inductive resistances.

Reactive power Q_L characterizes the amplitude of the power oscillations of the exchange of energy between the source and the magnetic field coil.

Active, reactive and apparent power can be obtained on complex values of voltage $\dot{U} = Ue^{j\psi_u}$ and current $\dot{I} = Ie^{j\psi_i}$. For this purpose it is necessary to take the conjugate complex of the current (indicated by an asterisk) $\overset{*}{I} = I^{-j\psi_i}$ and multiply it by the complex of voltage $\dot{U}$:

$$\dot{U} \cdot \overset{*}{I} = Ue^{j\psi_u} \cdot I^{-j\psi_i} = U \cdot Ie^{j(\psi_u - \psi_i)} = Se^{j\varphi_L} .$$

This product is marked by S and called the complex of apparent power

$$\underline{S} = \dot{U} \cdot \overset{*}{I} = Se^{j\varphi_L} = S\cos\varphi_L + jS\sin\varphi_L = P + jQ_L . \qquad (5.51)$$

From equation (5.51) it is shown that ***the real part of a complex of apparent power is the active power P, imaginary part – complex of reactive power jQ***.

5.6 Series connection of a resistor and an ideal capacitor

Let’s assume that in the circuit (fig. 5.7, *a*) consisting of a serially connected resistor and an ideal capacitor a sinusoidal current $i = I_m \sin(\omega t + \psi_i)$. The voltage at the input of this circuit according to the second Kirchhoff's law in complex form

$$\dot{U} = \dot{U}_R + \dot{U}_C .$$

The complex current value can be recorded for a given equation of the instantaneous value of current:

$$\dot{I} = \frac{\dot{I}_m}{\sqrt{2}} e^{j\psi_i} = Ie^{j\psi_i} .$$

Then the complexes of active and capacitive voltages

$$\dot{U}_R = R \cdot \dot{I} \text{ и } \dot{U}_C = -jX_C \cdot \dot{I} .$$

The complex of voltage at the input of circuit

$$\dot{U} = R \cdot \dot{I} - jX_C \cdot \dot{I} = (R - jX_C) \cdot \dot{I} = \underline{Z} \cdot \dot{I} .$$

From the last equation, we obtain the formula of Ohm's law in complex form for the studied circuit

$$\dot{I} = \dot{U} / \underline{Z} . \tag{5.52}$$

The complex of impedance of capacitive circuit

$$Z = R - jX_C = \sqrt{R^2 + X_C^2} e^{-j\varphi_C} = Ze^{-j\varphi_C} , \tag{5.53}$$

where: $\boldsymbol{Z} = \sqrt{\boldsymbol{R}^2 + \boldsymbol{X}_C^2}$ – the module of the complex of impedance of circuit, and $\varphi_C = arctg\, X_C / R$ – its argument.

If the expression (5.52) can be rewritten as

$$Ie^{j\psi_i} = \frac{Ue^{j\psi_U}}{Ze^{-j\varphi_C}} ,$$

it is possible to obtain two ratios:

$$I = U / Z , \tag{5.54}$$

and

$$\psi_i = \psi_u + \varphi_C . \tag{5.55}$$

Equation (5.54) represents the ratio between the modules of effective values of voltage and current at the input of the investigated circuit (fig.5.7, *a*).

From the expression (5.55) it is followed that the initial phase of the voltage ψ_u is less then the initial phase of the current ψ_i on the phase shift angle φ_c. Therefore, the voltage at the input of the considered capacitive circuit lags in phase from the current on phase shift angle φ_c. This can be illustrated with a triangle voltage, built on figure 5.7, *b* for a given capacitive circuit. To simplify the construction the initial phase of the current ψ_i is accepted equal to zero. The vector of current İ is directed along the axis +1. With the same phase active voltage vector $\dot{U}_R$ and behind it on the phase angle of phase shift π/2 the vector of the voltage on the capacitor $\dot{U}_C$. The resulting vector $\dot{U}$ of the voltage on the input of the circuit lags on the phase from the vector of current in the phase shift angle φ_C. The instantaneous value of this voltage

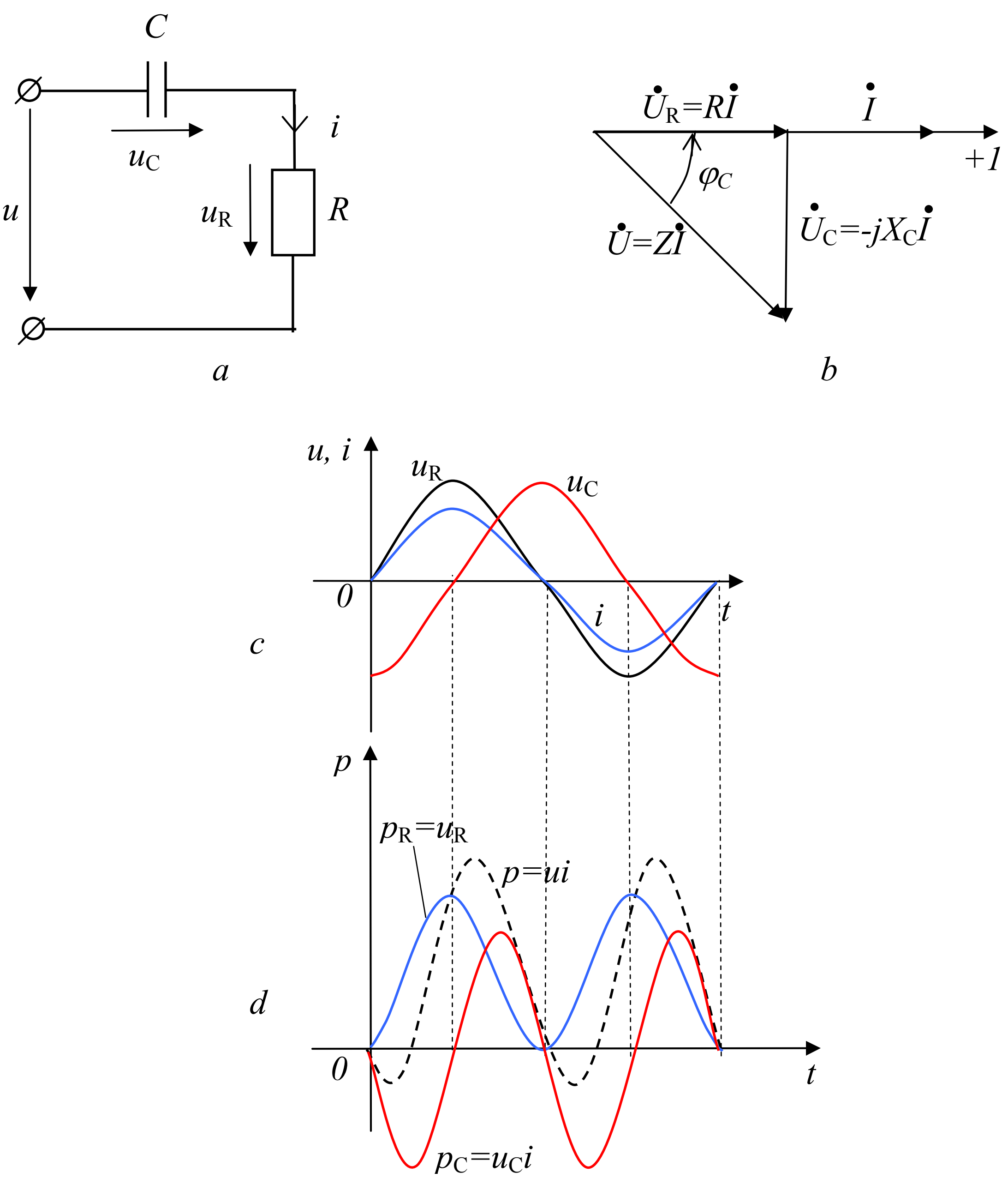


Figure 5.7: Scheme (*a*), graphs of instantaneous values of voltage, current and power (c, d), vector diagram (b) of circuit of series-connected resistor and capacitance

$$u = U_m \sin(\omega t + \varphi_C).$$

Instantaneous power of concerned capacitive circuit (fig. 5.7, *a*)

$$p = u \cdot i = U_m \cdot I_m \sin(\omega t - \varphi_C) \sin\omega t \,,$$

or

$$p = U \cdot I \, [\cos\varphi_C - \cos(2\omega t - \varphi_C)] \,.$$

The average value of power for the period

$$P = \frac{1}{T}\int_0^T p dt = U \cdot I \cdot \cos\varphi_C \,.$$

As for the inductive circuit (fig.5.4, *a*) the average value of the power two-terminal circuit is equal to its active power, as

$$P = U \cdot I \cdot \cos\varphi_C = Z \cdot I \cdot I \cdot \frac{R}{Z} = R \cdot I^2 .$$

Reactive power, which characterizes the amplitude of the power oscillations of the exchange of energy between the circuit and the electric field of the capacitor,

$$Q = X_C \cdot I^2 = U \cdot I \sin\varphi_C . \quad (5.56)$$

Apparent power

$$S = U \cdot I = \sqrt{P^2 + Q_C{}^2} . \quad (5.57)$$

Complex of apparent power

$$\underline{S} = \dot{U} \cdot \overset{*}{I} = Ue^{j\psi_u} \cdot Ie^{-j\psi_i} = U \cdot Ie^{j(\psi_u - \psi_i)} = U \cdot Ie^{-j\varphi_C} ,$$

or

$$\underline{S} = Se^{-j\varphi_C} = S\cos\varphi_C - jS\sin\varphi_c = P - jQ_C . \quad (5.58)$$

You should pay attention to the fact that the complex power is not an representation of a sinusoid, so its character does not put the point. The symbol of complex power, as well as a symbol of the complex resistance is stress.

According to equation (5.58) complex of capacitive reactive power is negative imaginary part of the complex of apparent power.

In figure 5.8, *b* and *d* the graphs of instantaneous values of current i, active u_R and capacitive u_C voltages were built, as well as active $p_R = u_R \cdot i$, reactive (capacitive) $p_C = u_C \cdot i$ and total $p = u \cdot i$ power.

The figure shows that during periods of time when the $p_C > 0$, there is simultaneous conversion of the energy incoming from the power source, into the thermal energy and the energy of the electric field of the capacitor.

During periods of time when $p_C < 0$, the energy of the electric field of the capacitor returns back in the circuit. When $p_C > p_R$ the part of this energy is returned to the source, and part is converted into heat energy. Power at this time is negative. When $p_C < p_R$ conversion of electric energy into thermal energy takes place at the expense of the energy incoming from the power source and the electric field of the capacitor.

5.7 Series connection of an inductive coil and a capacitor

Usually the real elements of electrical circuits contain inductive, capacitive and resistive components of resistance and can be represented by an equivalent scheme containing connected in series R, L and C (fig.5.8, *a*). If in this circuit a current $i = I_m \cdot \sin(\omega t + \varphi_i)$ passes, the complex value of which $\dot{I} = Ie^{j\psi_i}$, then according to the second Kirchhoff's law in complex form for the voltage on the input of the circuit you can write the equation

$$\dot{U} = \dot{U}_R + \dot{U}_L + \dot{U}_C . \quad (5.59)$$

If the complexes of voltage of resistive, inductive and capacitive sections are replaced by the product of complexes of resistance and current, equation (5.59) can be rewritten as follows:

$$\dot{U} = R \cdot \dot{I} + jX_L \cdot \dot{I} - jX_C \cdot \dot{I} = (R + jX_L - jX_C) \cdot \dot{I} = \underline{Z} \cdot \dot{I}, \qquad (5.60)$$

where: $\underline{z}$ the complex of impedance of a circuit defined by the ratio:

$$\underline{Z} = R + jX_L - jX_C = R + j(X_L - X_C) = R \pm jX, \qquad (5.61)$$

where: $\boldsymbol{X = X_L - X_C}$ – reactance of circuit.

Depending on the ratio between the inductive and capacitive reactances of the considered circuit the total reactance X can be: inductive ($X_L > X_C$), capacitive ($X_L < X_C$) and purely active ($X_L = X_C$).

Complexes of inpedances of the circuit in these cases are determined by the following equations:

1) $Z = R + \mathrm{j}X$ on $X_L > X_C$,
2) $Z = R - jX$ on $X_L < X_C$,
3) $Z = R$ on $X_L = X_C$.

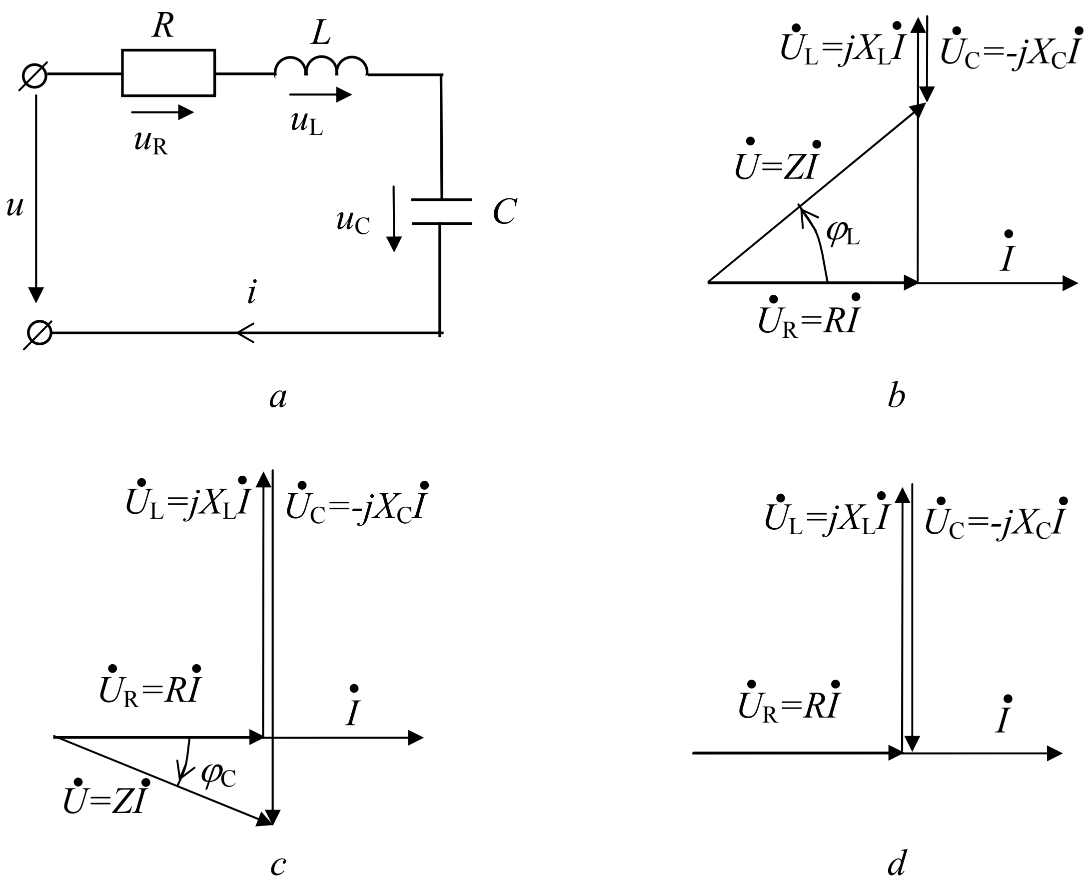


Figure 5.8: Scheme (*a*) and vector diagrams (*b*, *c, d*) of circuit consisting of series-connected elements *R*, *L* and *C*

In figures 5.8, *b*, *c* and *d* vector diagrams for indicated three cases are constructed. The initial phase of the current ψi on these charts has been taken as equal to zero.

Then the complex of impedance of element for all three cases is written in the form

$$\underline{Z} = R + jX_L - jX_C = R + jX. \tag{5.62}$$

The module of the impedance of element

$$Z = \sqrt{(R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X^2}\ . \tag{5.63}$$

The argument or phase shift angle φ is between the vectors of voltage and current

$$\mathrm{tg}\varphi = (X_L - jX_C)/R = X/R\ . \tag{5.64}$$

A positive value of this angle means that the reactance is inductive in nature. The voltage vector in it is ahead of the phase of the vector of current on angle φ (see fig. 5.8, *b*).

The negative value of the angle φ means that the reactance X is capacitive in nature. The voltage vector in this case is behind the phase of the vector current on angle φ (see fig. 5.8, *c*).

When $\varphi = 0$, the vectors of voltage and current of two-terminal circuit coincide with phase (see fig. 5.8, *d*).

The phenomenon, at which in the serial circuit from the elements *R, L* and *C* the total voltage of the circuit coincides in phase with its current, is called the ***resonance voltage***.

The resonance of voltages occurs when the reactance of the circuit is zero ($X = 0$), i.e. when the inductive impedance is equal to capacitive resistance of circuit ($X_L = X_C$). In this case, inductive and capacitive voltages compensate each other ($\dot{U}_C - \dot{U}_L = 0$), as they are equal in magnitude and opposite in phase. Values of current and power are maximum, from the source to the circuit acts only the active energy.

The same on the value of amplitude fluctuations of reactive powers p_L and p_C when the resonance of voltages are in antiphase. As for the energy of electric and magnetic fields, at those moments of time when the energy is stored in the electric field of the capacitor, this stock takes place due to the energy of the magnetic field of the coil. At other times there is a reverse transfer of energy from the electric field in the magnetic.

Complex of apparent power of the considered circuit

$$\underline{S} = \dot{U} \cdot \overset{*}{I} = Z \cdot \dot{I} \cdot \overset{*}{I} = (R + jX_L - jX_C) \cdot I^2 = R \cdot I^2 + jX_L \cdot I^2 - jX_C \cdot I^2,$$

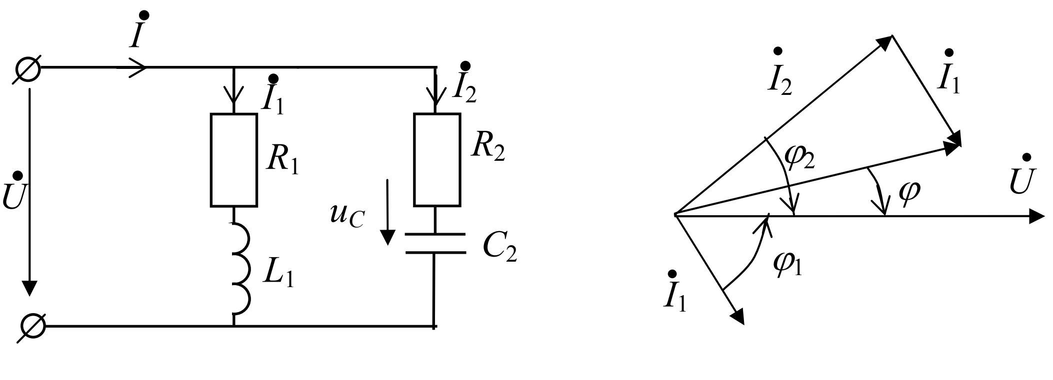
or

$$\underline{S} = P + jQ_L - jQ_C\ , \tag{5.65}$$

where: $\boldsymbol{Q}_L = X_L \cdot I^2$ – reactive power due to the presence in the circuit inductance; $\boldsymbol{Q}_C = X_C \cdot I^2$ – reactive power due to the presence in the circuit capacity.

5.8 Alternating current circuit with parallel-connected receivers

Consider the scheme of circuit in figure 5.9, *a* consisting of two parallel branches, the parameters of which R_1, L_1, R_2 and C_2 are set. Let the voltage U and frequency f of the source are known and it is necessary to determine the current, powers of circuit and its equivalent resistance relative to the input connection terminals.



a *b*

Figure 5.9: Scheme (*a*) and vector diagram (*b*) of circuit consisting of two parallel branches

The calculation can be started with the choice of the initial phase of the common voltage, and so it is convenient to send the voltage vector along a single axis, +1 or +j. Take $\dot{U} = U$ that corresponds to the direction of vector $\dot{U}$ on the axis +1.

The given settings of branches allow writing their complexes of impedances:

$$\underline{Z}_1 = R_1 + j\omega L_1 = Z_1 e^{j\varphi_1},$$

and

$$\underline{Z}_2 = R_2 - j\frac{1}{\omega C_2} = Z_2 e^{-j\varphi_2}.$$

Knowing complex values $\dot{U}$, $\underline{Z}_1$, and $\underline{Z}_2$ currents of the branches can be found, using Ohm's law in a complex form:

$$\dot{I}_1 = \dot{U}/\underline{Z}_1 \quad \text{and} \quad \dot{I}_2 = \dot{U}/\underline{Z}_2. \tag{5.66}$$

Total current of the unbranched part of the circuit is determined by the first Kirchhoff's law:

$$\dot{I} = \dot{I}_1 + \dot{I}_2. \tag{5.67}$$

Let us make up the balance of power of circuit by which a complex of apparent power of source

$$\underline{S}_{SOURCE} = \dot{U} \cdot \overset{*}{I} \tag{5.68}$$

must be equal to the sum of complexes of the apparent power of its individual branches:

$$\underline{S}_1 + \underline{S}_2 = \dot{U}\overset{*}{I}_1 + \dot{U}\overset{*}{I}_2 \ . \tag{5.69}$$

Powers of branches can be calculated and with other formulas:

$$\underline{S}_1 = \underline{Z}_1 \cdot I_1^2 = R_1 \cdot I_1^2 + jX_1 \cdot I_1^2 = P_1 + jQ_{L1},$$
$$\underline{S}_2 = \underline{Z}_2 \cdot I_2^2 = R_2 \cdot I_2^2 - jX_2 \cdot I_2^2 = P_2 - jQ_{C2}.$$

The apparent power of branches

$$\underline{S} = \underline{S}_1 + \underline{S}_2 = P_1 + P_2 + jQ_{L1} - jQ_{C2}$$

must be equal to the power calculated by the formula (5.68).

To determine the complex of equivalent impedance $\underline{Z}$ of the scheme let us take equation (5.61) and instead of currents let us substitute their values, expressed through voltage $\dot{U}$ and resistances $\underline{Z}$, $\underline{Z}_1$, and $\underline{Z}_2$:

$$\frac{\dot{U}}{\underline{Z}} = \frac{\dot{U}}{\underline{Z}_1} + \frac{\dot{U}}{\underline{Z}_2} \ .$$

From here

$$\frac{1}{\underline{Z}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}. \tag{5.70}$$

If there are n parallel branches, instead of the formula (5.70) it can be written in a more general formula for determination the equivalent resistance

$$\frac{1}{\underline{Z}} = \sum_{k=1}^{n} \frac{1}{\underline{Z}_k} \ . \tag{5.71}$$

Let us construct the vector diagram of currents of a given circuit (fig. 5.9, *b*). As a source let us take the voltage vector common to all branches. Let us direct this vector along the axis +1 and put with respect to it the vectors of currents $\dot{I}_1$ and $\dot{I}_2$ branches. The vector of current $\dot{I}_1$ lags in phase from the voltage vector on angle φ_1, and the vector current $\dot{I}_2$ is ahead of the phase the voltage vector on angle φ_2. The vector of current $\dot{I}$ of the unbranched section equals to the geometrical sum of the vectors of currents of branches, ahead of the phase the voltage vector on angle φ.

5.9 Active and reactive components of the conductivity and current

In circuits of sinusoidal current the value which is opposite to the complex of impedance $\underline{Z}$, is referred to as ***complex of full conductivity*** and denoted by $\underline{Y}$:

$$\underline{Y} = \frac{1}{\underline{Z}} \ . \tag{5.72}$$

Like any complex number, the complex of conductivity has the real part, which is denoted by G and is called the ***active conductance*** and the imaginary part, denoted by the letter B and is called the ***reactive conductance***.

If the circuit is active-inductive then its complex of impedance $\underline{Z} = R + jX_L$ and complex of conductivity

$$\underline{Y} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2} = \frac{R}{Z^2} - j\frac{X_L}{Z^2}, \tag{5.73}$$

or

$$\underline{Y} = G - jB_L, \tag{5.74}$$

where: the active ***G*** and reactive $\boldsymbol{B}_\mathrm{L}$ conductivity is determined by the ratios

$$G = R/Z^2 \text{ and } B_L = X_L/Z^2. \tag{5.75}$$

If the circuit is active-capacitive, then its complex of impedance $\underline{Z} = R - jX_C$ and complex of conductivity

$$\underline{Y} = \frac{1}{R - jX_C} = \frac{R + jX_C}{R^2 + X_C^2} = \frac{R}{Z^2} + j\frac{X_C}{Z^2}, \tag{5.76}$$

or

$$\underline{Y} = G + jB_C. \tag{5.77}$$

Comparison of (5.73) and (5.76) shows that the active G and reactive conductivity of the active-capacitive and active-inductive circuits are determined by the same formulas. The difference is that the ***imaginary part of the complex of conductivity is positive for capacitive circuit and a negative for inductive circuit***.

When using vector diagrams to analyze phenomena in circuits of sinusoidal current decomposition of the vector of current in its active $\dot{I}_A$ and reactive $\dot{I}_R$ components is used. This decomposition can be done graphically or analytically.

Consider the ***graphical method of decomposition*** of the current. Take the scheme of the circuit on figure 5.9, *a* consisting of two parallel branches, and on figure 5.10, *a* repeat its vector diagram, which was built earlier in figure 5.9, *b*.

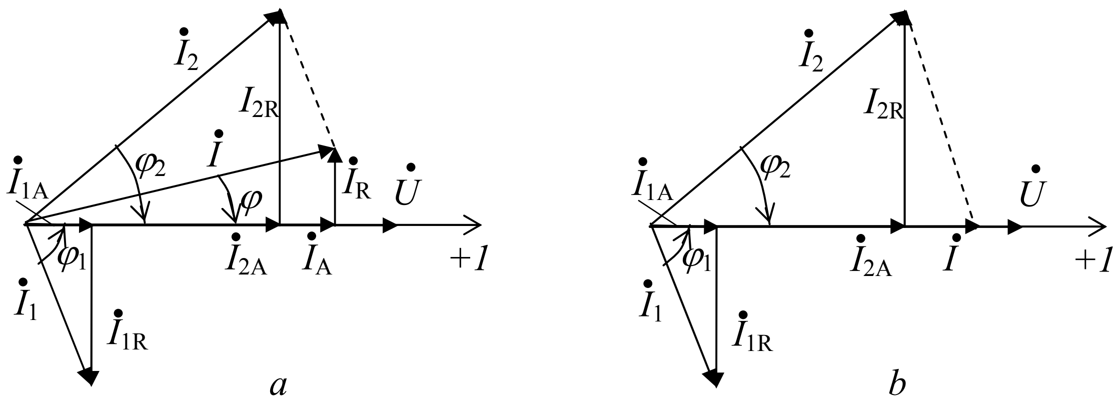


Figure 5.10: The decomposition of the total vector of current of branched circuit on the active and reactive components

Each of the current vectors on the diagram is decomposed into two components: active, coinciding in phase with the voltage vector, and a reactive which is perpendicular to the voltage vector. As the currents are the same in the phase with the voltage only in the active elements, and lag on the phase from the voltage or ahead of the phase the voltage on π/2 only in the reactive elements, then the components of the current $\dot{I}_A$ and $\dot{I}_R$ are called active and reactive.

Modules of active and reactive components of current $\dot{I}_1$ and $\dot{I}_2$

$$I_{1A} = I_1\cos\varphi_1, \qquad I_{1R} = I_1\sin\varphi_1,$$
$$I_{2A} = I_2\cos\varphi_2, \qquad I_{2R} = I_2\sin\varphi_2.$$

The components of current and conductivity can be used to determine the power of a circuit:

$$P = U{\cdot}I{\cdot}\cos\varphi = U{\cdot}I_a = G{\cdot}U^2,$$
$$Q_L = U{\cdot}I{\cdot}\sin\varphi_L = U{\cdot}I_{LR} = B_L{\cdot}U^2,$$
$$Q_C = U{\cdot}I{\cdot}\sin\varphi_C = U{\cdot}I_{CR} = B_C{\cdot}U^2.$$

In the ***analytical method of decomposition*** the current of some branch is reproduced by the product of complex values of voltage and conductivity:

$$\dot{I} = \underline{Y}\cdot\dot{U} = (G + jB)\cdot\dot{U} = G\cdot\dot{U} + jB\cdot\dot{U} = \dot{I}_A + \dot{I}_R\ . \qquad (5.78)$$

In equation (5.78) value $\dot{I}_A = G\cdot\dot{U}$ is called active, and the value $\dot{I}_R = jB\cdot\dot{U}$ – the reactive component of the current.

Using active and reactive components of the conductivity and current, it is convenient to analyze the modes of the branched circuit. As an example, let us return to the consideration of a circuit consisting of two parallel branches (see fig. 5.9, *a*).

The complex of equivalent total conductivity of this circuit

$$\underline{Y} = \underline{Y}_1 + \underline{Y}_2 = G_1 - jB_L + G_2 + jB_C,$$

or

$$\underline{Y} = G_1 + G_2 - j(B_L - B_C) = G_{eq} - jB_{eq}. \qquad (5.79)$$

If reactive conductivity of inductive branches is more than reactive conductivity of a capacitive branch ($B_L > B_C$), then

$$\underline{Y} = G_{eq} - jB_{eq}\ ,$$

and the circuit is active-inductive. Current of non-branched subcircuit of such a circuit equals to a current of the source of power, lags in phase from the voltage source. When $B_L < B_C$ the circuit is active-capacitive and the current is ahead of the phase voltage on this section.

In a parallel circuit with inductive and capacitive receivers (see fig. 5.9, *a*) it is possible that the total circuit of the current (current of the unbranched plot) and the voltage at the input of the circuit do not match in phase. This phenomenon is called ***resonance of current***.

Reactive components of the currents of inductive and capacitive branches on the resonance of currents are equal in magnitude and opposite in phase (see fig. 5.11, *b*). Therefore, the resonance of currents of any parallel circuit of its inductive reactive current and reactive capacitive current are mutually compensated. The circuit is an active resistance equivalent to the conductivity which is equal to the sum of active conductances of branches (fig.5.9, *a*):

$$\underline{Y} = G_{eq} = G_1 + G_2\ .$$

The circuit has only active components of the current

$$\dot{I} = \dot{I}_{1a} + \dot{I}_{2a}$$

and consumes only active energy, as reactive powers are proportional to the reactive components of their currents:

$$Q_L = U \cdot I_{LR} \qquad \text{and} \qquad Q_C = U \cdot I_{CR}\,,$$

and the fluctuations of these powers are in antiphase.

Therefore, the circuit at resonance of currents does not consume from the source reactive power. It is a mutual exchange of energy between the electric and magnetic fields. Power supply only compensates for the energy loss in the active resistances of the branches.

A chain of two parallel-connected ideal inductive and capacitive elements is of a particular interest (fig. 5.11, *a*). Conditions of resonance of currents of such a circuit $\dot{I}_{LR} = \dot{I}_{CR}$ or $B_L = B_C$, or $1/X_L = 1/X_C$ come to the condition $X_L = X_C$. Due to the lack of active resistances, the total current of this circuit is equal to zero ($\dot{I} = 0$), although each of the branches passes current $\dot{I}_{LR} = \dot{I}_{CR}$ (fig. 5.11, *b*).

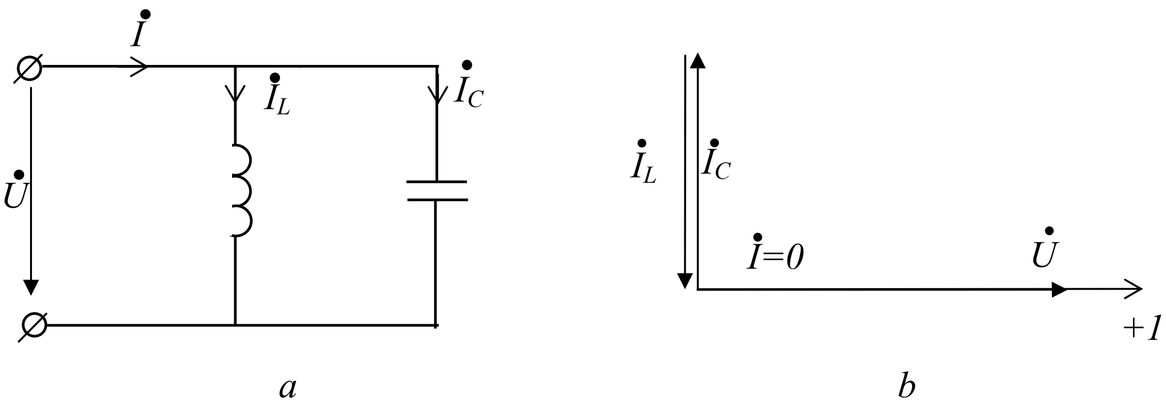


Figure 5.11: The equivalent scheme of parallel connection of two ideal inductive and capacitive elements (*a*) and its vector diagram (*b*)

5.10 Growth of power coefficient in AC circuits

Most of today's electricity consumers are AC inductive load currents which are lagging behind in phase power supply voltage. Active power of such consumers

$$P = U{\cdot}I{\cdot}\cos\varphi \tag{5.80}$$

at the given values of current and voltage also depends on cos*φ*.

If the voltage U and the active power P are set for the user, then with the change of $\cos\varphi$ the current user changes. With decreasing $\cos\varphi$ the consumer's current grows:

$$I = \frac{P}{U\cos\varphi} \quad . \tag{5.81}$$

The generators that power consumers expect a certain ***rated power*** $S_{rat} = U_{rat} \cdot I_{rat}$. At a given voltage U_{rat} they can be loaded with the current not exceeding the nominal value. Therefore, the increase of the consumer current due to the reduction of its $\cos\varphi$ it does not exceed certain limits. It is necessary to reduce the active power so that the current of the generator would not be above nominal at lowering of $\cos\varphi$ of the consumer. In this case, the generator is fully loaded on the current and underutilized on the active power.

To maintain the active power of the consumer while reduction of the $\cos\varphi$ it is possible to install the generator at a big rated power, so that the increase in current due to the reduction of cos does not exceed its nominal value. In this case, the active power $P = S_{rat}\cos\varphi$, at which the generator will be loaded is only a part of the rated power S_{rat}. For example, at lowering $\cos\varphi$ from 1 to 0.5, the load of the generator is only 50% of its rated power. Thus, ***cosφ describes how to use the rated power source, and therefore it is called the power coefficient***.

Operation of a consumer with a low power factor, in addition to the deterioration of the conditions of economic use of the power supply leads to the increase of power loss in transmission lines of electrical energy from source to consumer. If the resistance of wires of the line is R, then the power loss in it

$$\Delta P = R \cdot I^2 = R \cdot \frac{P^2}{U^2\cos^2\varphi} \quad . \tag{5.82}$$

Power of loss, as it can be seen from (5.82), the more the lower $\cos\varphi$ of the installation is. Therefore, ***the lower cosφ of the consumer is, the more expensive it will be to transfer the electricity to it***.

To increase the efficiency of power plants certain measures are taken to improve the power factor of the consumers.

The idea of increasing $\cos\varphi$ is that the total current of inductive consumer is considered to consist of active and reactive components. The active power of the consumer at this voltage is determined by the active current component: $P = U \cdot I_A$, therefore, for a given values of active power active component of the current must remain constant. It is possible to reduce the current of the user in this case only by reduction of the reactive current of the inductive consumer. This can be achieved by parallel connection to the load of any receiver with the capacitive current. This receiver can be the battery of special capacitors.

Let us consider the example of calculation of the capacity of the capacitor banks which is necessary to include in parallel with an inductive consumer with $\cos\varphi$ to bring the power coefficient of the installation to the specified $\cos\varphi$. The active power and the voltage of the user are specified.

In figure 5.12, *a* the equivalent scheme of the consumer $R_{con} - jX_{con}$ and batteries of capacitors *C* are shown, and on figure 5.12, *b* its vector diagram is seen. The chart shows that to obtain the phase shift angle of the desired value its capacitive branch must have a current, equal to the difference between the reactive components of a consumer's current before compensation of the phase shift angle $\dot{I}_{R.con}$ and after compensation of the phase shift angle $\dot{I}_R$:

$$\dot{I}_c = \dot{I}_{R.con} - \dot{I}_R . \tag{5.83}$$

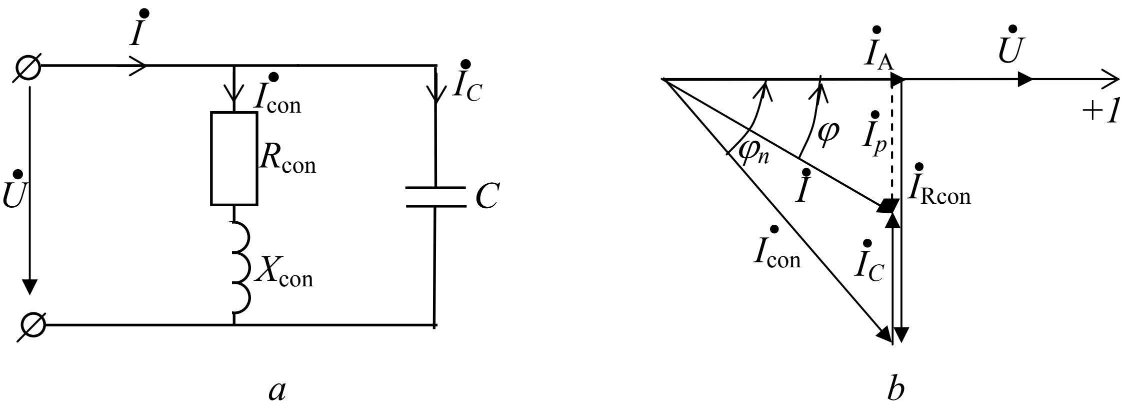


Figure 5.12 – The equivalent scheme of the consumer and batteries of capacitors (a) and its vector diagram (b)

From the vector diagrams these currents can be defined through the active component of the current of consumer $\dot{I}_A$:

$$I_{R.con} = I_A tg\varphi_{con} \quad \text{and} \quad I_R = I_{A.con} tg\varphi .$$

Therefore, the equation (5.83) can be rewritten in the form

$$I_C = I_{A.con}(tg\varphi_{con} - tg\varphi) . \tag{5.84}$$

The current I_C in this equation can be expressed in voltage and capacity ($I_C = U \cdot \omega C$), and current $I_{A.con}$ – through power and voltage ($I_{A.con} = P/U$). Therefore, instead of (5.84) we obtain another equation:

$$U \cdot \omega C = \frac{P}{U}(tg\varphi_{con} - tg\varphi),$$

from which you can determine the desired value of capacitance of capacitor banks:

$$C = \frac{P}{\omega U^2}(tg\varphi_{con} - tg\varphi) . \tag{5.85}$$

Usually with capacitor banks phase angle compensation is performed to the values cosφ 0,9–0,95.

Key findings

1. The average power in the circuit is equal to its active power.

2. In any direction of the current in the resistive element energy comes from the source in the circuit and is converted into heat energy.

3. Ideal voltage vector phase coil is ahead of the current vector by the angle equal to the phase shift π/2.

4. In a circuit with an ideal coil a continuous oscillation of energy between the source and the magnetic field of the coil without costs of the energy of the source happens.

5. The voltage vector on an ideal capacitor lags the phase from the vector of the current by a phase shift angle equal to π/2.

6. In circuit with an ideal capacitor there is a continuous oscillation of energy between the source and the electric field of the capacitor without costs of the energy of the source.

7. The active power in the circuit is equal to the product of the effective values of voltage and current on the cosine of the phase shift angle between them.

8. Apparent power characterizes the amplitude of the power oscillations around the average value of power.

9. Reactive power characterizes the amplitude of the power oscillations of the exchange of energy between the source and the magnetic field of coil (electric field of the capacitor).

10. When the inductive resistance of the circuit is equal to serially intercalated with it capacitive resistance, in the circuit resonance occurs the voltage, at which the current and power are maximum from the source to the circuit only the active energy acts.

11. When the inductive resistance of the circuit is equal to intercalated with it in parallel the capacitive resistance in the circuit resonance of the currents occurs, at which the total current in the circuit and the voltage at its input coincide with the phase and the circuit does not consume from the source reactive energy.

12. Cosφ characterizes the degree use of the apparent power.

Control questions

1. In what elements of electrical circuit irreversible conversion of electric energy happens?

2. Explain why at a constant current inclusion in the circuit of the capacitor is equal to rupture of circuit, and when alternating current circuit remains closed (the current passes through the capacitance)?

3. Write an equation for the instantaneous value of current in a circuit consisting of series-connected elements R and L, if voltage $u = U_m \cdot \sin(\omega t + \psi_u)$ of a circuit is applied to the connection terminals.

4. Write an equation for the instantaneous value of current in a circuit consisting of series-connected elements R and C, if voltage $U_m \cdot \sin(\omega t + \psi_u)$ of a circuit is applied to the connection terminals.

5. Write an expression for the instantaneous value of the voltage at the connection terminals of a circuit consisting of a coil of resistance R and inductance L, if the instantaneous value of current $i = I_m \cdot \sin(\omega t + \psi_i)$. Draw the vector diagram for this circuit.

6. Coil with parameters R and L are connected in parallel with a capacitor with capacitance C. The voltage at the connection terminals of a circuit $u = U_m \cdot \sin(\omega t + \psi_u)$. Write an equation for the instantaneous value of the current in the unbranched part of the circuit.

7. What are the angles of phase shift between the voltages of R, L and C elements connected in series?

8. What are the angles of phase shift between the currents of R, L and C elements connected in parallel?

9. Define the conditions for the resonance of voltages coming in the circuit and draw the vector diagram for this mode.

10. Write Ohm's law and Kirchhoff's laws in a complex form.

11. Prove that in AC circuits with sequential inclusion of several elements it is possible to create conditions under which the voltage on any of the items will exceed the voltage at the input circuit.

12. Prove that in AC circuits with parallel inclusion of multiple items it is possible to create conditions under which the current in any branch will exceed the current of non-branched section.

13. Write down the equation for the equivalent complex impedance for mixed connection of resistance.

14. Draw triangles of resistance and conductance, and display formulas of transition from resistance to conductivity and back.

15. Write the condition for the occurrence of resonance of currents expressed through the resistance of the parallel branches.

16. Build the voltage vector $\dot{U}$ and the vector of current $\dot{I}$ shifted between them on a phase angle $\varphi > 0$. Lay out the other vector on the active and reactive components.

17. Draw the triangle of powers and write formulas for the sides of the triangle.

18. Draw a graph of instantaneous power in the circuit at different receivers (active, inductive, capacitive, and mixed).

19. What are the features of the instantaneous power of a circuit? Prove that the instantaneous power can take positive and negative values.

20. Why and how do they tend to raise the power coefficient in electrical circuits?

6 THREE-PHASE AC CIRCUITS

Key concepts: phase of three-phase circuit, three-phase system of EMF, symmetrical three-phase system of EMF, direct (reverse) sequence of phases, neutral, phase EMF, linear EMF, star connection (triangle), balanced load, three-phase four-wire circuit, the active (reactive, total, complex) power of three-phase system.

6.1 Basic concepts and definitions

Integration of several similar in structure circuits of sinusoidal current of the same frequency with independent power sources in one circuit is widely used in engineering. United circuits of harmonic current are called ***phases***, and the whole united system of circuits is a ***polyphase system***. Thus, in electrical engineering, the term ***"phase" has two different meanings***: first, it is a ***parameter of the periodic process***, and secondly, it is ***the name of the component parts of a multiphase system of circuits of sinusoidal current***. The most spread one got the three-phase system.

The three-phase system was invented and developed in detail, including a three-phase transformer and induction motor, by outstanding Russian engineer M. O. Dolivo-Dobrovolsky (1862 – 1919) in 1891. At present three-phase systems are used for transmission and distribution of energy in most cases. ***A very important advantage of the three-phase system is also an exceptional simplicity and low cost of three-phase nonsynchronous motors***. In addition to the three-phase system six-phase system has practical value, for example in devices for rectification of alternating current, and in some automation devices a two-phase system is used.

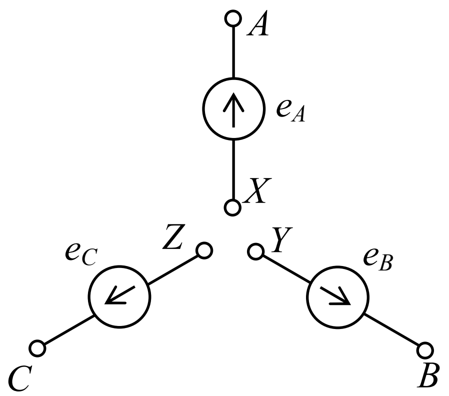


Figure 6.1: The designation of a three-phase generator

For designation of phase of three-phase systems letters of the Latin alphabet are used. The first phase is denoted by A or a – start phase X or x – end phase (capital letters refer to the source and lowercase refers to load. The entire phase is called the phase A, two others are phase B and phase C. The designation of a three-phase generator is shown in figure 6.1.

For the beginning phase, take the connection terminal through which the current flows in the external circuit when it has positive value. The ends of the phases of the source can be connected to each other, and then in the external circuit total EMF will act. Such system is called a ***bound***.

Three - phase of EMF is called symmetric, if the frequency and amplitude of the EMF of each phase are the same, sinusoidal and offset from each other by the angle 2π/3, i.e., 120° (fig. 6.2).

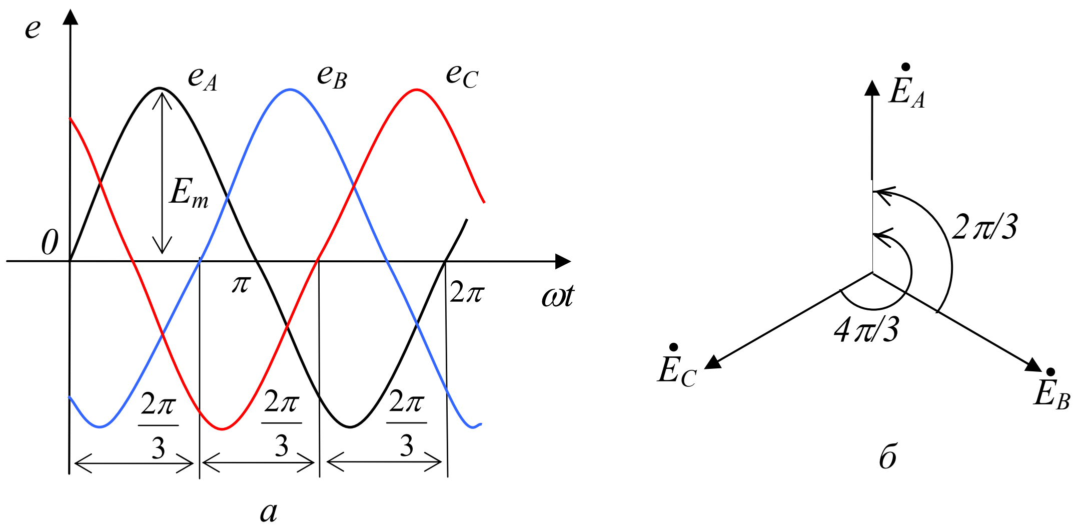


Figure 6.2: Graphical (*a*) and vector (*b*) representation of the three-phase symmetrical system of EMF

In an analytical form instant and effective values induced in the phase of EMF are written in the following form:

$$\left.\begin{array}{ll} e_A = E_{mA} \sin \omega t; & \dot{E}_A = E; \\ e_B = E_{mB} \sin(\omega t - \dfrac{2\pi}{3}); & \dot{E}_B = E \cdot e^{-j2\pi/3}; \\ e_C = E_{mC} \sin(\omega t - \dfrac{4\pi}{3}); & \dot{E}_C = E \cdot e^{-j4\pi/3} = E \cdot e^{j2\pi/3}. \end{array}\right\} \quad (6.1)$$

As it can be seen from figure 6.2, *a*, in a symmetric three-phase system, the sum of the instantaneous values of phase EMF at any point in time is equal to zero

$$e_A + e_B + e_C = 0. \quad (6.2)$$

Similarly, you can record for effective values of the vectors:

$$\dot{E}_A + \dot{E}_B + \dot{E}_C = 0. \quad (6.3)$$

On the vector diagram (fig. 6.2, *b*) phase B lags from phase A, and phase C lags from phase B. This phase sequence ABC is called ***direct sequence*** and phase sequence ACB is called ***reverse sequence***. The sequence of phases is determined by the special device known as phase rotation indicator.

As a three-phase source of electrical energy three-phase synchronous generators are mainly used to convert mechanical energy into electrical energy, each of the three windings of the armature of which is the source of single-phase sinusoidal EMF.

To three-phase consumers of electrical energy three-phase synchronous and asynchronous motors and transformers (with load), electric furnace, devices of electric lighting and others are concerned.

There are various ways of connecting the phases of three-phase power supply and three-phase consumers of electrical energy. The most common are connections "star" and "triangle". In this way the connections of the phases of the sources and phases of consumers in three-phase systems may be different. The phase of the source is usually connected in a "star" phase while consumers connect with either "star" or "triangle".

6.2 Connection schemes of windings of three-phase generator

Phase of winding of the three-phase generator can be connected in a star (fig. 6.3, *a*) or the triangle (fig. 6.3, *b*).

In a star connection the ends of the phases are merged into a single point N (fig. 6.3, *a*), which is called the ***zero*** or ***neutral***. The load can be connected to the connection terminals N – A, N – B, N - C or A – B, B – C, C – A.

There are ***phase*** E_A, E_B and E_C and ***linear*** E_{AB}, E_{BC} and E_{CA} EMF, which, as can be seen from figure 6.3, *c* are interconnected by expressions:

$$\left.\begin{aligned} \dot{E}_{AB} &= \dot{E}_A - \dot{E}_B; \\ \dot{E}_{BC} &= \dot{E}_B - \dot{E}_C; \\ \dot{E}_{CA} &= \dot{E}_C - \dot{E}_A. \end{aligned}\right\} \qquad (6.4)$$

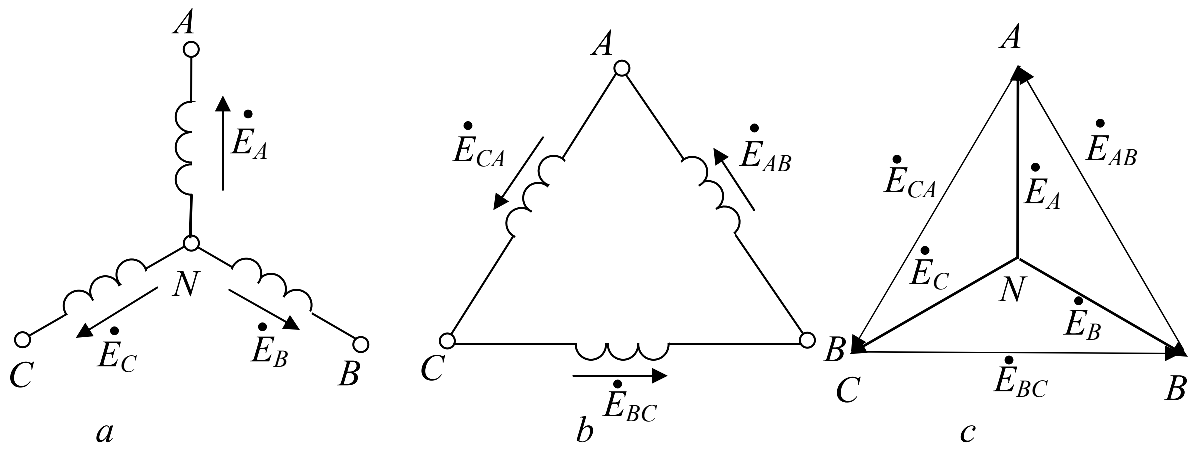


Figure 6.3: Connection schemes of windings of three-phase generator:
a – «star», b – "the triangle", c – vector chart

In a symmetric system, a system of linear EMF is symmetric $\dot{E}_{AB} + \dot{E}_{BC} + \dot{E}_{CA} = 0$. The ratio between phase and line EMF has the form

$$E_л = \sqrt{3}E_ф . \qquad (6.5)$$

The phases of the source in the triangle load are connected to its vertices (fig. 6.3, *b*). Meanwhile linear and phase EMF and voltage are equal: $E_{ph} = E_L$; $U_{ph} = U_L$. Such a connection is possible only with the symmetric source. In this case, the phases form a closed contour, the current in which is missing.

It is almost impossible to carry out all the windings being the same, i.e., the EMF is always asymmetrical. As a result undesirable surge currents occur. Therefore, almost always (with rare exceptions) the windings of the generator are connected by a star.

The electric energy receivers can be connected in a triangle and star. Usually the line voltage source is set. The standard stipulates the scale line voltage as 127, 220, 380, 500, 660 V.

6.3 Connection of three-phase consumers with a star

With connection of the phases of three-phase power source or the energy consumer by "star" (fig. 6.4) the ends of the phases of the source *X, Y, Z* are combined into a common neutral point *N,* but the beginnings of phase A, B, C are connected to the respective line wires *Aa, Bb, Cc*. Similarly, when the connection of three phase consumers unite in a neutral point *n* the ends of its phases *x, y, z*, meanwhile the beginnings of phases *a, b, c* are connected to the line wires.

Voltages U_A, U_B, U_C, acting between the beginnings and ends of phases of the power supply are the phase voltages, and voltage, U_a, U_b, U_c, acting between the beginnings and ends of the phases of the consumer, are phase voltages. Voltage U_{AB}, U_{BC}, U_{CA}, acting between the beginning phases of the source and voltage U_{ab}, U_{bc}, U_{ca}, acting between the beginnings phases of the consumer are line voltages.

In the scheme figure 6.4 the conditional positive direction of phase and line voltages are shown. ***Line currents*** I_l in the power supply lines (I_A, I_B, I_C) when connecting a three-phase power supply and three-phase electricity consumers by "star", conditional positive direction of which is shown in the scheme (fig. 6.4) are at the same time ***phase currents*** flowing through the phases of the consumer (I_A, I_B, I_C). Therefore, in the present case, in the presence of a symmetric three-phase system with junction of the phases of the consumer by "star" line currents are equal to the phase currents

$$I_{ph} = I_L \,. \qquad (6.6)$$

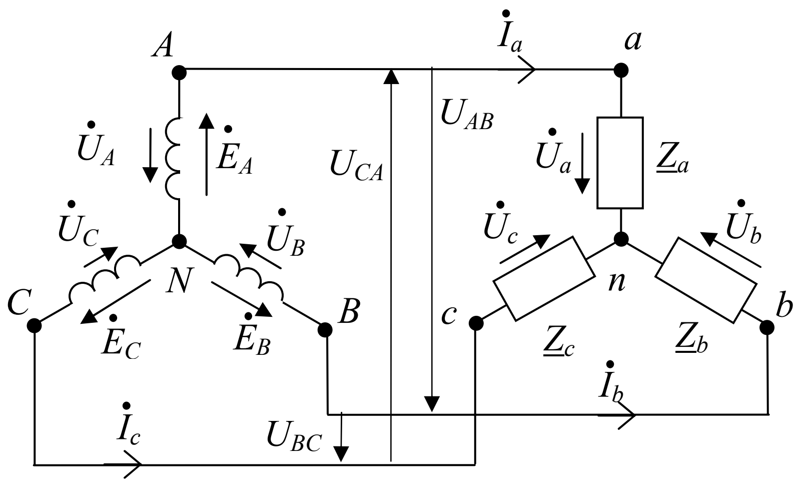


Figure 6.4: Connection of sources and consumers on the "star-star" scheme

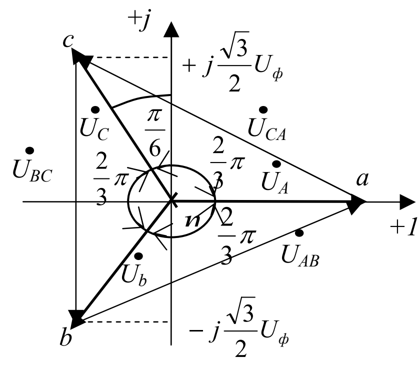


Figure 6.5: Vector diagram of the phase and line voltages

Three-phase power supplies are symmetric in most cases. In this case, the effective values of phase EMF $E_A = E_B = E_C = E_{ph}$, and phase voltage $U_A = U_B = U_C = U_{ph}$ are respectively equal and shifted relatively to each other in the phase by the angle $2\pi/3$. Meanwhile complex, active and inductive resistance of the phases are equal respectively, i.e. $\underline{Z}_A = \underline{Z}_B = \underline{Z}_C = \underline{Z}_{ph}$; $R_A = R_B = R_C = R_{ph}$; $X_A = X_B = X_C = X_{ph}$. Values of phase power coefficients $cos\varphi_A = cos\varphi_B = cos\varphi_C = cos\varphi_{ph}$ are also equal.

Three-phase electricity consumers can be symmetric and asymmetric. For symmetric consumers ratios obtained for three-phase symmetrical power supplies are fair. Meanwhile (see fig. 6.4) $U_a = U_b = U_c = U_{\text{ph}}$, $U_{AB} = U_{BC} = U_{CA} = U_{\text{L}}$, $\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}_{ph}$; $R_a = R_b = R_c = R_{ph}$; $X_a = X_b = X_c = X_{ph}$ $cos\varphi_{\text{a}} = cos\varphi_{\text{b}} = cos\varphi_{\text{c}} = cos\varphi_{\text{ph}}$. The relationship between phase and line voltages is determined by the equation

$$U_л = \sqrt{3}U_{ph} . \tag{6.7}$$

For ***asymmetric three-phase*** consumers not all of these ratios are observed.

In the analysis of three-phase electrical circuits method of complex numbers is widely used. With its help it is possible to carry out calculations that cannot be performed by other methods.

In figure 6.5 on the plane of complex numbers the vector diagram of the phase $\dot{U}_a$, $\dot{U}_b$, $\dot{U}_c$ and line voltage $\dot{U}_{AB}$, $\dot{U}_{BC}$, $\dot{U}_{CA}$ of electricity consumers are given, meanwhile the vector of phase voltage $\dot{U}_a$ directs along the real axis in the positive direction. With this in mind, the phase voltage of three-phase symmetrical consumer can be represented in complex notation:

$$\left.\begin{aligned}
&\dot{U}_a = U_a = U_{ph} = \frac{U_l}{\sqrt{3}} \ ;\\
&\dot{U}_b = U_b(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) = U_l(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) = \frac{U_l}{\sqrt{3}}(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) \ ;\\
&\dot{U}_c = U_c(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) = U_{ph}(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) = \frac{U_l}{\sqrt{3}}(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) .
\end{aligned}\right\} \tag{6.8}$$

In accordance with accepted conditional positive directions of phase and line voltages (see fig. 6.4) line voltage of electricity consumers is defined by equations composed in complex notation for the respective closed contours on the second Kirchhoff's law:

$$\dot{U}_{AB} = \dot{U}_a - \dot{U}_b ; \ \dot{U}_{BC} = \dot{U}_b - \dot{U}_c ; \ \dot{U}_{CA} = \dot{U}_c - \dot{U}_a \ .$$

From the vector diagram (fig. 6.5) it follows that the line voltage and phase voltage are shifted relatively to each other in a phase by the angle 2π/3. Meanwhile for a symmetric three-phase system the vector sum of the phase voltages is $\dot{U}_a + \dot{U}_b + \dot{U}_c = 0$ and the sum of the line voltage is $\dot{U}_{AB} + \dot{U}_{BC} + \dot{U}_{CA} = 0$.

Taking into account the above mentioned equations, linear voltage of consumer for a symmetric system can be represented by the following relations:

$$\dot{U}_{AB} = U_l(\frac{\sqrt{3}}{2} + j\frac{1}{2}) ; \ \dot{U}_{BC} = -jU_l ; \ \dot{U}_{CA} = U_l(-\frac{\sqrt{3}}{2} + j\frac{1}{2}) \ .$$

Similar equations can be written for the symmetric three-phase power source when connection of its phases is by "star".

If we neglect the resistance of the line wires connecting the three-phase power supply with three-phase electricity consumer, then the line voltages of consumers are equal to the corresponding line voltages of power supply: $\dot{U}_{ab} = \dot{U}_{AB}$, $\dot{U}_{bc} = \dot{U}_{BC}$, $\dot{U}_{ca} = \dot{U}_{CA}$.

With connection of the phases of the consumer by "star" and symmetric load complex phase currents are determined on the basis of the equations written on the Ohm's law for a subcircuit:

$$\dot{I}_A = \frac{\dot{U}_a}{\underline{Z}_a} \;;\; \dot{I}_B = \frac{\dot{U}_b}{\underline{Z}_b} \;;\; \dot{I}_C = \frac{\dot{U}_c}{\underline{Z}_c} \;.$$

As the phase voltage and the impedances of all phases of consumers are equal, phase and line currents will be also equal to:

$$I_A = I_B = I_C = I_{ph} = I_l \;. \tag{6.9}$$

6.4 Connection of three phase consumers by triangle

In three-phase systems along with a connection of three phase consumers by "star" the connection phase "triangle" is used. It does not matter how phase source is connected, by a "star" or "triangle".

Connection, where the beginning of one phase of the power consumer (or power source) is connected with the end of its another phase, the beginning of which is connected to the end of the third phase and the beginning of the third one is connected with the end of the first phase (meanwhile the beginnings of all phases are connected to the respective line wires) is called a ***triangle***.

With the "triangle" connection, as can be seen from the scheme (fig. 6.6), the phase voltages of the consumer are equal to line voltages ($U_{\text{ph}} = U_{\text{l}}$).

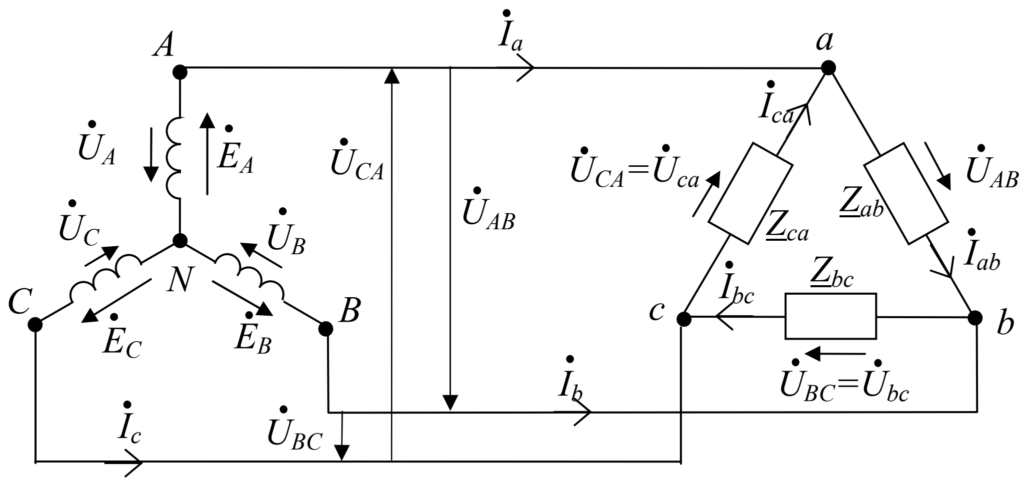


Figure 6.6: Connection of consumers by "triangle"

Neglecting the resistance of the line wires, line voltages of consumer can be equated to line voltages of power supply:

$$\dot{U}_{ab} = \dot{U}_{AB} \;;\; \dot{U}_{bc} = \dot{U}_{BC} \;;\; \dot{U}_{ca} = \dot{U}_{CA} \;.$$

For symmetrical supply system:

$$U_{ab} = U_{bc} = U_{ca} = U_{AB} = U_{BC} = U_{CA} = U_{ph} = U_l \,. \tag{6.10}$$

A vector diagram of voltages on the complex plane on symmetric supply for active-inductive load (φ > 0) is presented in figure 6.7. Here complex linear voltage directs on the positive real axis of the complex plane. Meanwhile complex linear voltages are written in the following form:

$$\left.\begin{aligned}
\dot{U}_{ab} &= U_{ab} = U_l; \\
\dot{U}_{bc} &= U_{bc}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = U_l\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right); \\
\dot{U}_{ca} &= U_{ca}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = U_l\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right).
\end{aligned}\right\} \qquad (6.11)$$

The relationship between line and phase currents on connection of consumers of electricity by triangle and symmetric load are determined from the equations developed for currents in accordance with the first Kirchhoff's law for nodes a, b, c of branching of electrical circuit (see fig. 6.6): $\dot{I}_A + \dot{I}_{ca} - \dot{I}_{ab} = 0$; $\dot{I}_B + \dot{I}_{ab} - \dot{I}_{bc} = 0$; $\dot{I}_C + \dot{I}_{bc} - \dot{I}_{ab} = 0$.

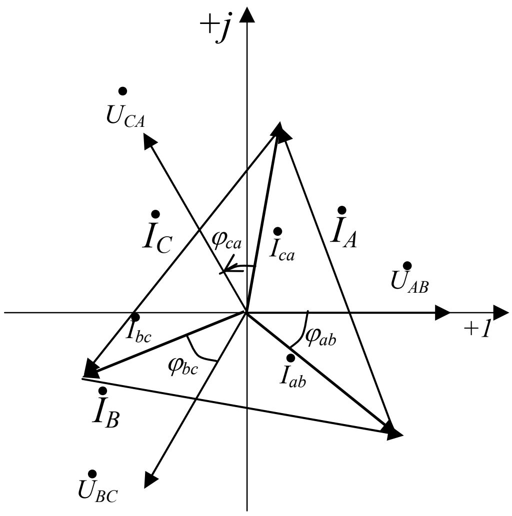


Figure 6.7: Vector diagram of currents and voltages at the consumer connection by "triangle"

For symmetrical load line currents $I_A = I_B = I_C$ and phase currents $I_{ab} = I_{bc} = I_{ca}$. Meanwhile the phase shift angle between phase currents and voltages $\varphi_{ab} = \varphi_{bc} = \varphi_{ca}$, as in this case, the power coefficient $\cos\varphi_{ab} = \cos\varphi_{bc} = \cos\varphi_{ca}$.

In accordance with these equations in figure 6.7 vector diagram of the phase and line currents of the consumer was built, from which it follows that on connection of the phases of the symmetric three-phase electricity consumers by "triangle" between the line and phase currents the ratio takes place

$$I_л = \sqrt{3} I_{ph}. \qquad (6.12)$$

6.5 Three-phase four-wire electrical circuits

In three-phase four-wire electrical circuits in the presence of linear wires connecting the beginnings of phases of the power source and consumer of electricity, there is also a neutral wire connecting the neutral point *N* of the source with the neutral point *n* of the consumer (fig. 6.8), which ensures the symmetry of the phase voltages of the source and the consumer, as the neutral wire equalizes the potentials of the neutral point *N* and *n*.

Three-phase four-wire systems of power supply of consumers are widespread in distribution circuits of industrial enterprises, residential and civil buildings. They allow getting two different $\sqrt{3}$ voltage - phase U_{ph} and linear $U_l=\sqrt{3}\ U_{ph}$. With mixed power and lighting loads power consumers are fed by line voltages $U_л$= 660; 380; 220 V. For lighting loads phase voltages U_l = 220; 127 V are used.

In four-wire electrical circuits phase source and phase of the consumer is always connected by "star".

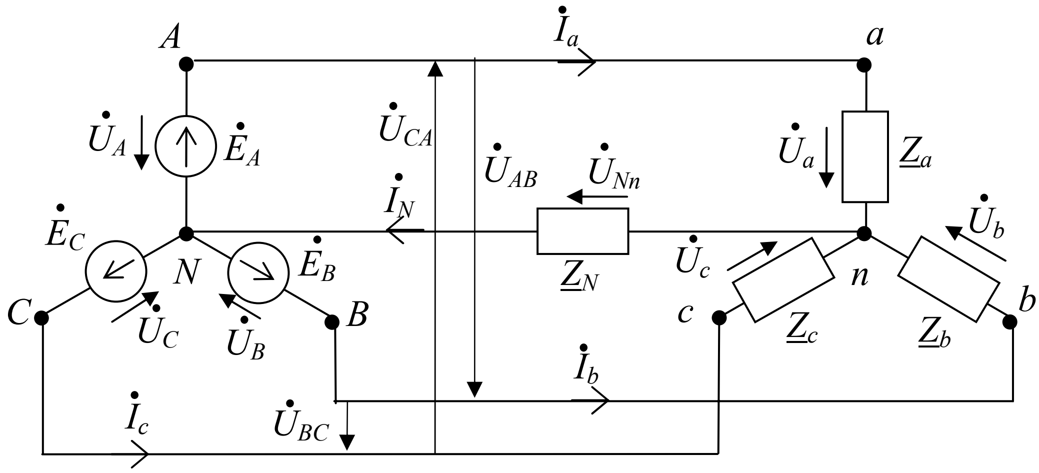


Figure 6.8: Three-phase four-wire electric circuit

With asymmetric load complex impedances of phases of the consumer are not the same ($\underline{Z}_a \neq \underline{Z}_b \neq \underline{Z}_c$) meanwhile complex voltage (U_{nN}) acting between neutral points N and n of the system, is determined by the method of two nodes

$$\dot{U}_{nN} = \frac{\dot{E}_A \underline{Y}_a + \dot{E}_B \underline{Y}_b + \dot{E}_C \underline{Y}_c}{\underline{Y}_a + \underline{Y}_b + \underline{Y}_c + \underline{Y}_N}, \tag{6.13}$$

where: $\dot{\boldsymbol{E}}_A, \dot{\boldsymbol{E}}_B, \dot{\boldsymbol{E}}_C$ are complex EMF of power source;

$\underline{\boldsymbol{Y}}_a, \underline{\boldsymbol{Y}}_b, \underline{\boldsymbol{Y}}_c, \underline{\boldsymbol{Y}}_N$ are the complex conductivity of the phases of the consumer and neutral wire.

With symmetrical load $Z_a = Z_b = Z_c$ sum of complex currents at the point n of branching of the circuit, recorded in accordance with the first Kirchhoff's law: $\dot{I}_A + \dot{I}_B + \dot{I}_C = \dot{I}_N = 0$, as the current in the neutral wire $I_N = 0$. Meanwhile the voltage acting between neutral points: $\dot{U}_{nN} = \underline{Z}_N \dot{I}_N = 0$.

Neglecting the internal resistance of the symmetric power supply, and taking into account that EMF $E_A = E_B = E_C = E_{ph} = \sqrt{3}U_l$, complex voltage acting between the neutral point of the system, is determined from the equation

$$U_{Nn} = \frac{U_n(\underline{Y}_a + a^2\underline{Y}_b + a\underline{Y}_c)}{\sqrt{3}(\underline{Y}_a + \underline{Y}_b + \underline{Y}_c + \underline{Y}_N)}, \tag{6.14}$$

where $a = e^{j\frac{2\pi}{3}} = (-\frac{1}{2} + j\frac{\sqrt{3}}{2})$, $a^2 = e^{-j\frac{2\pi}{3}} = (-\frac{1}{2} - j\frac{\sqrt{3}}{2})$ – turning multipliers (operators).

Complex phase voltages of electricity consumers are found from equations composed by the second Kirchhoff's law for the corresponding closed contour of the system (fig. 6.9):

$$\dot{U}_a = \dot{E}_A - \dot{U}_{nN};\ \dot{U}_b = \dot{E}_B - \dot{U}_{nN};\ \dot{U}_C = \dot{E}_C - \dot{U}_{nN}.$$

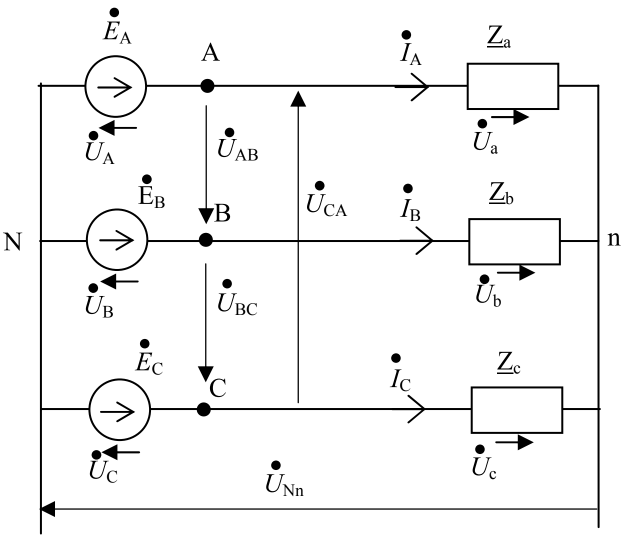


Figure 6.9: Calculation scheme

Meanwhile complex phase currents of the consumer are determined by the Ohm's law for the relevant subcircuits:

$$\dot{I}_A = \dot{U}_a / \underline{Z}_a; \qquad \dot{I}_B = \dot{U}_B / \underline{Z}_B;$$

$$\dot{I}_C = \dot{U}_C / \underline{Z}_C.$$

The complex current in the neutral wire is found in accordance with the equation made by the first Kirchhoff's law for the neutral point n of the circuit: $\dot{I}_N = \dot{I}_A + \dot{I}_B + \dot{I}_C$.

With symmetric load phase voltages: $U_a = U_b = U_c = U_{ph}$, meanwhile

$$U_{ph} = \frac{U_l}{\sqrt{3}}; \quad I_A = I_B = I_C = I_{ph} = \frac{U_{ph}}{Z_{ph}} = \frac{U_l}{\sqrt{3} Z_{ph}}.$$

At the rupture of neutral wire its impedance $\underline{Z}_N = \infty$, and the total conductivity $\underline{Y}_N = 0$.

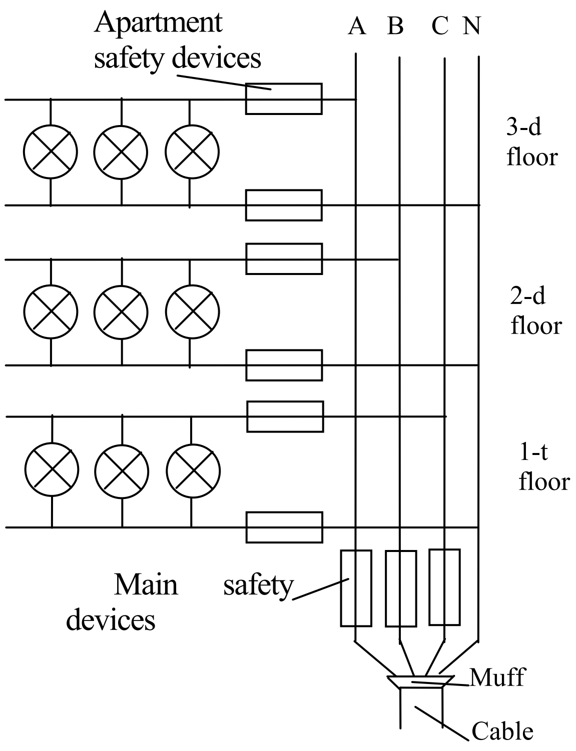


Figure 6.10: Four-wire electrical network

With the asymmetric load of electricity consumers ($\underline{Z}_a \neq \underline{Z}_b \neq \underline{Z}_c$) on a vector chart is offset from the neutral point n of the consumer relative to the neutral point N of the source, which leads to distortion of the phase voltages of the consumer. As a result on some phases of the consumer voltage will be greater than on others, which in many cases is unacceptable, particularly at the power of the lighting load when one of the illuminators is under tension, less then nominal, and the other is under tension, a greater than nominal, which leads to premature device failure. In connection with it, ***neutral wire four-wire electric circuit prohibites the installation of safety devices or switches*** (fig. 6.10), as when disconnected neutral wire phase voltages are unequal. As a result, the voltage of one phase may be less than the nominal, and others are more than nominal. As a result, for example in circuits of lighting installations underheating of lamps in phases with low voltage will occur, and overheating and premature burnout of lamps in phases with increased voltage as well. A burnout is one of the main safety device will disable the receiving terminals of the corresponding line. A fuse burnout of the main power consumers will disable the corresponding line.

6.6 Active, reactive and apparent power of three-phase electric circuit

The active (reactive, apparent) power of three-phase system is the sum of active (reactive, apparent) powers of all phases of the energy source equal to the sum of active (reactive, apparent) powers of all phases of the receiver.

Three-phase four-wire system provides electricity consumers with a symmetric power. Meanwhile active, reactive and apparent power can be determined by the following formula taking into account the sign of the reactance:

$$\left.\begin{aligned} &P = I_A^2 R_a + I_B^2 R_b + I_C^2 R_c = I_A \cdot U_a \cdot \cos\varphi_a + I_B \cdot U_b \cdot \cos\varphi_b + I_C \cdot U_c \cdot \cos\varphi_c; \\ &Q = I_A^2 \cdot X_a + I_B^2 \cdot X_b + I_C^2 \cdot X_c = I_A \cdot U_a \cdot \sin\varphi_a + I_B \cdot U_b \cdot \sin\varphi_b + I_C \cdot U_c \cdot \sin\varphi_c; \\ &S = \sqrt{P^2 + Q^2}, \end{aligned}\right\} \quad (6.15)$$

where: $\boldsymbol{cos\varphi_a = R_a/Z_a}$; $\boldsymbol{cos\varphi_b = R_b/Z_b}$; $\boldsymbol{cos\varphi_c = R_c/Z_c}$; $\boldsymbol{sin\varphi_a = X_a/Z_a}$; $\boldsymbol{sin\varphi_b = X_b/Z_b}$; $\boldsymbol{sin\varphi_c = X_c/Z_c}$.

For symmetrical load the formulas are as follows:

$$\left.\begin{aligned} &P = 3I_{ph}^2 R_{ph} = \sqrt{3}U_l I_l \cos\varphi_{ph}; \\ &Q = 3I_{ph}^2 X_{ph} = \sqrt{3}U_l I_l \sin\varphi_{ph}; \\ &S = \sqrt{P^2 + Q^2} = \sqrt{3}U_l I_l, \end{aligned}\right\} \quad (6.16)$$

where: $\cos\varphi_{ph} = R_{ph} / Z_{ph}$; $\sin\varphi_{ph} = X_{ph} / Z_{ph}$.

For calculation of difficult AC circuits the concept of ***complex power of three-phase circuit*** is used, which is considered as ***the sum of the complex powers of all phases of the energy source equal to the sum of the complex powers of all phases of the receiver.***

In complex notation the apparent (complex) power of three-phase electrical circuit is:

$$\dot{S} = P \pm jQ. \quad (6.17)$$

Apparent power of each of the phases of the consumer can be defined by the formulas:

$$\left.\begin{aligned} &\dot{S}_a = P_a \pm jQ_a = \dot{U}_a \cdot \overset{*}{I}_a; \\ &\dot{S}_b = P_b \pm jQ_b = \dot{U}_b \cdot \overset{*}{I}_B; \\ &\dot{S}_c = P_c \pm jQ_c = \dot{U}_c \cdot \overset{*}{I}_C, \end{aligned}\right\} \quad (6.18)$$

where: $\overset{*}{\boldsymbol{I}}_A$, $\overset{*}{\boldsymbol{I}}_B$, $\overset{*}{\boldsymbol{I}}_C$ are accordingly conjugate complex currents in phases.

6.7 Comparison of conditions of the receiver work when connection of its phases is made by "triangle" and "star"

The scheme of connection of the three phases of the receiver does not depend on the connection schemes of three phases of generator. The connection of phase of the receiver by triangle is frequently switched to the connection by star to change the current and power, for example to reduce the starting current of three-phase motors, to change temperature of the three-phase electric furnaces, etc.

Consider the changes in the effective values of the currents of the symmetric receiver with phase impedance Z_{ph} with switching of phase from star to triangle, for example, three-pole switch (fig. 6.11).

With the connection of phases of receiver by star between the operating values of phase and line currents (6.6) and voltages (6.7) the following ratios are fair

$$I_{phY} = U_{phY} / Z_{ph} = I_{lY} \ ,$$

$$U_{phY} = U_l / \sqrt{3} \ ,$$

from which it follows that

$$I_{lY} = \frac{U_l}{\sqrt{3} Z_{ph}} . \tag{6.19}$$

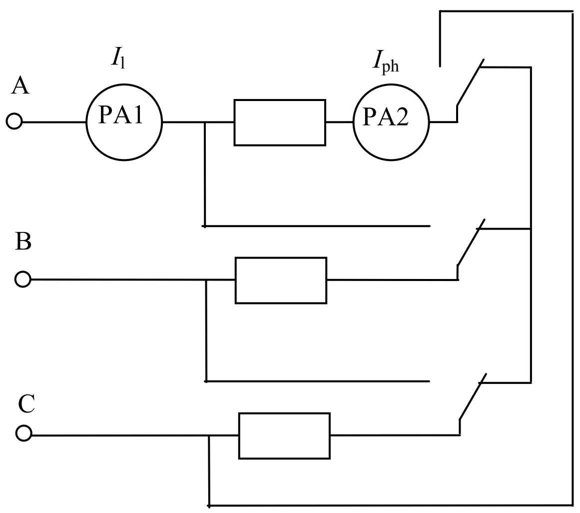


Figure 6.11: Scheme of switching of three-phase receiver from star to triangle

With the connection of the phases of the receiver by triangle between the operating values of phase and line currents (6.12) and voltages (6.11) the following ratios are fair

$$I_{ph\Delta} = \frac{U_{ph\Delta}}{Z_{ph}} = \frac{I_{l\Delta}}{\sqrt{3}} ; \qquad U_{ph\Delta} = U_l$$

from which it follows that

$$I_{l\Delta} = \frac{\sqrt{3} U_l}{Z_{ph}} \ . \tag{6.20}$$

Comparing the expressions for the operating values of line currents on connection of the phases of receiver by "star" (6.19) and "triangle" (6.20), we obtain the same operating value of the line voltage $U_л$ and the same phase impedances $Z_ф$.

$$I_{l\Delta} = 3 I_{lY} \ ,$$

and for operating values of phase currents

$$I_{ph\Delta} = \sqrt{3} I_{phY} \ .$$

Active power of three-phase symmetrical receiver when any of the schemes of connection (6.15) is equal to

$$P = \sqrt{3} U_l I_l \cos\varphi \ .$$

Due to the reduction of the current value of the line current when switching the phase of the receiver with the "triangle" to "star" power is reduced by 3 times, i.e.

$$P_{\Delta} = 3P_{Y} \ . \tag{6.21}$$

Key findings

1. In three-phase electrical systems two schemes of connection of consumers – "star" and "triangle" are used. Sources of electrical energy, as a rule, are connected in a “star” scheme.

2. In a symmetrical three-phase system:
linear (phase) voltage is shifted relative to each other in phase by the angle 2π/3;
the vector sum of phase (line) voltage is equal to zero.

3. In a symmetrical three-phase system with connection of the phases of the consumer by "star":
linear currents are equal to the phase currents;
linear voltages in $\sqrt{3}$ are more than in phase ones.

4. In a symmetrical three-phase system with connection of the phases of the consumer by "triangle":
phase voltage is equal to line voltage;
linear currents in $\sqrt{3}$ are more than phase ones.

5. In three-phase four-wire electrical circuits:
phase source and phase of the consumer are always connected by "star";
neutral wire provides symmetry of phase voltage of source and the consumer.

6. When switching of phases of the receiver from the "triangle" to the "star" its active power is reduced by 3 times.

Control questions

1. What are the meanings of the term phase in electrical engineering?
2. What is the three-phase system?
3. When is the three-phase system symmetrical?
4. What is direct (reverse) sequence of alternation of phases?
5. What is the neutral of the three-phase circuit?
6. What schemes of phase connection are used in three-phase circuits?
7. Write down the complex form of the phase voltage of three-phase symmetric consumers united with the “star” scheme.
8. Draw the vector diagram of phase and line voltages of the consumer included in the “star” scheme.
9. Draw the vector diagram of voltages and currents of the consumer included in a "triangle" scheme.
10. Explain the features of three-phase four-wire circuits.
11. Write the ratios for active, reactive and apparent power of three-phase symmetrical circuit.

SECTION 3.
ELECTRICAL MEASUREMENTS

7 ELECTRICAL MEASURING INSTRUMENTS

Key concepts: measurement, tools of electrical measuring, measure, master form, check gage, working measure, electrical measuring instrument (EMI), measuring transducer, electrical measuring installation, electrical measuring information system, normalizing value of EMI, division value of EMI, sensitivity of EMI, threshold of sensitivity, accuracy, ammeter, voltmeter.

7.1 Basic concepts

Methods, ways and means of ensuring the uniformity of obtaining the required accuracy of measurements of physical quantities are studied by the science of Metrology. ***Unity of measurements*** is the comparability of results regardless of where, when, by whom and by what means the physical quantity was measured.

Currently, for measuring of physical quantities of different nature in science and on production electrical methods are widely used. Their main advantages are: high sensitivity, possibility of automation of the measurement process, and low energy consumption. The use of transducers of non-electrical quantities in electric ones allows us to perform measurements of almost all parameters of the technological processes of the construction industry.

The number of mechanical, thermal and other non-electrical quantities of interest to science and industry to be measured is many times more than the number of all possible electric and magnetic quantities. Measurement of nonelectrical quantities by electrical methods has now reached a high level of development and forms a large branched area of modern technology which provides the ability to perform any necessary measurements.

Measurement is a process that involves the comparison of the measured physical quantity with a certain value taken as a unit by experiment. Or, in other words, the measurement is the process of finding the values of physical quantities by experience by using special technical means.

Generally, measurement results can be written as follows

$$X=n(x)\,, \qquad (7.1)$$

where: $\boldsymbol{X}$ is the measured value; n is the quantitative characteristic of measurements; x is a unit.

There is difference between the ***true*** and ***actual value*** of the measured values. The true value is free from measurement error. The actual value refers to the value obtained by measurement with admissible accuracy. If it is possible to neglect the measurement error, then the true value coincides with the actual $\boldsymbol{X_{TRUE}}\approx\boldsymbol{X_{A}}$.

Means of electrical measurements is a technical instrument used in electrical measurements. The instrument has some normalized metrological characteristics. The means of electrical measurements are measures of electrical measuring instruments, measuring transducers, electrical measuring installation and measuring information systems.

The measure is a measurement tool designed to reproduce physical quantities of specified value. The main measures of electrical quantities are measures of electromotive force, electric current, electrical resistance, inductance, capacitance, etc.

Depending on the degree of accuracy and application domain the measures are divided into standards, check gages and working measures. ***The standards*** provide reproduction and storage of a unit of physical quantities for transmission of its size to other means of measurement. ***Standard measures*** are used for the verification and calibration of working measures and measuring instruments. ***Working measures*** are used for calibration of measuring instruments, and to measure in scientific organizations and industrial enterprises.

Electrical measuring instrument (EMI) is the means of electrical measurements, designed to generate a signal of measurement information in a form accessible to direct perception by an observer. EMI includes ammeter, voltmeter, wattmeter, and counter.

Measuring converters are the means of electrical measurements, designed to generate a signal of electric information in a form suitable for transmission, further conversion, processing and storage, but beyond the immediate perception of the observer. They are divided into converters of electrical quantities in electrical (shunts, voltage dividers, instrument transformers, and so on), the converters of non-electrical quantities in electrical, sensing devices (thermistors, thermocouples, resistance strain gauges, capacitive and inductive transducers, and so on).

Electrical measuring installation is a set of functionally integrated measurements (measures, measuring instruments, measuring transducers) and auxiliary devices, designed to generate signals of measurement data in a form convenient for the direct perception by the observer and located in one place.

Measuring information system is a set of measuring instruments and auxiliary devices connected by communication channels designed to generate signal of measurement information from a number of sources in a form convenient for processing, transmission and use in automatic control systems.

7.2 Classification of electrical measuring instruments

EMI are used for measurement of electric circuits parameters: current, voltage, powers, resistance, capacitance and inductance.

Classification of EMI is carried out by several principles.

By the action EMI are divided into the following groups: magnetoelectric, electromagnetic, electrodynamic, electrostatic, and ferrodynamic. The most widespread are the first two groups.

By the nature of the measured current they are divided into EMI DC and AC.

By referring to the measured parameter they are divided into ammeters – current measurement; voltmeters – voltage measurement; wattmeters – measurement of active power; ohmmeters – active resistance measurement, etc.

By the way of presenting the results of the measurements there are analog and digital instruments.

By destination there are instruments for industrial applications, instruments included in complex information systems, laboratory instruments.

There are also ***combined instruments*** to measure several parameters of electrical circuits.

In addition, the instruments are divided into showing, registering (recording) and summarizing (counters, integrators).

In the indicating instruments the readout of measured values is produced by the position of the arrow on the scale of the instrument.

Self-recording instruments provide a continuous record of the measured value on paper, and direct registrations reading for the measurement of the arrow.

Summing (integrating) instruments provide a summation of the readings for a certain required period of work.

7.3 General technical specifications of electrical measuring instruments

While making electrical measurements it is necessary to consider the minimum and maximum limits of the measurements, the division value of the instrument, its sensitivity, measurement error, input impedance, and power consumption.

The upper limit of the measuring instrument is called a ***normalizing value*** X_N.

The arrow of the multirange EMI shows the measured value in the divisions.

To go to the largest measurement it is necessary to determine the ***division value*** (number of units of the parameter being measured in the same division of the scale EMI):

$$C = \frac{X_N}{N} , \tag{7.2}$$

where: ***N*** – the number of divisions of the scale.

Value of physical quantity is defined as follows:

$$X = C \cdot n , \tag{7.3}$$

where: ***n*** – the number of deflection instrument divisions.

Sensitivity of EMI is the ratio of signal changes at the output of the instrument to changes in the measured value. The smaller the measured value the instrument will note, the more opportunities for high-precision measurement will be.

There is ***absolute*** S_{ab} and ***relative*** S_{rel} sensitivity:

$$S_{ab} = \frac{\Delta l}{\Delta X} \quad ; \quad S_{rel} = \frac{\Delta l}{\Delta X / X} \tag{7.4}$$

where: Δl – change of the signal on output of the instrument;
ΔX – change of the measured value;
X – measured value.

Changing the measured value that causes the least movement of the pointer instrument, which can be seen in the normal method of reference, is called the ***threshold of sensitivity***.

The sensitivity S is associated with the division value of instrument C by the following ratio:

$$S = \frac{1}{C} . \tag{7.5}$$

The limits of permissible errors (basic and additional) can be expressed in the forms of absolute, relative and reduced error. More errors of the means and methods of measurements are discussed in chapter 8.1.

7.4 The magnetoelectric system instruments

Instruments of magnetoelectric system are used in DC circuits for measuring current and voltage; they have a number of advantages: high sensitivity and precision, uniformity of scale and low power consumption.

The principle of action of EMI magnetoelectric system bases on the interaction of the magnetic field generated by the measured current in the moving coil with the magnetic field of the permanent magnet, in result of which the frame is rotated by an angle proportional to the measured current. Figure 7.1 shows the instrument of EMI of magnetoelectric system with the inside frame magnet. The instrument has a permanent magnet 1 and the annular magnetic core 2 made of soft magnetic steel.

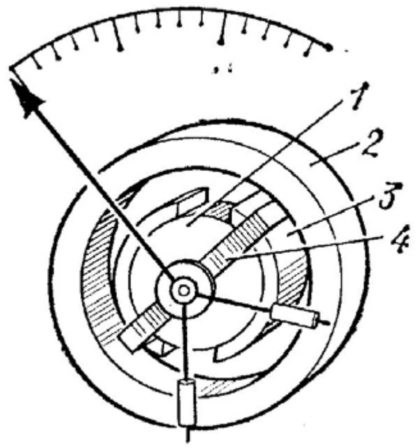


Figure 7.1: Instrument EMI of magnetoelectric system

Due to the uneven magnetic flux of the permanent magnet in different parts of the air gap uneven magnetic field would be created. To overcome this defect the soft magnetic steel plates 3 are set, which allow creating a uniform radial magnetic field.

In the air gap between the magnet 1 with steel plates 3 and the magnetic core 2 there is movable coil 4 made in the form of a frame of insulated copper wire. A frame is fixed on the center pads and it rolls freely on its axis.

A spiral spring, connected to the axis of the instrument, turns the frame and creates a counteracting moment.

When switching on the instrument into the electric circuit in the frame of the instrument an electric current arises. The interaction of the current in frame with the magnetic field of the permanent magnet causes the generation of the torque of frame, proportional to the current

$$M_{\text{torque}} = k_1 I, \tag{7.6}$$

where: $\mathbf{M_{вр}}$ is torque;
I is the current in frame;
$\boldsymbol{k_1}$ is coefficient of proportionality, which depends on the size of frame, number of turns, quantity of the magnetic induction in the air gap between the tips of the permanent magnet and the core and the system of units.

When the torque frame is rotated, the spiral springs begin to spin and create a counter moment

$$M_{\text{counter}} = k_2\,\alpha \tag{7.7}$$

where: $\boldsymbol{k_2}$ is a coefficient that depends on the elastic properties of the spring; $\boldsymbol{\alpha}$ is the angle of torsion of spring (angle of turn of frame).

In case of equal and opposing torque moments acting on the frame, it comes equilibrium, $M_{\text{torque}} = M_{\text{counter}}$, i.e., $k_1 \cdot I = k_2 \cdot \alpha$, where the rotation angle of the frame with the arrow equals $\alpha = (k_1/k_2)\, I = k \cdot I$.

From the last equation it follows that ***the angle of rotation of arrow of the magnetoelectric instrument is directly proportional to the value of the current passing through the frame, and the scale of the instrument is uniform***.

Instruments of this system are strictly polar and for the proper inclusion in the electrical circuit of the connection terminals are marked "+" and "-".

7.4.1 DC current measurement. Instruments used to measure current are called ***ammeters***. Ammeter is included in the electrical circuit sequentially, with the resistance of the instrument being much less than the resistance of the electric circuit.

For measuring currents above the maximum current of the instrument shunts are used. The shunt represents the resistance R_{sh} included parallel to the measuring instrument. In some cases, for convenience, the instrument has several shunts to measure small and large currents. The smaller the resistance of the shunt compared with the internal resistance of the ammeter, the less current flows through the instrument.

As in parallel connected electrical circuits voltage drops are equal to each other, then the following equation for shunt and ammeter is true (fig. 7.2): $I_0 R_0 = I_{sh} R_{sh}$, from which $R_{sh} = I_0 R_0 / R_{sh}$.

According to Kirchhoff's law the total current in the circuit is $I = I_0 + I_{sh}$, then, substituting the value of current I, we define the resistance of the shunt $R_{sh} = I_0 R_0/(I - I_0)$, where I_0 is the current of full deflection of the ammeter, R_0 is the internal resistance of the ammeter, I is the measured current in the circuit.

7.4.2 Measurement of DC voltage. Instrument for measuring voltage is called a ***voltmeter***. Voltmeter measures DC voltage and consists of an indicating instrument of magnetoelectric system, in series with which the additional resistance is included (fig. 7.3).

The additional resistance is selected so that with the limiting value of the measured voltage arrow instrument would have a maximum deviation. The amount of the additional resistance is determined by the formula

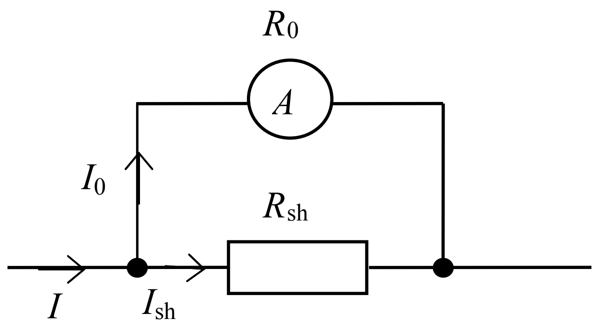


Figure 7.2: Scheme of switching on ammeter with shunt

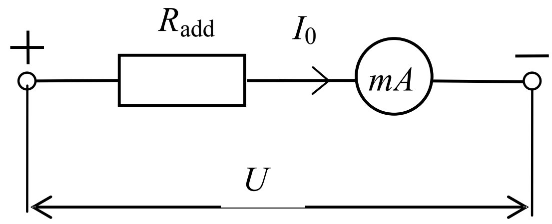


Figure 7.3: Electric scheme of voltmeter

$$R_{add} = (U_L/I_0) - R_0, \tag{7.8}$$

where: $\boldsymbol{U_L}$ is the limiting value of the measured voltage;
$\boldsymbol{I_0}$ is the current of full deflection of instrument;
$\boldsymbol{R_0}$ is the internal resistance of the instrument.

When measuring voltage the voltmeter is connected in parallel to measured subcircuit of electrical circuit.

7.5 The electromagnetic system instruments

Instruments of electromagnetic systems are widely used in circuits of DC and AC currents. Advantages of electromagnetic system instruments are simplicity of design, reliability and resistance to overloads.

The principle of EMI of electromagnetic system is based on the interaction between the two ferromagnetic cores, magnetizable under the action of magnetic field coils, through which the measured current flows.

The measuring mechanism of an electromagnetic system with a circular coil is shown in figure 7.4. Inside the coil 2 with the screen 1 there are two ferromagnetic sector cores: mobile 3, fixed on the axis and fixed 4.

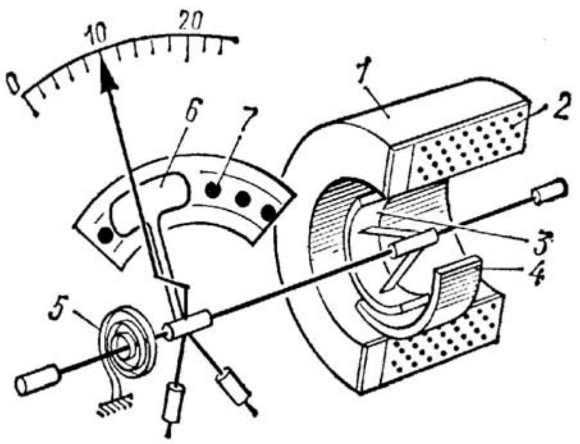


Figure 7.4: Measuring mechanism of an electromagnetic system

When flowing through the coil 2 of the measured current cores 3 and 4 are magnetized similarly and therefore push off each other. As a consequence, the torque is created and indicating arrow of instrument is deflected at a certain angle.

Opposite moment is generated by the spring 5. Magnetoinductive damper of instrument has a movable aluminum sector 6 and the permanent magnets 7.

Simultaneously with the measurement arrow of the instrument when sector 6 is moving wherein eddy currents are induced. As a result of interaction of these currents with the magnetic field of the permanent magnet 7 the power breaking the movement of the sector 6 is created.

Torque moment of measuring mechanism of the instrument in the AC circuit is proportional to the square of the effective value of current $M_{torque} \approx C \cdot I^2$. As a result, ***the scale of the instrument is quadratic***, which is a disadvantage. By choosing the shape of the ferromagnetic cores the scale can be obtained, which is irregular only in the initial part.

Improvement of sensitivity, accuracy and lifetime of EMI has been achieved in the development of fundamentally new magnetic system, the transition from the core pillars of moving parts on fastening with braces and implementation of new design liquid damper. For example, standardized measuring mechanism has a 50 times higher sensitivity compared to other types of instruments with the same overall dimensions.

Design of a unified measuring mechanism is shown in figure 7.5. The coil 2 is installed on the fixed magnetic core 3, on the ends of which there are two pairs of pole plates 1 and 4. Plate in pairs form the gaps 6 and 7, in which is core 5movable.

Electromagnetic instruments are used to measure in circuits of DC and AC currents as ammeters and voltmeters.

Ammeters produce single-range and multirange by partitioning of the coil. Voltmeters are usually done on several limits of measurement using a number of additional resistors.

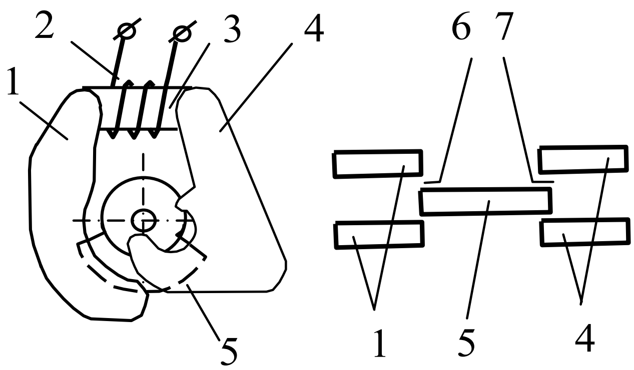


Figure 7.5: The design of the universal measuring mechanism

Electromagnetic instruments are among the most common panel instruments for measurement in AC circuits: they are simple in design and reliable, and endure overloads relatively well. Among the disadvantages of these instruments are relatively low accuracy and large private consumption (ammeters to 5 watt, voltmeters 1.5–12 watts), a limited frequency range, the influence on the readings of external magnetic fields.

Panel ammeters are produced by classes 1.0; 1.5; 2.5 currents up to 300 A of direct connection (with built-current transformers) and up to 15 kA with external current transformers. Panel voltmeters of the same classes are produced for voltages up to 600 V of direct connection and up to 450 kV with voltage transformers.

7.6 Electrodynamic and ferrodynamic instruments

7.6.1 Electrodynamic EMI. The principle of work of EMI of electrodynamic system is based on the interaction of the magnetic fields of the fixed and movable coils, through which the measured current flows.

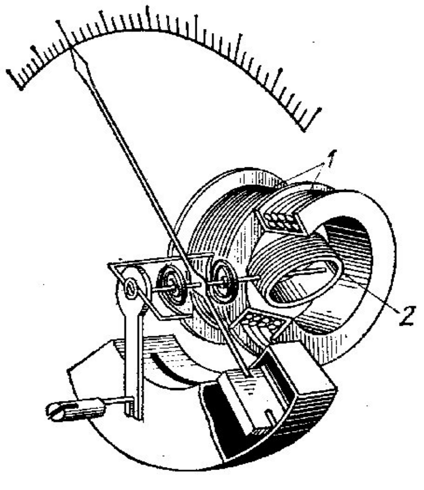


Figure 7.6: EMI instrument of electrodynamic system

Electrodynamic measuring mechanism (fig. 7.6) consists of two coils: 1 fixed and 2 mobile. Coil 2 is mounted on the braces (or axes) and can be rotated around the axis within the two sections of the fixed coil. If there are constant currents I_1 and I_2 in the coils electromagnetic forces occur tending to rotate the spool 2 in alignment with the coil 1. Torque occurs

$$M_{\text{torque}} = k\, I_1\, I_2\,. \qquad (7.9)$$

When a sinusoidal currents torque of electrodynamic measuring mechanism is proportional to the product of the effective values of currents in the coils I_1 and I_2 and the cosine of the phase shift angle between them $M_{\text{torque}} = k{\cdot}I_1{\cdot}I_2{\cdot}\cos\delta$.

Electrodynamic instruments are used in circuits of DC and AC currents as ammeters, voltmeters and wattmeters.

7.6.2 Power measurement. Electrodynamic wattmeters are used to measure the power in circuits of DC and AC currents. When using the wattmeter in the DC circuit (fig. 7.7) stationary coil is included in the circuit of current I, and the movable coil with series-connected additional resistor R_A - parallel to a load instrument with a resistance R_L. In a parallel circuit of the wattmeter current $I_V = U/R_V$ is formed, where R_V is the resistance of this circuit: $R_V = R_{WV} + R_{add}$; R_{WV} is resistance of winding voltage of the instrument. Then, substituting in (7.9)

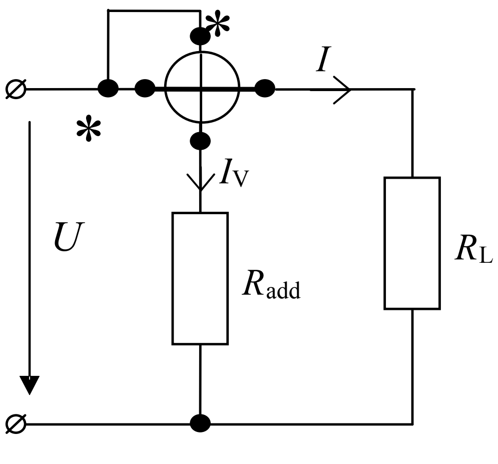


Figure 7.7: Scheme of inclusion of electrodynamic wattmeter

$I_1 = I,\ I_2 = I_V$, we get

$$M = k\, I\, I_V = k\, U/\, R_V \qquad (7.10)$$

or

$$\mathrm{M} = C\, U\, I = C\, P\,, \qquad (7.11)$$

where: $\boldsymbol{C}$ is the proportionality coefficient.

Thus, torque is proportional to the power consumed by the load resistance R_L.

The scheme of inclusion of the wattmeter in the alternating current circuit is similar to the scheme shown in figure 7.7.

7.6.3 Ferrodynamic EMI. Ferrodynamic measuring mechanism (fig. 7.8) differs from the electrodynamic by stronger magnetic field produced due to the presence of the magnetic system consisting of a magnetic core 3 and the stationary cylinder 4. Stationary coil 1 generates a magnetic field in the gap, in which the movable coil 2 can rotate. In this mechanism a torque stronger than in electrodynamic one is created, due to the presence of the magnetic core.

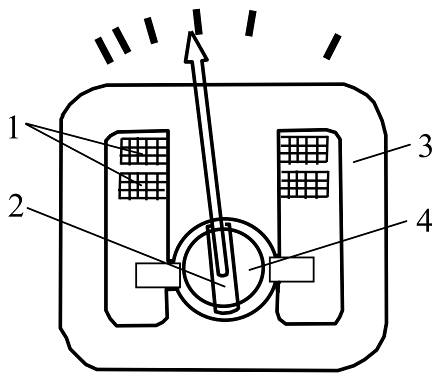


Figure 7.8: Ferrodynamic measuring mechanism

Ferrodynamic instruments (ammeters, voltmeters, wattmeters) are mainly used in circuits of AC as panel and portable instruments. They are less susceptible to external magnetic fields, have high sensitivity and consume their own energy less. Ferrodynamic instruments have a few disadvantages such as a relatively low accuracy and limited frequency range.

7.7 Measuring transducers

Measuring transducers are a large group of measuring instruments intended for conversion of the measured physical quantity X in handy for fixing value (usually voltage or current) Y (fig. 7.9).

In electrical engineering the following conversion instruments are used: shunts and additional resistance; measuring transformers of current and voltage; the inverter of the current type.

Shunts are resistors, connected in series in the circuit of the measured current and in parallel with the measuring mechanism. They are made of manganin.

The connection terminals of the shunt, which the current is connected to, are called the current connection terminals. On the schemes they are indicated by (*). The connection terminals which the measuring mechanism is connected to, are called potential, on figure 7.10 they are marked by (V).

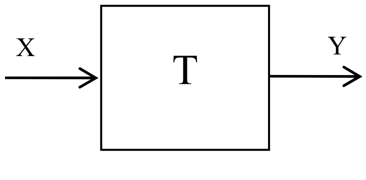


Figure 7.9: Measuring transducer

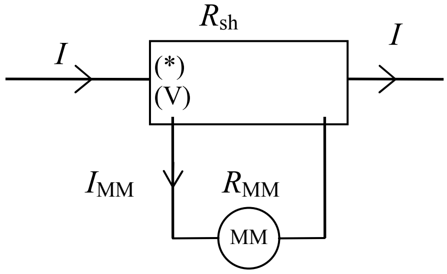


Figure 7.10: Scheme of shunt inclusion

Shunts are characterized by the nominal values of the input current and output voltage. The ratio of nominal voltage of nominal current is determined by the nominal resistance of the shunt.

Rated current in a circuit is defined as:

$$I_{MM} = I\frac{R_{sh}}{R_{MM} + R_{sh}}, \qquad I = I_{MM}\left(\frac{R_{MM}}{R_{sh}} + 1\right) = I_{MM} \cdot p \ , \qquad (7.12)$$

where: $\boldsymbol{p} = \dfrac{\boldsymbol{R_{MM}}}{\boldsymbol{R_{sh}}} + \boldsymbol{1}$ is a coefficient, which is called ***shunt multiplier***. Shows how many times the measured current in the circuit is higher than the current instrument, or how many times the measure current limit extends

$$p = \frac{I}{I_{MM}} \ . \qquad (7.13)$$

From (7.12) we can obtain the ratio to determine the shunt resistance

$$R_{sh} = \frac{R_{MM}}{p-1}. \qquad (7.14)$$

Shunts can be internal and external; single-range and multirange (see fig. 7.11). By accuracy they are divided into classes: 0.02; 0.05; 0.16; 0.2; 0.5; 1.

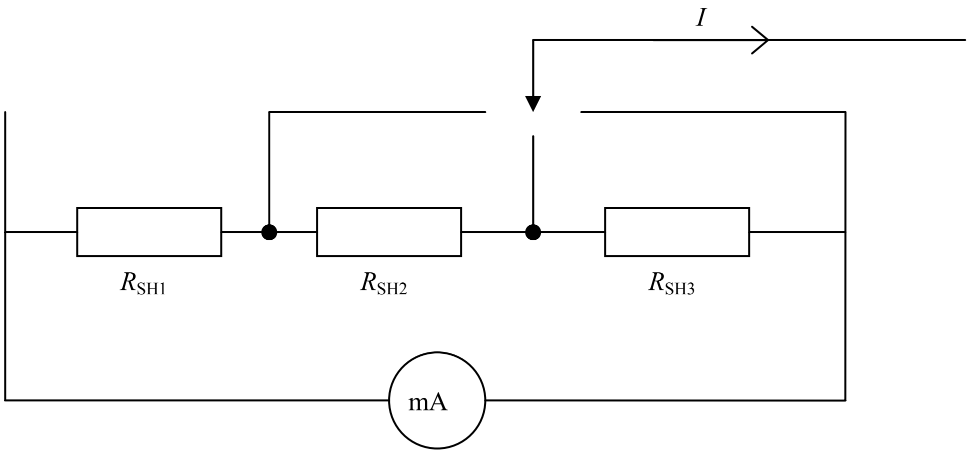


Figure 7.11: Scheme of a multirange shunt

Shunts are usually used in DC circuits. On AC current distribution in parallel branches depends on the inductance and frequency, so it brings additional error in measurement.

Additional resistances are used to limit the extension of the measurement voltage. They are included sequentially with measuring mechanism (fig. 7.12), which excludes the influence of temperature on the resistance of the instrument.

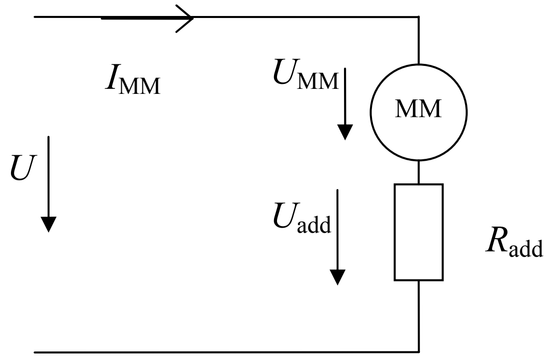


Figure 7.12: Inclusion scheme of additional resistance

In accordance with figure 7.10 we can write

$$U = U_{MM} + U_{add} = I_{MM}\left(R_{MM} + R_{add}\right). \qquad (7.15)$$

From (7.15) we can determine the value of additional resistance

$$R_{add} = \frac{U - I_{MM}R_{MM}}{I_{MM}} = R_{MM}(p-1), \qquad (7.16)$$

where: $\boldsymbol{p} = \boldsymbol{U/U_{TR}}$ is the division coefficient on the voltage.

From (7.16), it follows that the additional resistance must be in the (p - 1) times greater than the resistance of the instrument.

To obtain multirange voltmeters additional resistance is often applied, consisting of several resistors (fig. 7.13).

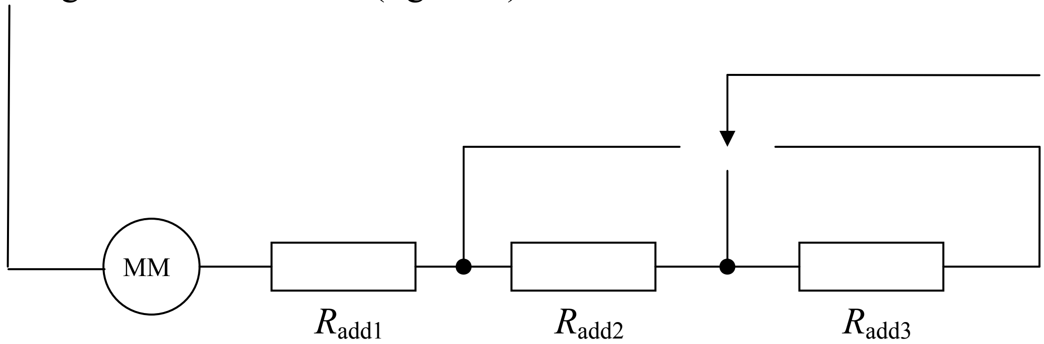


Figure 7.13: Scheme of multirange additional resistor

Voltage dividers work on the same principle, i.e., from each resistor the output is made with which the required voltage can be taken off.

Additional resistances have accuracy classes: 0.02; 0.05; 0.1; 0.2; 0.5; 1 and manufacture on rated currents: 0.5; 1; 3; 5; 7; 15 A and on 30 mA.

Measuring transformers of current and voltage are used as converters of large alternating currents and voltages in a relatively small currents and voltages (see chapter 9.6), valid for measuring by instruments with small standard nominal values (for example, 5 A, 100 V).

On the inclusion scheme in the measuring circuit (see fig. 7.14) and on the conditions of work the current transformers and of voltage are different from each other. The current transformer primary winding is included in the measuring circuit in series. The primary winding of the voltage transformer is included in the measuring circuit in parallel. Instruments attached to the secondary winding are ammeter and voltmeter respectively.

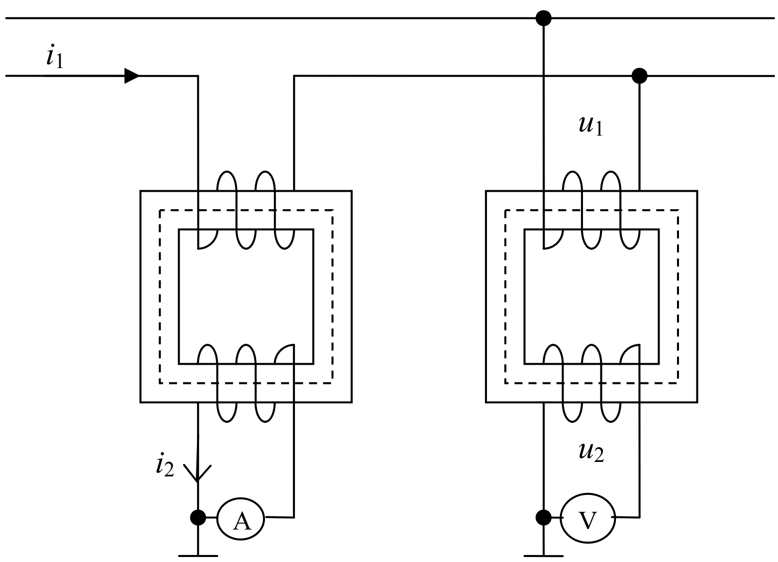


Figure 7.14: Inclusion schemes of the current transformers and of voltage

According to the readings of instruments included in the secondary windings, we can determine the values of the measured values. To do so, their readings are multiplied by the nominal transformation ratio (indicated in the passport transformers) for the current transformer

$$k_{I_R} = \frac{I_{1R}}{I_{2R}} , \tag{7.17}$$

and for the voltage transformer

$$k_{U_R} = \frac{U_{1R}}{U_{2R}} . \tag{7.18}$$

The actual transformation coefficients k_U and k_I depend on the resistance values of the secondary circuit, the voltage and current in the primary circuit. Because of this there are errors on the coefficient of transformation:

$$\delta_U = \frac{k_{U_R} - k_U}{k_U} \cdot 100\%, \quad \delta_I = \frac{k_{I_R} - k_I}{k_I} \cdot 100\% . \tag{7.19}$$

The measured current I_1 and the voltage U_1 are determined by the formulas:

$$I_1 = k_{I_R} I_2, \qquad U_1 = k_{U_R} U_2 . \tag{7.20}$$

Autotransformers are used for converting in circuit of alternating current of one AC voltage to another (see also chapter 9.5). They have one winding wound on the core. Usually the coils w1 are designed for voltage of 220 V. Part of windings in the coil is separated by means of the engine and are the turns of the secondary winding w_2. When the engine moves, the voltage is changed from "0" to the circuit voltage (see fig. 7.15).

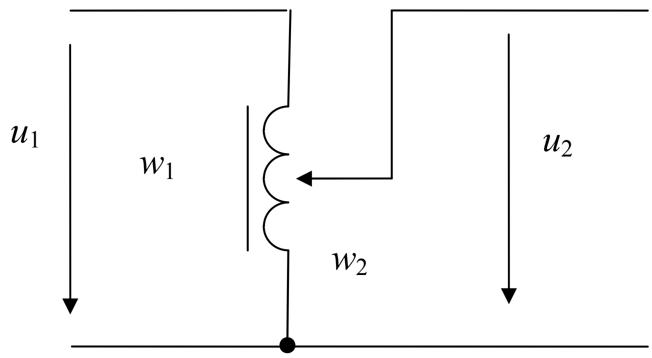


Figure 7.15: Scheme of autotransformer

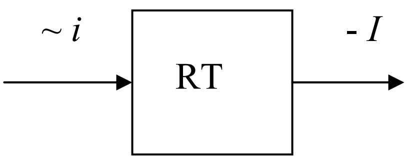


Figure 7.16: Functional scheme of the rectifier transducer

Rectifier transducers are used in EMI for converting AC to DC (see fig. 7.16) and are made on the basis of semiconductor technology.

The main characteristic of the rectifier transducer is the coefficient rectification k_{rec}, which characterizes the ratio of the direct current through the diode to reverse.

$$k_{rect} = \frac{I_{direct}}{I_{reverse}} = \frac{R_{reverse}}{R_{direct}} \; . \qquad (7.21)$$

The coefficient of rectification depends on the voltage, ambient temperature, and frequency of the rectified AC.

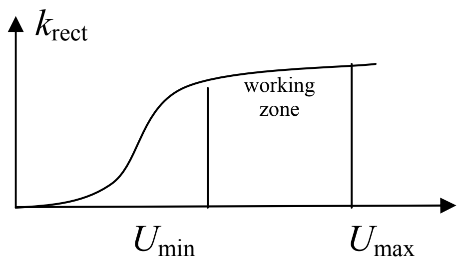


Figure 7.17: Dependence $k_{rect}=f(U)$

Figure 7.17 shows the dependence of the coefficient of rectification on the applied voltage. When the voltage $U < U_{min}$ rectification does not occur. When the voltage $U > U_{max}$ there is a breakdown of diode.

For germanium diode

$$k_{rect} = (4 \div 5) \cdot 10^3,$$

for a silicon diode $k_{rect}=10^5 \div 10^6$.

The rectified current is measured by magnetoelectric instruments. Semiconductor rectifier can be switched on according to the scheme of half-wave or full-wave rectification (see section 13.2).

Instruments with rectifiers measure the average current and voltage. To recalculate in the active value the shape coefficient of current curves and voltage must be considered. To reduce ripple in the DC circuits filters are put: capacitive, inductive, and combined.

7.8 Electronic analogue electrical measuring instruments

Electronic analogue EMI is a means of measurement, in which the conversion of the signal measurement information is performed using analogue electronic instruments. The readings of these instruments are continuous function of change of the measured quantity. Electronic voltmeters are an example of this group of instruments.

In electronic voltmeters measured voltage is converted using an analogue electronic instruments in DC, which is moved to the electromagnetic measuring mechanism with a scale calibrated in units of voltage.

There are voltmeters of direct and alternating voltages. Structural scheme of the electronic voltmeter of DC voltage is presented in figure 7.18, where ID is the input instrument in the form of a high-impedance resistive voltage divider, ACV is an amplifier of constant voltage, EMD is the electromagnetic measuring instrument.

The input instrument provides high input impedance and a value of the measured voltage required for conversion. ACV is used to increase the sensitivity of a voltmeter and a power amplification of the measured signal in order to activate the electromagnetic measuring mechanism.

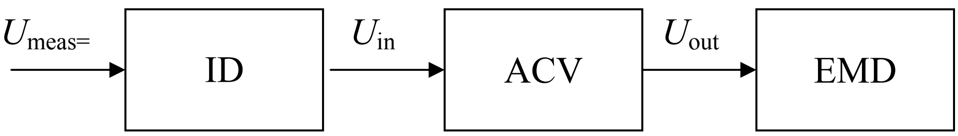


Figure 7.18: Structural scheme of the electronic voltmeter of direct voltage

A distinctive feature of electronic voltmeters of alternating voltage is the presence of the converter Cr (see fig. 7.19) of alternating voltage to direct. Depending on the value of the alternating voltage needed to be measured (average, active, amplitude) converters of average, active and peak values of the alternating voltage are respectively distinguished.

The advantages of modern electronic voltmeters are: large input impedance (>1 MW), low power consumption, high sensitivity, wide range of measured voltages (from tens of nanovolt DC to tens of kilovolts), and wide frequency range (from DC to hundreds of MHz).

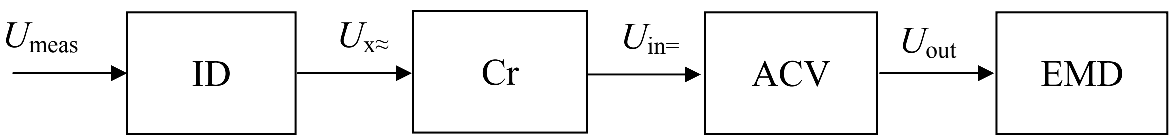


Figure 7.19: Structural scheme of the electronic AC voltmeter

7.9 Digital electrical measuring instrument

Achievements in the area of semiconductor and microprocessor technology have enabled the production of a wide range of digital measuring instruments for different purposes. They are used to measure the electrical and mechanical parameters in scientific research, laboratory and production conditions.

Digital measuring instruments have a number of significant advantages over conventional (analogue) instruments: high accuracy, the ability to store, transfer and enter results of measurements in automated measuring complexes and systems.

The principle of work of digital measuring instruments is based on the continuous transformation of the measured value into a sequence of pulses (digital code), followed by processing this code. A continuous transformation of the measured values is carried out using analogue-to-digital converters. To build them three methods are used: time pulse, pulse frequency and method of balancing servo.

In the time-pulse convert instruments the measured value is converted into proportional time interval measured by completion it by sequence of pulses of the reference frequency.

On the pulse frequency conversion the measured value is proportional to the frequency of the pulse determined by counting the number of pulses for a calibrated period of time.

Method of balancing servo means alternate comparison of the measured value with the sum of the exemplary discrete values that change according to a certain law.

Key findings

1. Electrical measuring instruments are divided into instruments of direct estimation and the instruments working according to the method of comparison. The first group includes, in particular, the most common instruments: ammeters, voltmeters, and wattmeters.
2. Instruments of direct estimation are divided into system depending on what principle the torque generated in the electric mechanism. The most common systems are magnetoelectric, electromagnetic, electrodynamic, and induction.
3. Electrical measuring instruments are classified by the nature of the measured current, the type of measured parameter (current, voltage, power, and so on), by the method of presentation of the measured value.
4. To extend the measuring range of ammeters and voltmeters additional resistance is used. In ammeter this resistance (shunt resistor) is connected in parallel with the resistance of the instrument, in the voltmeter it is connected consistently to the resistance of the instrument.
5. Digital measuring instruments in comparison with analogue ones have great opportunities in technological process automation, programming and accounting of energy resources by displaying the results of measurements on the computer.

Control questions

1. What is EMI classification?
2. What is meant by the accuracy of the EMI?
3. What is meant by the error of EMI? What kinds of errors exit?
4. How are ammeters and voltmeters included in the electrical circuit?
5. Explain the structure and working principle of magnetoelectric EMI.
6. Explain the structure and working principle of electromagnetic EMI.
7. Explain the structure and working principle of electrodynamic EMI.
8. What is the difference between ferrodynamic EMI?
9. Explain the principle of shunt operation.
10. Explain the schemes of the inclusion of measuring transformers of current and voltage.
11. What are the advantages of digital EMI?

8 METHODS OF ELECTRICAL MEASUREMENTS

Key conctps: accuracy (systematic, random, rough, absolute, relative, reduced), sensitivity, accuracy class of EMI, direct and indirect measurements, the method of direct evaluation, comparison methods (zero, differential, substitution).

8.1 Errors and measurement errors

Under ***the error*** is understood the deviation of the expected measurement result from the true value. Depending on the nature of the change there are systematic, random and rough errors.

Systematic errors change naturally, with repeated measurements remain constant and are only detected when testing the device.

Random errors can be detected by repeated measurements in view of the dispersion of results.

Rough errors give a sharp difference from the expected result and they do not take them into account.

Depending on the source of the origination error measurements are divided into hardware, methodical and subjective.

Hardware or instrumental errors depend on the errors of used measuring instruments. ***Methodological or theoretical errors*** appear due to the imperfection of the measurement methods, the use of approximate relationships put in the basis of the selected method of measurement, not account for the influence of several factors on the measurement accuracy. ***Subjective errors*** are errors caused by the imperfection of the senses of the operator, his inattention in conducting the measurements.

Depending on the value of the measured value X of the ***errors*** are divided into ***additive***, the absolute value of which does not depend on X, and ***multiplicative***, the absolute value of which is proportional to X.

Sources of additive error can be: offset of indicator of devices from zero up to implementation of measurement, the friction in the bearings moving parts of EMI, the inaccuracy of the grading scale. The reasons for the multiplicative error is the influence of external factors (temperature change, external electromagnetic fields) and ageing of components and nodes of EMI.

8.1.1 Measurement errors. The limits of permissible errors of EMI are expressed in absolute, relative and reduced errors.

Absolute error is the difference between the measured and the actual value

$$\Delta X = X - X_A \tag{8.1}$$

Absolute error is measured in units of the measured value, and can be both positive and negative values.

Relative error is the ratio of absolute error to the actual value of the measured quantity

$$\delta = \frac{\Delta X}{X_A} = \frac{\Delta X}{X_A} \cdot 100\% . \tag{8.2}$$

The relative error measures the accuracy of the measurement, expressed either in relative units or in percent and can be both positive and negative values.

8.1.2 The errors of measuring instruments. It is necessary to distinguish measurement errors and the errors of measuring instruments, due to its accuracy class.

By way of expression it is distinguished the following error measurement: absolute and relative (similar absolute (8.1) and relative (8.2) measurement errors), as well as the reduced.

Absolute error, taken with the opposite sign, is called the correction

$$П = -\Delta X = X_0 - X \,. \tag{8.3}$$

The reduced error is the ratio of absolute error to the normalizing value

$$\gamma = \frac{\Delta X}{X_N} = \frac{\Delta X}{X_N} \cdot 100\% \tag{8.4}$$

where: $\boldsymbol{X}_{\mathbf{N}}$ – normalizing value that may be equal to upper limit of scale, measurement range, the length scale and so on.

For most devices $X_N = X_R$, where X_R is the limit of measurement EMI or the nominal value of the measuring.

As a relative, reduced error can be expressed either in relative units or in percent.

On the basis of character changes are distinguished between systematic and random error of EMI, on the basis of the conditions of application of EMI – primary and secondary.

The main error occurs in EMI under normal operating conditions established by state standard 2261-82: ambient temperature 20±50 Celsius; atmospheric pressure is 750±30 mm mercury column, relative humidity 65±15% voltage of supply circuit 220±4.4 V the circuit with a frequency of 50 Hz, the normal position of the instrument scale, the absence of external electric and magnetic fields, except terrestrial, and so on

Additional errors of EMI occur when the deviation influencing factors (parameters listed above) from normal values.

For the characteristics of the tools and methods of measurement also it is used the concept of ***sensitivity*** - the minimum value of the controlled parameter changes that can react to the measuring device.

8.1.3 Accuracy classes of EMI. ***Accuracy class of EMI*** is the generalized characteristic defined by the limits of the permissible basic and additional errors, as well as other properties of measurement tools that affect the accuracy, the value of which is established in the standards for certain types of measuring instruments.

The main methods of normalization of permissible errors and designation of the class of accuracy established by state standard 8.401-80. On the scale of the device it is marked the value of the accuracy class of the instrument in the form of a number, indicating normalizing error value in %.

For measuring instruments measuring electrical quantities and having the upper limits, the accuracy class is set according to the given error. For measuring instruments that do not have upper limits, the accuracy class is set according to the relative error.

In accordance with state standard 8.401-80 measurement tools when determining the accuracy class is divided into four major groups:

1. Measurement tools from which it has dominated the additive component of error. It is indicating and recording devices with additive error from friction, changing the position in space, etc. For this group it is normalized error value, expressed as a percentage (8.4), which is used to describe the accuracy class.

Indicating EMIs have 8 classes of accuracy: 0.05; 0.1; 0.2; 0.5 – laboratory instruments; 1.0; 1.5; 2.5; 4 – technical devices. Figure characterizing the accuracy class, defines expressed in percent of the maximum, the basic reduced error of the instrument. The smaller the number, indicating the accuracy class, the higher the accuracy class of the instrument.

The relative error of EMI is determined by the ratio

$$\delta = \pm\gamma \frac{X_R}{X}, \tag{8.5}$$

where $\boldsymbol{\gamma}$ – the accuracy class of the instrument.

From the expression (8.5) it follows that the relative error of measurement, which characterizes the accuracy of the measurement depends not only on the accuracy class of the instrument, but also on what part of the scale is measured.

Any indicating EMI is advisable to use only in the last quarter of the scale of the instrument. Otherwise, even at the device high precision relative measurement error may be large enough.

2. Measurement tools, which dominates the multiplicative component of the error: the voltage dividers, shunts, instrument transformers of current and voltage, etc. In this group is normalized limit relatively permissible error in percent, and the accuracy class is indicated by a number, placed in a circle (for example, 1.5, which shows that the relative error in any point does not exceed ±1,5%.

3. Measurement means in which the additive and multiplicative components of the error are comparable. This is a digital devices, devices of comparison with manual and automatic balancing (bridges, compensators). For this group of devices limit the relative permissible basic error is expressed as the ratio

$$\delta = \pm\left[c + d\left(\frac{X_R}{X} - 1\right)\right]\%, \tag{8.6}$$

where: $\boldsymbol{X_R}$ – the limit of measurements;

X – measured value;

$\boldsymbol{d = \gamma_a = \Delta X_a / X_R \cdot 100\%}$ – the reduced value of the additive component of the error, expressed in percent; and $c = \delta_м + \gamma_a$ – the relative value of the multiplicative component of the error in percent;

c and ***d*** – a constant numbers, and the ratio *c/d* – the accuracy class of the instrument, for example 0.02/0.01. The first member *c* is equal to the relative error of measurement tools in optimal conditions, when $X = X_L$, and the second member *d* characterizes the increase in the relative measurement errors with decrease *X*, i.e. the influence of the additive component of the error.

4. Measuring instruments, in which it is dominated the additive component of the error, and which have a markedly pronounced non-uniform scale, for example, hyperbolic or logarithmic. In this case, it is normalized reduced value of the errors with respect to the range of the scale. The accuracy class is designated as the number placed between the two lines, arranged at an angle, for example 1.5 . Number of accuracy class means the limit of the allowable reduced errors, expressed in percentage, relative to the scale of the instrument in millimeters.

8.2 Classification of methods of electrical measurements

Depending on the method of receiving of the result the measurement is divided into two types: direct and indirect.

Direct measurements are call those measurements in which the target value of the physical quantity is determined directly on reading of device (measuring current with an ammeter, power with meter, voltage with voltmeter and others).

Indirect measurements are called those measurements in which the target value of the physical quantities are found on known functional dependency between this value and the values obtained by direct measurements. An example is the determination of the electrical resistance of the readings of the ammeter and voltmeter.

Depending on the combination of techniques of usage of principles and measuring instruments all methods are divided into methods of direct evaluation and comparison methods.

Under the method of direct evaluation it is understood the method by which the value of the measured values it is determined directly under the reading of indicating device of the measuring device of direct action (current value – the reading of the ammeter, the voltage value – on the reading of the voltmeter and others).

The method of comparison is called the method in which the measured value in a special measuring circuit is compared with the value reproduced by measure. Comparison methods are divided into zero, differential and substitution.

Zero method – a method of comparison of the measured value with the measure in which the resulting effect of influence of the compared values on the device of comparison is brought to zero.

Differential method – comparison method, in which on the measuring device it is influenced the difference between the measured values and the values reproduced by measure (for example, the measurement of electrical resistance using an unbalanced bridge).

Substitution method – method of comparison in which the measured value is substituted in the measuring system by known value, reproducible by measure. Thus by changing the known quantity it is achieved the same readings, which was on action of the measured value (for example, comparison of the resistance of a resistor with resistance of exemplary coil included alternately in one and the same arm of the bridge).

8.3 The measuring schemes

The measuring schemes and the inverters are used for the specific inclusion of sensors to measure controlled values, and conversion of the received signal in suitable one for further use and processing.

As already mentioned, the measurement error depends on the accuracy class of the measuring device and the relationship of values, on which it is designed the device, to the actual value of the measured quantity.

The measuring schemes are characterized by high ***sensitivity***. When measuring of small quantities sensitivity of measurement method is of particular importance, often determining the ability of the measurement itself. The sensitivity of the method of measurement is determined from the expression

$$S = S_{\text{sch}} \cdot S_{\text{inst}} \tag{8.7}$$

where: $\boldsymbol{S}_{\mathbf{sch}}$ – sensitivity of scheme;

$\boldsymbol{S}_{\mathbf{inst}}$ – sensitivity of measuring instrument.

Thus, to increase the sensitivity of the method measurement it can as an growth of sensitivity of the measuring scheme, so and the selection of appropriate measuring equipment.

In practice, to increase the sensitivity of the measurements they are applied pavement, compensation and differential measuring scheme.

8.3.1 Bridge measuring scheme. The scheme has four shoulder, to one diagonal of which is brought the feeding voltage, and from the other diagonal it is removed the output voltage, is called a bridge measuring scheme or just the bridge (fig. 8.1). It is used to convert the resistance changes of the sensor to the change in the magnitude or amplitude of the voltage. The resistance of the sensor (active, inductive or capacitive) changes proportionally to the changes of the controlled process parameter, for example temperature.

There are two main types of bridge scheme: equilibrium or balanced, bridged scheme, providing a null method of measurement; non-equilibrium, or unbalanced, bridged scheme, providing measurement by method of direct reading on the measuring instrument, included in the diagonal of the bridge. For measuring of non-electrical quantities by electrical methods most often it is used the second type of bridge scheme. In cases where the task of the bridge scheme is not measurement, and control of any process, mainly used for the first type of the bridge scheme.

Bridge schemes can operate on DC and AC current.

In the operation of the bridge scheme an important moment is equilibrium condition, which for the scheme in DC (fig. 8.1) has the form

$$R_1 \cdot R_4 = R_2 \cdot R_3\ . \tag{8.8}$$

On accomplishment of the condition (8.8.) the current in the diagonal of the bridge. $I_{\text{inst}} = 0$. In one of the shoulder of the bridge we insert the sensor, the resistance of which changes in proportion to the change in the monitored parameter. The condition of balance is disturbed and in the diagonal of the bridge *bd* through the measuring device it is passed current whose value is proportional to the magnitude of the controlled parameter.

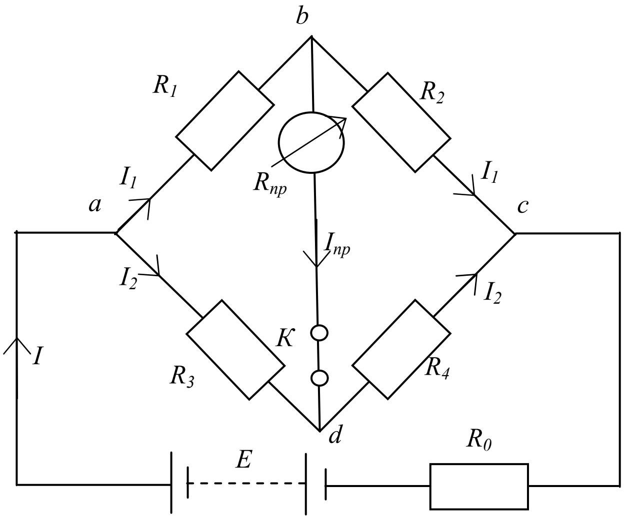


Figure 8.1: Bridge scheme

In the case of bridge scheme, feeding by alternating current, an equilibrium condition has the form:

$$Z_1 \cdot Z_4 = Z_2\, Z_3, \tag{8.9}$$

where Z_1, Z_2, Z_3, Z_4 – complex (total) resistance of the shoulders of the bridge scheme.

Because of the complex impedances Z_1, Z_2, Z_3, Z_4 it can be represented in the following form

$$Z_1 = R_1 + \mathrm{j}\, X_1,$$

$$Z_2 = R_2 + \mathrm{j}\, X_2,$$

$$Z_3 = R_3 + \mathrm{j}\, X_3,$$

$$Z_4 = R_4 + \mathrm{j}\, X_4\,, \tag{8.10}$$

where: $\boldsymbol{R_1}$**,** $\boldsymbol{R_2}$**,** $\boldsymbol{R_3}$**,** $\boldsymbol{R_4}$ – active resistances of branches of schemes;
$\boldsymbol{X_1}$**,** $\boldsymbol{X_2}$**,** $\boldsymbol{X_3}$**,** $\boldsymbol{X_4}$ – reactive resistance of the branches of the scheme, then the equilibrium condition (8.8) for the scheme in alternating current becomes more complicated:

$$\left.\begin{aligned} R_1 \cdot R_4 - X_1 \cdot X_4 \cdot &= R_2 \cdot R_3 - X_2 \cdot X_3; \\ R_1 \cdot X_4 + R_4 \cdot X_1 &= R_2 \cdot X_3 + R_3 \cdot X_2\,. \end{aligned}\right\} \tag{8.11}$$

The difficulty of controlling of the bridge scheme on alternating current is that after run of equilibrium condition (8.9) it is necessary to fulfill the condition (8.11), without violating the first condition. Such regulation it is possible only by the method of successive approximations, when at first it is achieved the minimum force of current adjusting one parameter, then achieve even greater reduction i_{inst}, adjusting another parameter, and so on, and successively reducing i_{inst} to zero.

The regulation of the equilibrium bridge with alternating current is simplified in the following cases:

1. If there is only reactive resistance when active resistances are equal to zero (which it is possible only with the use of capacitors). In this case, there is only one equilibrium condition

$$X_1 \cdot X_4 \cdot = X_2 \cdot X_3. \tag{8.12}$$

2. If the shoulders of the bridge it is included only active resistances, then the equilibrium condition is determined only by the equality

$$R_1 \cdot R_4 \cdot = R_2 \cdot R_3\,. \tag{8.13}$$

3. If two adjacent shoulders have only active, while the other two - only reactance (which it is possible when using capacitors). In this case, there is only one condition:

$$X_1 \cdot R_4 \cdot = R_2 \cdot X_3\,, \tag{8.14}$$

if $R_1 \cdot = R_3 \cdot = X_2 \cdot = X_4 = 0$.

The sensitivity of the equilibrium of the bridge scheme with a variable resistance R on current it is determined from the expression

$$S''_{sch} = \Delta i_{inst}/\Delta R_1 = i \cdot R_4/N = U \cdot R_4/M \qquad (8.15)$$

and on voltage

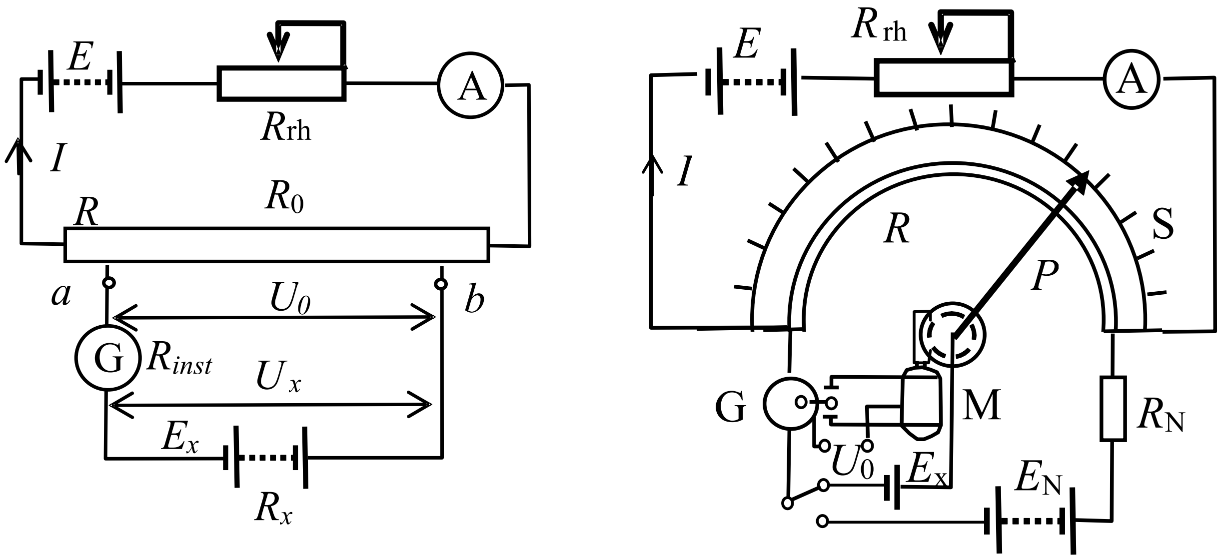
$$S''_{sch} = \Delta U_{inst}/\Delta R_1 = \Delta i_{inst} \cdot R_{inst}/\ \Delta R_1 = i \cdot R_{inst} \cdot R/N = U \cdot R_{inst} \cdot R_4/M \qquad (8.16)$$

where: $\Delta \boldsymbol{R_1}$ – change of the resistance of R_1 (one of arm);
M, N – the resistance of the shoulders, depending on how the inclusion of sensors;
U – voltage of feeding of the bridge scheme;
i – current strength consumed by the bridge scheme from the power source.

Thus, *the increased sensitivity of the bridge scheme may occur due to the increase of supply voltage and how to incorporate sensors.*

8.3.2 The compensation scheme. The principle of compensation is that the measured EMF (or voltage) are balanced by equal and opposite in sign the voltage drop, the value of which can be determined with high accuracy. Balancing voltage drop goes off from the potentiometer (rheochord) and fixed by the position of the slider (fig. 8.2).

Slider are set either manually checking the reading on the galvanometer (fig. 8.2, *a*), or it is set automatically by an electric motor M, managed by the galvanometer G (fig. 8.2, *b*). The latter scheme is called autocompensation, and it is used in almost all devices operating on the compensation method.



a *b*

Figure 8.2: Compensation measuring scheme

The mobile system of zero galvanometer G (fig. 8.2, *b*) instead of the arrow has a contact that at appearance in the galvanometer current depending on its direction is closed with the upper or lower stationary contact C. Meanwhile it is turned on the electric motor M and moved the engine of rheochord R_{rh} up until the current strength in the galvanometer again becomes equal to zero. Then the contacts will open, the electric motor will stop and the engine of rheochord will remain in the position corresponding to the condition of compensation until measured EMF again will not change its value. Then the described process will be repeated and the engine jumps to the new position corresponding to new position of compensation. The movement of the slider is mechanically transmitted to the pointer P, indicating on a scale S the value of the measured EMF, or on the carriage with the recording pen mechanism (or both simultaneously).

The condition of compensation can be written as:

$$\Delta i_{пр} \cdot = (U_x - U_0)/(R_{аб} + R_x \cdot + R_{пр}) = 0, \qquad (8.17)$$

i.e. $U_x - U_0 = 0$, from where $U_x = U_0 = i \cdot R_x$, where i = const. The voltage U_x (EMF of thermocouple) at the moment of the compensation are always proportional to R_0, and then the movement of the engine.

8.4 Measurement of current and voltage

As noted in chapter 7.4.1 to measure current in any element of the circuit (fig. 8.3, *a*) sequentially with him it is included meter DC - ammeter (fig. 8.3, *b*).

When measuring small DC currents (less than 10^{-3}A) it is used direct and indirect measurement methods. In the first case, the current is measured by analog magnetoelectric devices, to increase the sensitivity of which it is usually used amplifiers DC.

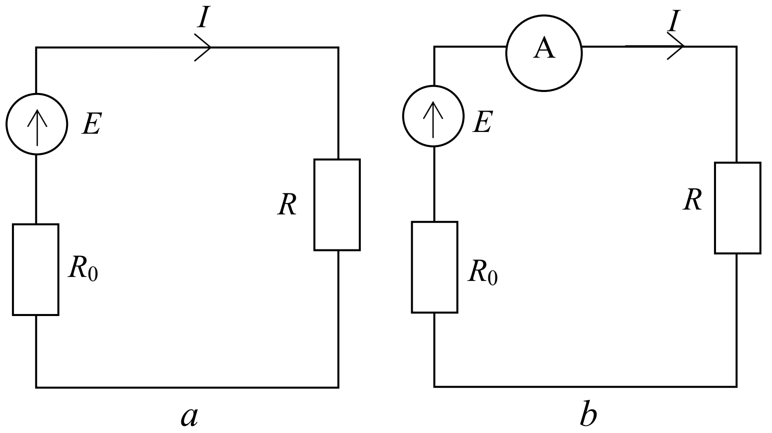


Figure 8.3: Measurement of current in circuit with ammeter

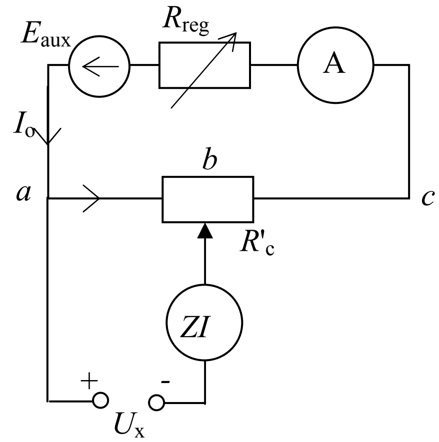


Figure 8.4: An indirect measurement of the current by compensation method

More accurate is the indirect measurement of current at which in the measured circuit it is included an exemplary resistor with resistance $R_к$ and on it it is measured the voltage drop by compensation method. On figure 8.4 it is shown a fundamental scheme of the measurement of the unknown voltage U_x by compensation method. In the upper contour under the action of EMF of auxiliary power source E_{aux} it is created operating current I_o. Its value is regulated by the resistor R_{reg} and is measured by the ammeter *A*. At the lower contour the measured unknown voltage U_x is balanced by the voltage drop on the compensating resistor R'_c by changing the position of the slider *b*. When compensation when $U_x = R' \cdot I_o$, the current in the zero indicator *ZI* becomes zero, which corresponds to an infinitely large input resistance of the measuring device.

Compensation measuring circuit operates without energy extraction from the object of measurement. Knowing R'_c and I_o, you can define U_x.

Devices that implement the compensation method of measurement (see chapter 8.3.2), are called potentiometers. In the last working current is monitored not by the ammeter, but by the compensation method using normal item, EMF $E_{нэ}$ of which is known with a high degree of accuracy (fig. 8.5). By adjusting the resistance of the resistor R_{reg} it is achieved absence of current in the zero indicator *ZI* (switch in position *1*). In this case, true equality

$$R_N \cdot I_p = E_{нэ}, \tag{8.18}$$

where $\boldsymbol{R_N}$ – the resistance of the exemplary resistor.

Because EMF of normal element and the resistance R_N are known with high accuracy, the current value $I_p = E_{NE}/R_N$ also known with high accuracy.

At position *2* of the switch measured voltage U_X is compared with a compensating voltage U_c generated by the current I_o in the presence of compensating resistor R'_c. In the absence of current at ZI voltage U_X it was balanced by voltage U_c, i.e.

$$U_x = U_K = R_k^{'} \cdot I_P = \frac{R_k^{'}}{R_N} E_{NE} \ . \tag{8.19}$$

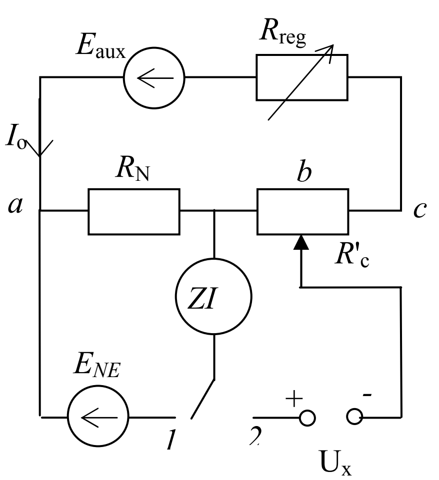


Figure 8.5: Scheme of the potentiometer

From (8.19) we see that the accuracy of measurement U_x, at this method of measurement is determined by the accuracy values U_c, precision of its comparing with U_x, i.e., the sensitivity of ZI, the constancy of the working current I_o – stability E_{aux}. In turn, the accuracy $U_к$ depends on the precision of making of the resistors $R_к$. The latest in modern potentiometers are in the form of a highly multidecade shops of resistance. Produced by industry potentiometers have accuracy classes: 0.0005; 0.001; 0.002; 0.005; 0.01; 0.02; 0.05; 0.1; 0.2; 0.5. The maximum voltage measured by the potentiometer directly, is 2.12111 V.

Permanent currents of the order 10^{-3}–10^{2} A also measured by direct and indirect methods. When direct measurements using a milliammeter and ammeters magnetoelectric, electromagnetic and electrodynamic systems and also electronic analog and digital devices. At the indirect measurement the current is determined by measuring voltage drop across the exemplary resistor with a potentiometer DC and digital voltmeters.

Measuring of high DC currents (more than 100 A) is usually carried out by the ammeters of magnetoelectric system using shunts (fig. 7.2).

When measuring of alternating currents it is necessary to remember, what the current value is measured by a specific instrument: active, amplitude or average. This is because all devices are typically graduate in operating values of sinusoidal current, and the movable parts of the measuring mechanisms of different systems react to values different from the existing ones.

The alternating currents up to 100 microamperes are usually measured with a digital microammeters, currents over 100 microamperes – rectifier microammeters. For the measurement of alternating currents in the range 10 microamperes – 100 A it is used electromagnetic, electrodynamic and rectifier devices operating in the frequency range up to tens of kilohertz, and thermoelectric devices in the frequency range up to hundreds of megahertz. Measurement of large alternating currents is carried out by the same devices, but using the measuring current transformers (fig. 7.14).

The alternating currents are measured and indirectly. In this case, the exemplary resistor is included consistently in the measuring circuit, and the voltage drop on it is measured by the voltmeter. The accuracy of the measurement increases in it, but in relation to the measurement accuracy on DC it lower.

When direct and indirect current measurements inclusion in the measured circuit of the meter with an internal resistance R_A changes the operation mode of the analyzed circuit. Figure 8.3 shows a scheme of circuit before and after the inclusion of the ammeter. The current I after inclusion of the meter is equal to I_A. The relative measurement of the current in this case is characterized by the ***error of measurement method or methodological error in current measurement***:

$$\delta_I = \frac{-1}{1 + R_{IN.A} / R_A} \ , \tag{8.20}$$

i.e. ***the inclusion of the ammeter reduces the measured current on value, depending on the relationship*** $R_{IN.A} / R_A$.

As noted in chapter 7.4.2 to measure the EMF and the voltage U on any suncircuit of the electric circuit (fig. 8.6, *a*) a voltage meter is included by parallel to this plot (fig. 8.6, *b*).

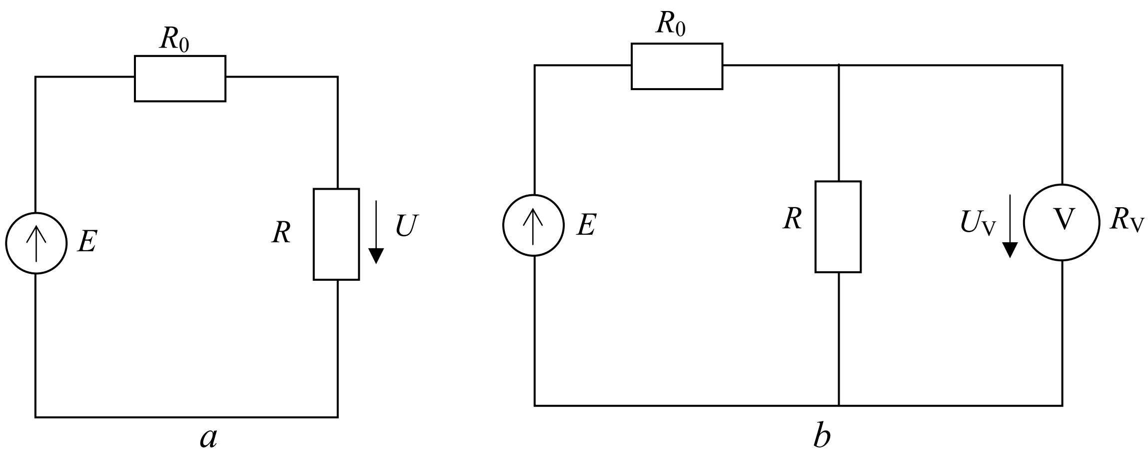


Figure 8.6: Measuring voltage with a voltmeter

When measuring of small direct voltages (of the order of 10^{-7}–10^{-8} V) it is used magnetoelectric galvanometers. More accurate results when measuring voltages in the range of 1–1000 microvolts is got through a potentiometer of DC and digital microvoltmeter.

The value of the constant voltages from tens of millivolts to hundreds of volts is measured by devices of magnetoelectric, electrodynamic, electromagnetic systems, electronic analog and digital voltmeters, potentiometers of DC using additional resistors and voltage dividers. For the measurement of direct voltages up to several kilovolts it is applied mainly electrostatic voltmeters, as well as devices from other systems with voltage dividers.

Small alternating voltages (up to units of volts) is measured by instruments of rectifier systems, analog electronic voltmeters. Higher accuracy is obtained when the measurement of voltage by potentiometers of AC, by digital voltmeters.

To measure AC voltages from units to hundreds of volts in range of frequencies up to tens of kilohertz it is used devices electromagnetic, electrodynamic and rectifier systems, potentiometers of AC. In the range of the frequencies up to tens of megahertz measurement of voltages is made by electrostatic devices and thermoelectric systems, digital voltmeters.

For measuring large alternating voltages it is used the same instruments, but with the use of measuring of voltage transformers (see fig. 7.14). Measuring transformers except transformation of the alternating voltage are isolated secondary circuit from the primary, under high voltage.

When you connect a voltage meter to subcircuit of the electric circuit with resistance R (fig. 8.6) it is changed the mode of its operation. The voltage on this section becomes equal to U_V. The relative change in the voltage is characterized ***by the error of the measurement method or methodological error of measurement of voltage***:

$$\delta_V = \frac{-1}{1 + R_V / R_{IN.V}} \, , \tag{8.21}$$

i.e. inclusion of the voltmeter reduces the measured voltage to a value dependent from relationships $R_V/R_{IN.V}$. Note that the ***measurement of the voltage by the potentiometer does not have a methodological error of voltage measurement***.

Currently, wide spread occurrence is got automatic potentiometers using the compensation method of measuring and widely used for measuring non-electrical quantities, which were previously converted into voltage.

8.5 Measurement of power and electric energy

Power measurement is performed using direct and indirect methods. When the direct method is used wattmeters, with indirect – ammeters and voltmeters.

8.5.1 Measurement of power in circuits of DC. In the DC circuits power is measured by the method of the ammeter – voltmeter. Measuring by ammeter current I and by voltmeter voltage U (see fig. 8.7), it is computed the power of the receiver

$$P = U \cdot I. \tag{8.22}$$

To reduce errors due to the influence of the internal resistance of the instruments scheme (fig. 8.7, *a*) should be used for small values of resistance *R*, and the scheme (fig. 8.7, *b*) for large *R*.

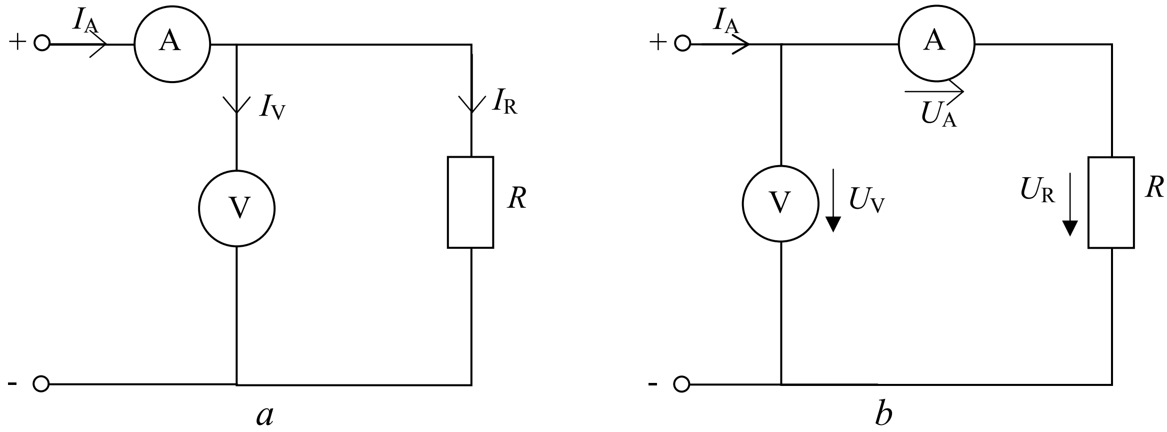


Figure 8.7: Power measurement by method ammeter-voltmeter

Power measurement by the wattmeter in DC circuit is produced rarely.

8.5.2 Measurement of power in single-phase AC circuits. The total power of the receiver is measured, usually by the method of the ammeter - voltmeter:

$$S = U \cdot I, \tag{8.23}$$

where ***U*** and ***I*** – operating voltage and current.

Active $P = U \cdot I \cos\varphi$ and reactive $Q = U \cdot I \sin\varphi$ power of receivers is measured using a wattmeters and varmeters. As wattmeters it is used electrodynamic and ferrodynamic devices, as varmeters - electrodynamic devices.

The scheme of inclusion of the wattmeter to measure active power in circuits of single-phase AC is shown in figure 7.7.

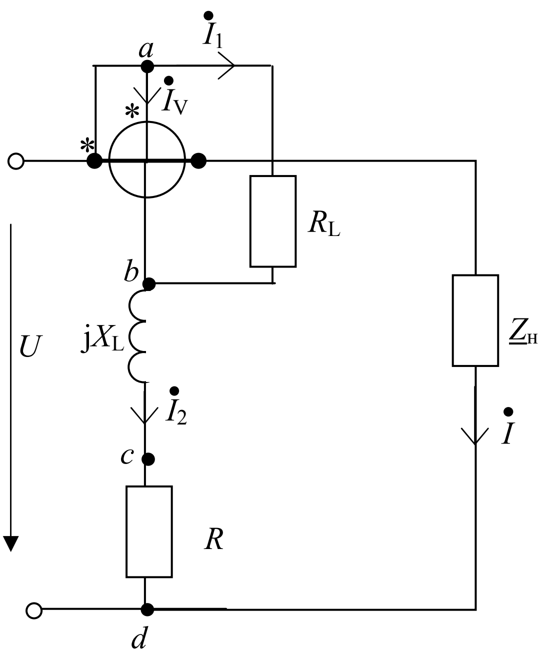


Figure 8.8: Scheme of inclusion of varmeter

Measurement of reactive power in single-phase circuits is performed using the reactive wattmeters, called varmeters. In these devices by circuit way it is created an artificial phase shift on 90° between the voltage *U* at the receiver and the current in the winding of voltage of device I_V. From the diagram figure 8.8 it is shown that in series with the parallel winding R_{W_V} of device and the additional resistor R_1 inductive coil with resistance X_L was included, and in parallel to this winding (connection terminals *a* and *b*) a resistor was connected with resistance R_L. Then the current $\dot I_V = \dot I_2 - \dot I_1$ at a suitable choice of parameters $X_L << (R_{W_V} + R) = R_V$ is shifted in phase relative to the voltage exactly on 90°. As a result the torque is obtained is proportional sinφ (where φ is the phase shift angle between voltage and current of the receiver), i.e., the torque is proportional to reactive power $Q = U{\cdot}I \sin\varphi$.

8.5.3 Measurement of power in three phase circuits. Power of three-phase system is equal to the amount of power consumed by the loads of each phase:

$$P = P_A + P_B + P_C. \tag{8.24}$$

In the case of uniform load total active power is equal to triple power of any phase:

$$P = 3P_{ph} = 3I_{PH}{\cdot}U_{PH}{\cdot}\cos\varphi\ , \tag{8.25}$$

where $\boldsymbol{I_{ph}}$ and $\boldsymbol{U_{ph}}$ – phase current and voltage.

If the phase values of current and voltage to express through line, you will get:

$$P = \sqrt{3}U \cdot I \cdot \cos\varphi\ , \tag{8.26}$$

where $\boldsymbol{I}$ and $\boldsymbol{U}$ – linear current and voltage.

Measurement of active power. To measure active power of three-phase systems they are used various ways:

Method of one wattmeter is used to measure the power with balanced load in a four-wire or three-wire line, if to connect the neutral (zero) point load is available (fig. 8.10). The total power equal to three times the reading of the wattmeter:

$$P = 3I_{ph}{\cdot}U_{ph}{\cdot}\cos\varphi\ . \tag{8.27}$$

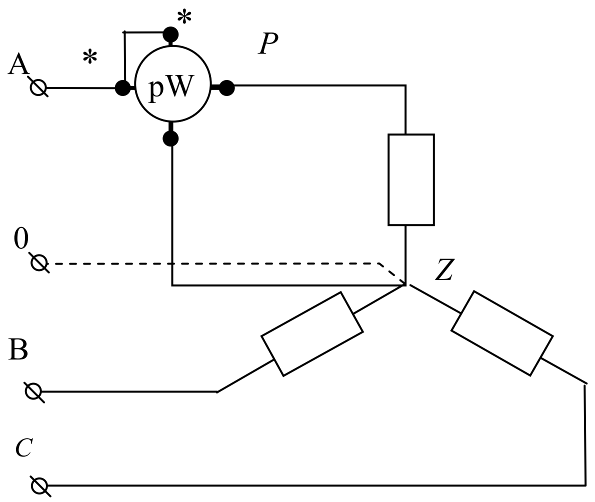


Figure 8.9: Way of one wattmeter

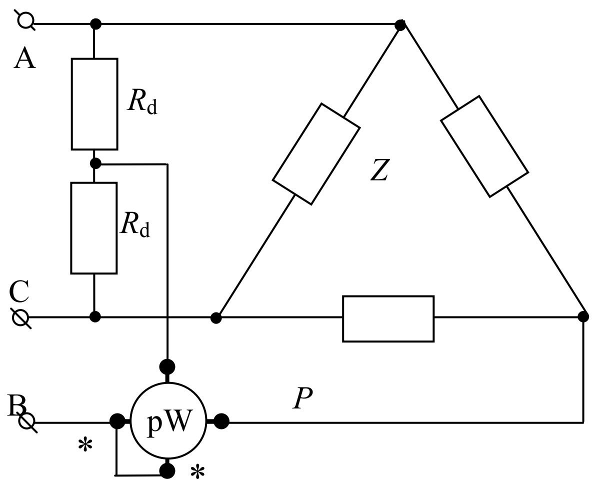


Figure 8.10: Way of one wattmeter with an artificial zero point

Method of one wattmeter with creation of the artificial zero point is used to measure the power with symmetrical load in cases where zero (neutral) point of the receiver is unavailable or in general is absent (for example, in connection by a triangle). Meanwhile in one of the phases it is included the current winding of the wattmeter and a zero (neutral point) is obtained by inclusion of two identical resistances R_d between two other phases (fig. 8.10). In this case, the total power equal to triple reading of the wattmeter.

Method of three wattmeters is used to measure the power with an uneven load in the four-wire line (fig. 8.11). The total power is the sum of the readings of all three wattmeters.

Method of two wattmeters can be applied in three lines in all cases when measuring three-phase power receivers (fig. 8.12). According to this scheme the windings of current of wattmeters are included in any two phases and windings of voltage between the third (unoccupied) phase and those phase, in which it is included the winding of current of this wattmeter. Meanwhile the total power is the sum of the readings of the two wattmeters.

You need to keep in mind that when the phase shift of 60° (the work of many electric cars is in idle mode) arrow of the first wattmeter will deflect in the opposite direction from zero. For counting of negative values of the power by the first wattmeter connection terminals of one of its windings (current or winding of voltage) are switched, and total power in this case is equal to the difference of the readings of the wattmeters

$$P = P_2 - P_1 . \qquad (8.28)$$

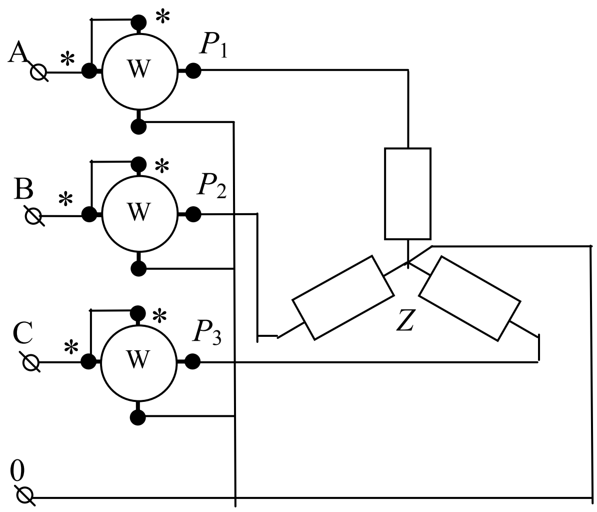


Figure 8.11: Way of three wattmeters

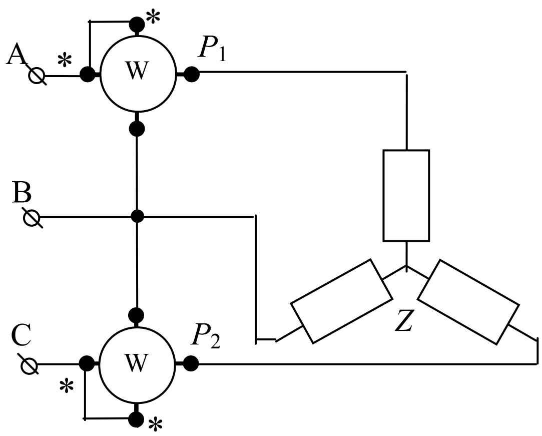


Figure 8.12: Way of two wattmeters

Using the method of two wattmeters with uniform load, you can use the readings to calculate *tg*φ, then φ *cos*φ

$$tg\varphi = \sqrt{3}\frac{P_1 - P_2}{P_1 + P_2} . \qquad (8.29)$$

Method of two wattmeters are widely used in practice. However, it is much easier to measure the power of three-phase receivers by two-element three-phase wattmeter, which combines two single-phase wattmeters, acting torques on an axis common to both measuring systems.

Measurement of reactive power. For measuring reactive power of three-phase system with a uniform load, you can use one wattmeter of active power, meanwhile the current winding includes on one of phase, but winding of voltage - between the other two phases (fig. 8.13, *a*).

Vector diagram (fig. 8.13, *b*) explains the principle of measurement of reactive power for this case. The power measured by the wattmeter will be determined by the following ratio

$$P = I_A \cdot U_{BC} \cdot \cos(90^0 - \varphi) = I_L \cdot U_L \cdot \sin\varphi \ . \quad (8.30)$$

Multiplying the readings of the wattmeter on $\sqrt{3}$ it is obtained the total reactive power of three-phase circuit

$$Q = \sqrt{3}U \cdot I \cdot \sin\varphi \ . \quad (8.31)$$

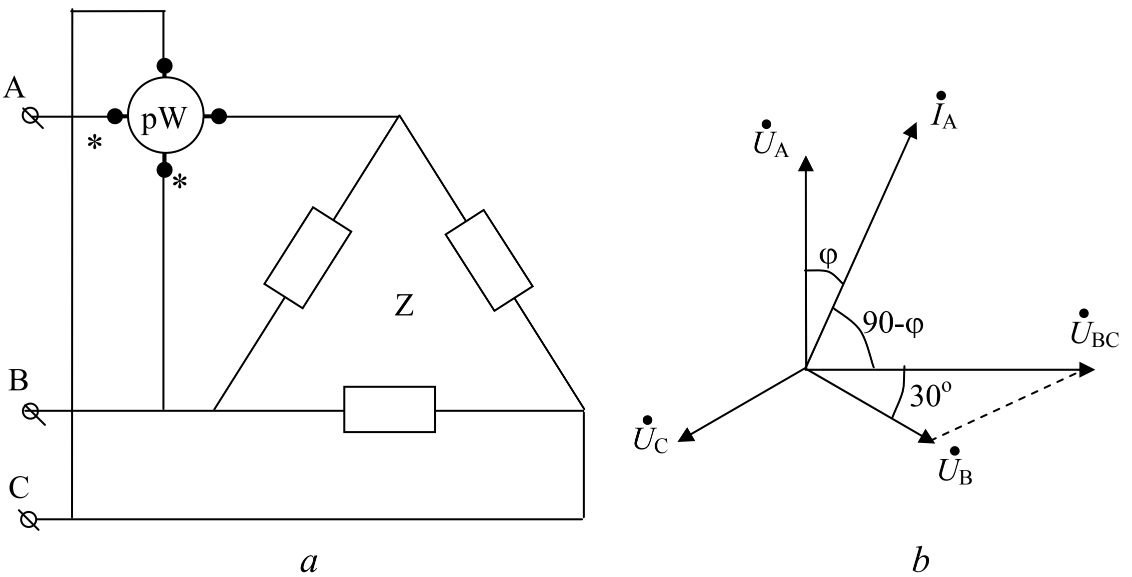


Figure 8.13: Measurement of reactive power by one wattmeter

In practice, three-phase reactive power meter are used based on scheme of two wattmeters (fig. 8.14).

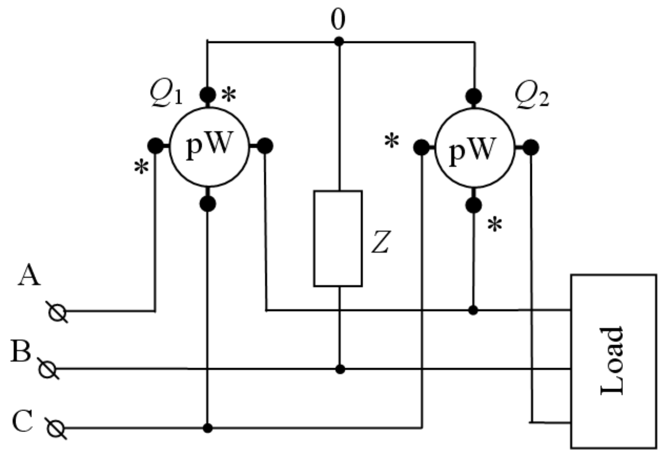


Figure 8.14: Measurement of reactive power of three-phase electricity consumer

For inclusion of windings of voltage on phase voltage here there are artificial zero point 0, formed by the resistances of the two windings of voltage and additional resistor Z. the Total reactive power of three-phase load is defined as the sum of the readings of two wattmeters multiplied by $\sqrt{3}$:

$$Q = \sqrt{3}(Q_1 + Q_2) \ . \quad (8.32)$$

8.5.4 The measurement of electrical energy. Active electric energy in AC circuits is measured by the induction counters that are included in schemes similar to the inclusion scheme of wattmeters. Figure 8.15 shows the scheme of inclusion of a single phase induction counter for active energy.

Induction counters are available with single phase and three phase. The extension of the limits of measurement is achieved by including the counter into circuit via measuring transformers.

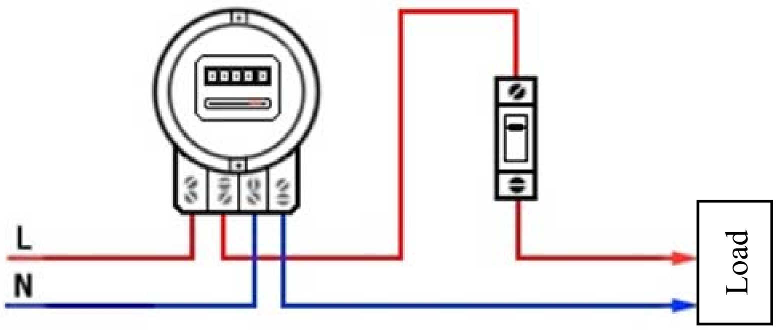


Figure 8.15: Scheme of inclusion of single-phase induction counter

For accounting of active energy in three-phase circuits are widely used two-element counters, which represent themselves a collection of two single-phase induction counters, torques of which interact on a common axis and counting mechanism. It is based on a method that gives the possibility of using two counters (wattmeters) to determine the total energy (power) consumed in a three phase system.

In practice it is used also and three-phase counters of reactive energy. Accounting of the reactive power is dictated by the need to determine the average value of cosφ, forms the basis of encouraging and penalty rate, which is set for consumers of electrical energy. The penalty tariff for exceeding the limit of consumption of reactive energy promotes to the reduction of large industrial consumers of reactive power of their installations, and, consequently, reduction of power losses in high-voltage distribution circuits.

On the accuracy the counters are divided into classes 1.0; 2.0; 2.5 (counters for active energy) and 2.0; 3.0 (reactive energy counters).

8.6 Measurement of non-electrical quantities

In engineering practice in the control of various industrial processes it often has to deal with the measurement of non-electrical quantities: mechanical (force, pressure, speed, etc.), thermal (temperature, heat capacity, etc.), light (illuminance, luminous flux, and others).

For control of non-electrical quantities and management them currently it is widely used electrical methods and electrical appliances. They allow you to retrieve data with a high degree of accuracy and in a wide range of change of values of quantities, to determine the characteristics of objects at large distances and in hard to reach places, study of fast processes, to remember the results of measurement using computers or information systems, etc.

In order for a that or other non-electrical quantity to be measured, it must be converted into an electric signal. This conversion is performed by means of sensors or sensing devices. Figure 8.16 shows a structural diagram of a device for measuring non-electrical quantities by the electrical method. Figure 8.16 shows a structural scheme of a device for measuring non-electrical quantities by the electrical method. The scheme shows: SD – sensing device, EC – electrical measuring circuit, OD – output device. Measured non-electrical quantity x ifs acts to the input of SD, the output of which is an electric signal $y=f(x)$. Next, this signal is converted to EC in another electrical signal $U(x)$, which is perceived by OD, as a result of which at the output of the device is obtained, for example, the deviation of index $\alpha(x)$. Scale of output device calibrated directly in values of non-electrical quantities x.

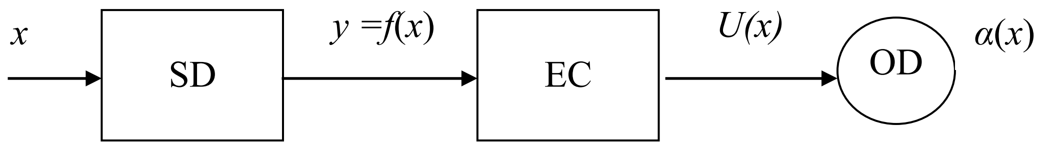


Figure 8.16: Structural scheme of a device for measuring of non-electrical quantities

Sensing device (SD) used in the measurements, are extremely diverse in structure and principle of action. They are divided into generative and parametric.

Generative SD produce EMF or current and for their work, as a rule, it is not required additional power source. To the generative SD it is concerned thermoelectric converters (thermocouples), inductive, piezoelectric, galvanic converters and several others.

Parametric SD convert the change of measured non-electrical quantities in the changes that or other parameter of the electric circuit (R, L, M, C) and they require an additional power source for their work. To parametric SD is concerned: thermistors, resistance strain gauges, rheostat, inductive and capacitive transducers, etc. The same non-electrical quantity often can be measured using various SD.

Electrical measuring circuit (EC) in these devices typically consist of bridges or measuring potentiometers. In the simplest case EC may be absent, and the signal $Y = \mathrm{E}(\theta)$ acts directly to the output device.

Output devices (OD) used in the measurement of non-electrical quantities, are very different - from the arrow magnetoelectric millivoltmeter to a recording device with record on the diagram ribbon or computer. When a large number of simultaneously controlled values of the signals from all OD arrive at the control room or in the information-measuring system.

Key findings

1. All measurements of electrical and non-electrical quantities are performed with some error determined by the method of measurement, sensitivity and accuracy class of the measuring device.

2. For assessment of the accuracy of the measurements it is used the concepts of absolute, relative and reduced error.

3. To estimate the errors of the EMI it is set accuracy classes. Indicating EMI have 8 classes of accuracy. Figure characterizing the accuracy class, defines expressed in percent the maximum reduced error of the instrument.

4. To increase the sensitivity it is used in the measuring scheme (bridge, compensation and other).

5. To raise the limits of the measured values of current and voltage it is used to the shunt, the additional resistances, the measuring transformers.

6. For measuring of non-electrical quantities it is used various sensors (primary converters), which convert the measurement of the controlled physical quantity in a change in the electrical signal (voltage or current).

Control questions

1. What is the difference between accuracy and sensitivity of the device?

2. How to expand the limits of measurement of the measuring instrument of current and voltage in circuits of DC and AC currents?

3. How to switch on electrodynamic wattmeter in the circuit when measuring active power in single-phase (three-phase circuit)?

4. How by a wattmeter to measure reactive power three-phase circuit?

5. By what is it stipulated the high measurement accuracy by compensation method?

6. How to choose the ammeter (voltmeter) to reduce the methodical error of measurement of the current (voltage)?

7. How using two wattmeters to measure in three-phase three-wire circuit active and reactive power receiver?

8. With what sensing devices are automatically bridges and potentiometers used?

9. What are the advantages of electrical methods for measuring non-electrical quantities?

SECTION 4.

TRANSFORMERS AND ELECTRICAL MACHINES

Transformers and electrical machines are devices which convert energy: in transformers – electrical energy of one class of voltage or current in another class of voltage or current; electric machines – mechanical energy into electrical energy or electrical into mechanical.

The electric machine is an electromechanical device that performs mutual conversion of mechanical and electrical energy. Devices that converts mechanical energy into electrical energy, are called ***generators***, and devices that convert electrical energy into mechanical energy – ***electric motors***. Under the ***transformer*** is understood a device for converting electrical energy of alternating current.

The ability to convert electrical energy into mechanical energy was first installed by Michae Faraday, created in 1821 the first model of the electric motor, in which the electric current flowing in a copper wire, called its movement around the vertical permanent magnet. However, further works on creation of the electric motor for more than ten years did not bring satisfactory results. Only in 1834 by russian academician B. S. Jacobi was created the design, which served as the prototype of the modern electric motor.

The ability to create an electric generator arose only after the discovery of M. Faraday in 1831 of law of electromagnetic induction. Using this discovery, the brothers Pixie in 1832 created the design of the first electric generator with rotating permanent magnets and a switch for rectifying current. The first time the development of electric motors and generators walked independently from each other. In 1833 by E. Lenz it was formulated the principle of reversibility of electrical machines, and in 1838 this principle was practically implemented. A further step in the development of generators was the replacement of permanent magnets by electromagnets.

The initial period of development of electric cars is associated mainly with direct current. The reason is that the consumers of electric power was installations, working exclusively on direct current (arc lamp, the installation of the galvanoplasty and so on). The use of electric lighting in large cities demanded the rise of power of electric generators and their further improvement.

In 1867 century, Werner von Siemens has applied the principle of self-excitation of generators of series excitation. In the same year James Clerk Maxwell for the first time gave a mathematical theory of electrical machines with self-excitation, thus laid the foundations of the theory of electrical machines.

In 1870, Zénobe Théophile Gramme built the machine with the annular anchor, and 1873 Friedrich von Hefner-Alteneck and W. Siemens designed a car with a "drum" anchor.

The development of electric railways has significantly increased the demand for electric motors and generators that promoted to their further improvement.

In 80-ies of the XIX century there was a need to transmit power at a distance. In 1882 experiments were taken on the transmission of electricity at a constant current at heightened voltages. However, the high voltage in DC generators made worse the work of the collector, which often led to accidents. All this reinforced the interest of electrical engineers of that time to an alternating current. A great merit in the development of technology AC belongs to the Russian scientist P. N. Yablochkov, who widely used AC power invented by hem electric candles. In 1876 P. N. Yablochkov used to power these candles transformers with open core, thereby initiating the practical use of transformers. Transformers with a closed core like a modern transformers, appeared later, in 1884.

The beginning of the practical application of alternating current for the purposes of the electric drive should be considered 1889, when Russian engineer M. O. Dolivo-Dobrovolsky proposed for the practical application three-phase alternating current system and built a three-phase asynchronous motor and three-phase transformer.

The first line of electricity transmission of three-phase alternating current with a length of 175 kilometres at a voltage of 15 thousand volts with the use of three-phase transformers was built Dolivo-Dobrovolsky in 1891. The results of the tests of this line confirmed the applicability of three-phase current to transfer significant quantities of electricity at a relatively high COP.

By the beginning of XX century all the main types of electrical machines were created and developed the foundations of their theory. Since that time, by rapid rates electrification of industry and transport happens.

Currently, transformers and electrical machines are used in almost all sectors of the economy, particularly on the enterprises of the construction industry and building grounds.

In the textbook it was selected adopted by many authors the sequence of presentation of the course, which is determined by the logic of the study of physical processes in electrical equipment: transformers (single-phase, three-phase), electrical machines (DC machines, anisochronous machines, synchronous machines).

To understand the principle of operation of electrical machines it is necessary the knowledge of the law of electromagnetic induction (Faraday's law) and the law of electromagnetic interaction (Biot-Savart-Laplace).

The law of electromagnetic induction is formulated as follows: the magnitude of the EMF e, induced in a closed conductor is proportional to the rate of change of the magnetic flux Φ, penetrating this contour

$$e = \frac{-d\Phi}{dt}.$$

The minus sign reflects the Lenz's law, on which induced current always seeks to prevent change of the magnetic flux of contour (frame).

In accordance with the law of electromagnetic induction an EMF is induced in the contour in the following cases:

during the rotation of contour in a magnetic field (DC machine);

when a fixed contour and a rotating magnetic field (synchronous machines);

when rotate and magnetic field, and a contour (asynchronous machine);

when stationary in space of the magnetic field and the contour, but variable on the value in time of the magnetic field (transformers).

The principle of operation of generator is that when moving with velocity V m/s of conductor, having a length of l m, perpendicular to the magnetic lines in a magnetic field with induction B in it by the law of electromagnetic induction EMF occurs

$$e = B \cdot l \cdot V, \text{ V}.$$

If you close the ends of the conductor through the resistance, or in a short circuit, under the influence of EMF on it current I, A will flow. The direction of the EMF and current are the same. The current I, interacting with the magnetic flux, in which it is located, creates ejected the conductors from the magnetic field strength, which is in accordance with the law of electromagnetic interaction (the law of Biot-Sawart-Laplace) is equal to

$$F = B \cdot I \cdot l, \text{ N}.$$

The force of interaction becomes apparent as a reaction to an external force applied to the conductor. It is equal and opposite to the latter.

The principle of operation of the motor is that when current passes through the conductor located in a magnetic field, on the conductor the force F affects, under the action of which it will move.

9 TRANSFORMERS

Key concepts: the transformer, primary and secondary winding, a magnetic core, idle, short-circuit conditions, step-up and step-down transformer, transformation coefficient, external characteristics, autotransformer, the measuring transformer.

9.1 General information about transformers

The transformer is an electronic device for converting alternating current of one voltage into alternating current of another voltage of the same frequency.

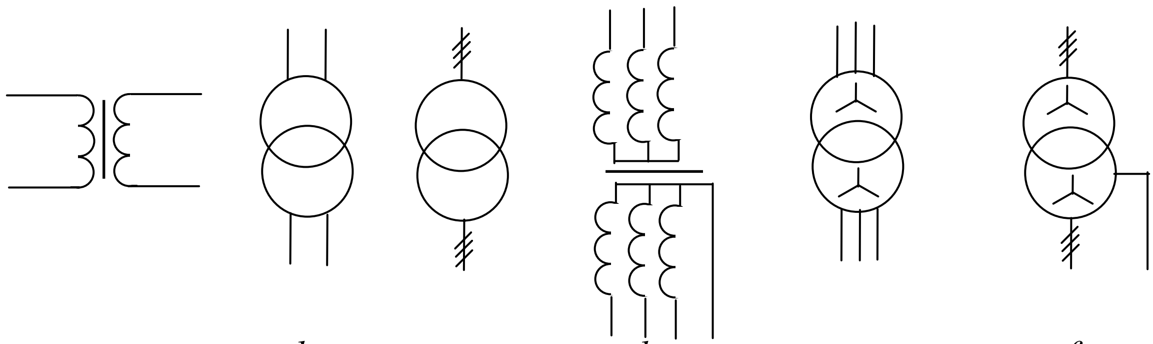
The transformer consists of a steel core and windings. The core is assembled from thin sheets of electrotechnical steel, isolated from each other to reduce power loss by hysteresis and eddy currents.

Conversion of voltage in transformers is put into effect by variable magnetic flux inductively related between each other windings. Winding connected to a source of electrical energy, is called the ***primary*** winding, but winding, to which connected a load – the ***secondary***. All parameters of the transformer, belonging to the primary winding (number of turns, voltage, current and so on) are called the primary and in their letter designations it is used the index 1. Accordingly, the parameters of the secondary winding are called the secondary and recorded with index 2.

If through the transformer it is necessary to carry out power of two or more loads with different voltage, then several secondary windings are satisfied.

On the purpose transformers are divided into power and special purpose (welding, measuring, matching and others).

Power transformers are single-phase (for circuits of single-phase current) and three-phase (for three-phase circuits), step-up or step-down. Figure 9.1 shows the graphic symbols of a single-phase (*a, b, c*) and three-phase (*d, e, f*) transformers.



a *b* *c* *d* *e* *f*

Figure 9.1: Conditional graphic symbols of transformers

By way of cooling transformers are divided into dry and oil. The oil transformers windings are immersed in a steel tank filled with oil.

On the desk of the transformer it is indicated its nominal settings: primary and secondary voltage; rated apparent power; current at rated apparent power; frequency; the number of phases; scheme of winding connection; mode (long or short); cooling method.

9.2 Single-phase transformer

9.2.1 The principle of action of single phase transformer. At the core of the single-phase transformer (fig. 9.2) in the simplest case there are two windings, made of insulated wire. To the primary winding is fed to the supply voltage U_1. With the secondary winding is gone off voltage U_2, which is supplied to the consumer of electrical energy Z_L.

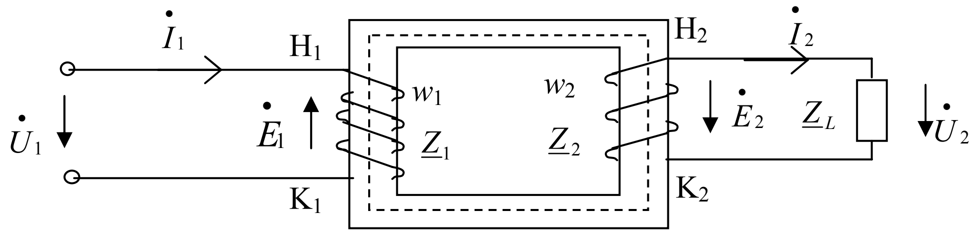


Figure 9.2: The principle of action of single phase transformer

The alternating current $\dot{I}_1$ passing through coils of transformer primary winding w_1, creates in the core of a magnetic core an alternating magnetic flux Φ. Changing with time in a sinusoidal law $\Phi = \Phi_m \cdot \sin\omega t$, this thread penetrates the coils of the secondary winding of the transformer. However, in accordance with the law of electromagnetic induction it is induced in it EMF e_2

$$e_2 = w_2 \frac{d\Phi}{dt} = E_{m2} \sin(\omega t + \frac{\pi}{2}), \tag{9.1}$$

where: $\boldsymbol{w_2}$ – the number of turns of the secondary winding of the transformer;
$\boldsymbol{E_{m2}}$ – peak value of EMF in the secondary winding.

Under the action of EMF e_2 in the secondary circuit of the transformer, closed to the load current $\dot{I}_2$ flows.

The ratio of the EMF of the primary winding of the transformer to the EMF of the secondary winding is equal to the ratio of the number of turns of the respective windings is called the ***transformation coefficient*** of the transformer

$$n = \frac{E_1}{E_2} = \frac{w_1}{w_2}. \tag{9.2}$$

If $E_1 < E_2$, then the transformer is ***step-up***; when $E_1 > E_2$, it is ***step-down***.

9.2.2 The modes of work of the transformer. Depending on the load resistance value there are ***three modes of work of the transformer***: $Z_L = \infty$ - idle mode; $0 < Z_L < \infty$ - load mode; $Z_L = 0$ short-circuit conditions.

In idle mode the secondary winding of the transformer is open. The current of the primary winding of the transformer when the disconnected consumer of electric power is ***the no-load current*** I_0

$$I_0 = I_{m0} \sin(\omega t + \alpha).$$

Included in the equation the magnetic loss angle α (the angle of phase shift between the current and the magnetic flux of the transformer) is caused by the power loss in the magnetic core of the transformer. The value of the angle α for modern electrical steels are usually small and is of the order of 4...6°.

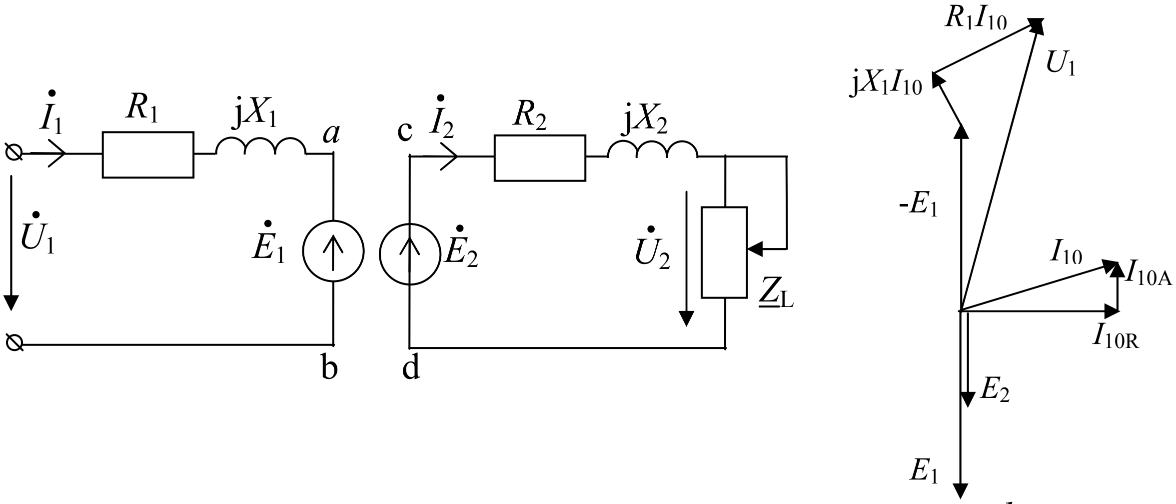
The voltage applied in the idle mode to the transformer, in accordance with the second Kirchhoff's law can be represented as the sum of the voltage drop in the primary circuit

$$\dot{U}_1 = \dot{E}_1 + R_1 \dot{I}_0 + jX_1 \dot{I}_0 \tag{9.3}$$

where: $\boldsymbol{R_1}$ – active resistance of the primary winding;
$\boldsymbol{X_1}$ – inductive resistance of the primary winding;
$\boldsymbol{E_1}$ – EMF induced in the primary winding of the magnetic flux

$$e_1 = w_1 \frac{d\Phi}{dt} = E_{m1} \sin(\omega t + \frac{\pi}{2}).$$

Based on the equations of electrical equilibrium (9.3), we can construct the vector diagram of transformer for idle mode (fig. 9.3).



a *b*

Figure 9.3: Equivalent scheme and vector diagram of idling

In consequence of the magnetization of the magnetic core in it is appeared power loss, which are called ***idle loss***. Power P_0 consumed from the circuit in idle mode, spent mainly on cover of the loss in the magnetic core, as losses in the winding $R_1 I_{10}^2$ are small. The idle current I_{10} contains active and reactive components $I_{10} = \sqrt{I_{10A}^2 + I_{10R}^2}$.

Mode of short circuit for power transformer is an emergency. However, some special transformers are calculated for operation in a mode close to a short circuit. These are welding transformers, measuring transformers of current.

At work of the transformer in the load mode ($Z_L \neq 0$) in the secondary circuit under the influence $\dot{E}_2$ appears current $\dot{I}_2$. The main magnetic flux Φ_0 is created by joint action of the magnetomotive force of the primary and secondary windings. The resultant magneto motive force F_p is equal to their geometric sum (fig. 9.3, *c*)

$$\dot{F}_p = w_1\dot{I}_1 + w_2\dot{I}_2 = w_1\dot{I}_{10}\ . \tag{9.4}$$

With a glance the active resistance of the windings the equations of electrical state primary and secondary circuits are of the form:

$$\begin{aligned} \dot{U}_1 &= -\dot{E}_1 + (R_1 + jX_1)\dot{I}_1, \\ \dot{E}_2 &= (R_2 + jX_2)\dot{I}_2 + \underline{Z}_L\dot{I}_2 \end{aligned}\ . \tag{9.5}$$

9.2.3 Equivalent scheme of the transformer. For studies of the modes of work of transformers it is reasonable magnetic coupling between the primary and secondary windings to replace by the electrical connection. Connection by jumpers ac and bd in the scheme in figure 9.3, *a* maybe, if $\dot{U}_{ab} = \dot{U}_{cd}$. This requirement satisfies the condition $\dot{E}_2^{'} = -\dot{E}_1 = \dot{E}_2\frac{w_1}{w_2}$, where $\dot{E}_2^{'}$ is called the reduced electromotive force.

The equivalence of energy ratios in the transformer and its equivalent scheme will not be broken if apparent power $S_2 = S_2^{'}$ $(E_2I_2 = E_2^{'}I_2^{'})$, active power $P_2 = P_2^{'}$ $(R_2I_2^2 = R_2^{'}I_2^{'2})$ and reactive power $Q_2 = Q_2^{'}$ $(X_2I_2^2 = X_2^{'}I_2^{'2})$ and also power in the load $S_H = S_H^{'}$ $(U_2I_2 = U_2^{'}I_2^{'})$ will remain unchanged. From the last equalities we obtain the values of parameters of the equivalent scheme, called reduced (to the number of coils w_1).

$$I_2^{'} = I_2\frac{1}{n};\quad R_2^{'} = R_2n^2;\quad X_2^{'} = X_2n^2;\quad U_2^{'} = U_2n;\quad Z_2^{'} = Z_2n^2\ . \tag{9.6}$$

Thus, the scheme of the transformer (fig. 9.3, *a*) can be represented as the equivalent scheme shown in figure 9.4, *a*. The complete system of equations of the electric and magnetic state of the transformer considering reduction of the secondary winding to the primary ones on number of turns and $\dot{U} = -\dot{E}_1 = \dot{E}_2^{'}$ looks like

$$\begin{aligned} \dot{U}_1 &= \dot{U} + \underline{Z}_1\dot{I}_1; \\ \dot{U} &= \underline{Z}_2^{'}\dot{I}_2^{'} + \dot{U}_2^{'}; \\ \dot{I}_1 &= \dot{I}_{10} + \dot{I}_2^{'}. \end{aligned} \tag{9.7}$$

These equations describe the electromagnetic processes in the double-circuit scheme, which is called the equivalent scheme of the transformer. In figure 9.4, *a* there is the T-shaped equivalent scheme of the transformer. In cases when $\underline{Z}_1\dot{I}_1$ is smallish compared with $\dot{U}_1$, suppose $U_1 \approx$ U, the equivalent scheme is simplified (fig. 9.5, *b*). This scheme is called the Γ-shaped. Here $X_к = X_1 + X_2^{'}$; $R_{sh.c} = R_1 + R_2^{'}$.

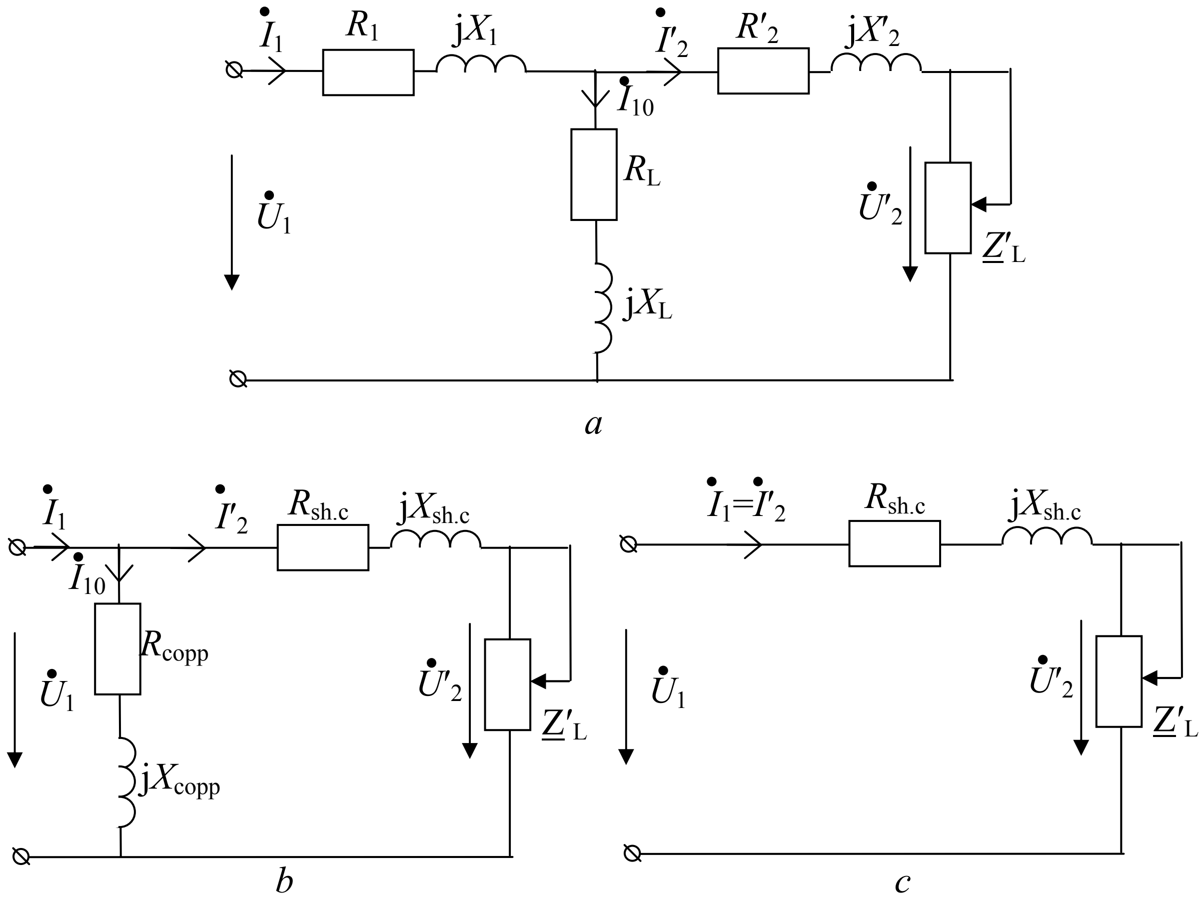


Figure 9.4: Equivalent scheme of single-phase transformer:
a – T-shaped; *b* – Γ-shaped, *c* – mode of short-circuit conditions

In the modes of operation of the transformer close to a short circuit, when $I_{10} << I_1$, from the scheme of figure 9.4, *b* it is excluded branch $R_M - X_M$ (see fig. 9.4, *c*).

The equivalent schemes of the transformer are used for the analysis and calculations of the modes of its work, therefore they are called by the calculation equivalent schemes of the transformer.

9.3 Passport parameters and external characteristics of transformer

9.3.1 Basic parameters. In the process of converting the voltage in the transformer it is arised the losses of electrical energy in the windings and the magnetic core caused by heating. The larger the load current and source voltage, the more the windings and the magnetic core of the transformer heat. Prolonged overheating of the windings can cause aging and isolation degradation, interturn short circuit and failure of the transformer, therefore, to ensure continuous operation of the power transformer under load by the factory it is set parameters, called ***passport data***: nominal power S_n, the transformation coefficient *n*, voltages U_{1n} and U_{2n}, short circuit voltage $U_{sh.c}$, currents I_{1n}, I_{2n}, and idle current I_{10n}, frequency f_n, mode of operation (continuous or short-time), the losses in steel of magnetic core ΔP_{0n} and in the windings ΔP_w and other During long-term operation, especially when a current overload, the oscillations of the primary voltage and frequency, high humidity and environment temperature isolation properties deteriorate, energy losses rise. It is therefore necessary to periodically check the basic parameters of the transformer, to which concern $U_{sh.c}$ and I_{10n}, characterizing the energy losses in the windings, isolation and steel of the magnetic core.

For this purpose there are two experiences: the experience of idle and experience of short circuit. On the basis of these experiments they are also determined the parameters of equivalent schemes.

9.3.2 The experience of idle. The scheme of inclusion of measuring instruments with the experience of idle is presented in figure 9.5, *a*. In the process of the experience it is measured: U_1, U_2, I_{10}, P_0 when change U_1 from 0 to U_{1L}. When $U_1 = U_{1L}$ it is defined: loss in the steel of the magnetic core ΔP_{0L}, the transformation coefficient n = U_1/U_2, idle current I_{10L}. Obtained data allow to calculate the parameters of the equivalent scheme:

$$R_{copp} = \frac{\Delta P_{0L}}{I_{10L}^2};\quad Z_{copp} = \frac{U_{1L}}{I_{10L}};\quad X_{coop} = \sqrt{Z_{copp}^2 - R_{copp}^2};\quad \cos\varphi_{0L} = \frac{\Delta P_{0L}}{I_{10L}U_{1L}}\ . \quad (9.8)$$

When calculation the expected loss in the winding are small, as I_{10L} is 5–10% from I_{1L}.

9.3.3 Experience of short circuit. When experience of short circuit (fig. 9.5, *b*) a secondary winding of the transformer is closed in a short circuit, and the primary winding is switched on via the voltage regulator of VR on such low voltage $U_{1sh.c}$ on which in the windings of the transformer nominal currents carry. This voltage is called ***the voltage of short circuit***. In the experiment they are measured I_1, I_2, $U_1 = U_{\text{sh.c}}$ and P_2 when change the current I_1 from 0 to $I_{1\text{L}}$. When $I_1 = I_{1\text{L}}$, it is determined loss of short circuit $\Delta P_{sh.c.n}$, voltage of short circuit $U_{\text{sh.c.n}}$.

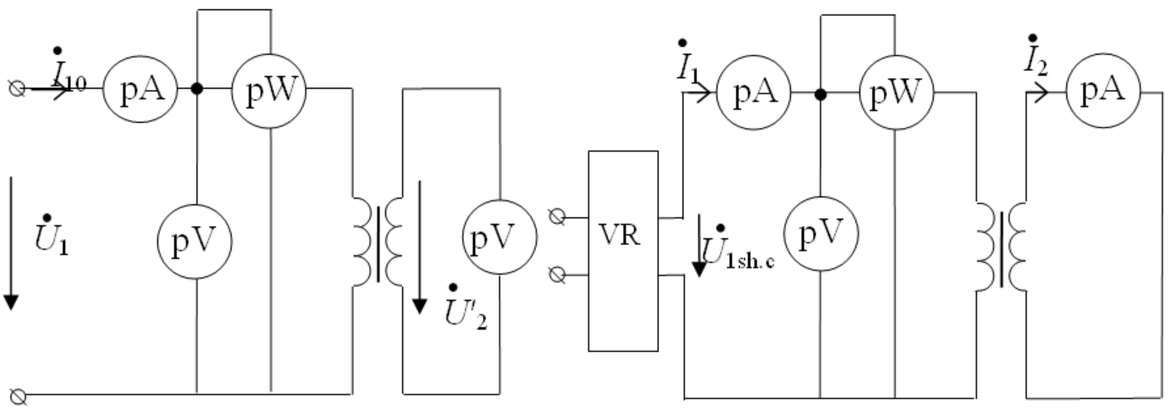


Figure 9.5: Schemes of the experiment of idle (*a*) and short circuit (*b*)

According to the experience of a short circuit it is calculated parameters of the equivalent scheme:

$$R_{sh.c} = \frac{\Delta P_{sh.c.L}}{I_{1L}^2}; \qquad Z_{sh.c} = \frac{U_{sh.c.L}}{I_{1L}};$$
$$X_{sh.c} = \sqrt{Z_{sh.c}^2 - R_{sh.c}^2}; \quad cos\,\varphi_{sh.c} = \frac{R_{sh.c}}{Z_{sh.c}}, \tag{9.9}$$

where: $\boldsymbol{R_{sh.c} = R_1 + R_2'}$ and $\boldsymbol{X_{sh.c} = X_1 + X_2'}$ active and inductive resistance of short circuit of the transformer.

When calculation it is supposed that at low voltage magnetic flux and magnetizing current are small, i.e., $I_{10} \approx 0$. So we can assume that the magnetomotive force of the primary and secondary windings of the transformer are equal to

$$w_1 I_1 = w_2 I_2 \text{ or } I_1 = I_2' \tag{9.10}$$

and, therefore, the wattmeter measures power loss only in the windings.

Short circuit voltage and its active and reactive components usually are expressed in percents:

$$U_{sh.c.\%} = \frac{U_{sh.c.L}}{U_{1L}} 100\% \ . \tag{9.11}$$

On the value $U_{\mathrm{sh.c}}$ it is possible to calculate the short-circuit current $I_{\mathrm{sh.c}}$ in emergency mode:

$$I_{1sh.c} = \frac{U_{1L}}{Z_{sh.c}} = I_{1L}\frac{U_{1L}}{U_{sh.c.L}} = I_{1L}\frac{100}{U_{sh.c\%}} \ . \tag{9.12}$$

9.3.4 External characteristics of the transformer. Working properties of the transformer are characterized by the dependence of the voltage on load U_2 and the COP η from the current I_2.

The dependence $U_2(I_2)$ under different load character (active, reactive, capacitive) is called the ***external characteristic of the transformer***.

External characteristics of the transformer $U_2(I_2)$ and the dependence $\eta(I_2)$ can be obtained experimentally or calculated on the equivalent scheme. In the last case, the equation of electric state obtained from the Γ-shaped equivalent scheme (see fig. 9.4, *b*) has the form

$$\dot{U}_2' = \dot{U}_1 - (R_{sh.c} + jX_{sh.c})\dot{I}_2' \ . \tag{9.13}$$

Dependence $U_2(I_2)$ is determined by the character of the load. Thus, when the capacitive character of the load (cosφ < 0) with increase of current I_2, the voltage U_2 increases, and when the inductive character (cosφ > 0) decreases (fig. 9.6).

The coefficient of performance of the transformer η is equal to the ratio of useful active power P_2 to the entire active power coming from the circuit:

$$\eta = \frac{P_2}{P_1} = \frac{P_2}{P_2 + \Delta P_{st} + \Delta P_{copp}}, \tag{9.14}$$

where: $\boldsymbol{\Delta P_{\mathrm{st}}}$ – the power loss in magnetic core steel;
$\boldsymbol{\Delta P_{\mathrm{copp}}}$ – the power of loss in the copper of windings.

The useful power of the transformer at any load character

$$P_2 = U_2 I_2 \cos\varphi_2 = \beta \cdot S_L \cos\varphi_2, \tag{9.15}$$

where: $\boldsymbol{S}_L$ – apparent power of transformer, VA;
$\boldsymbol{\beta} = I_2/I_{2L}$ – the load coefficient.

Losses in steel ΔP_{st} not depend on the load and are equal to the losses of idle. Losses in the windings ΔP_{copp} are proportional to the square of the current

$$\Delta P_{copp} = R_{sh.c} I_2^2 = R_{sh.c} I_L^2 \beta^2 = \Delta P_{copp.L} \beta^2 ,$$

where $R_{sh.c}$ – the active resistance of the winding.

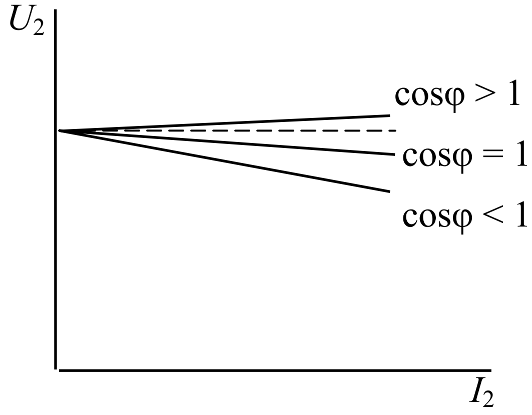


Figure 9.6: Dependence $U_2(I_2)$ under different load character

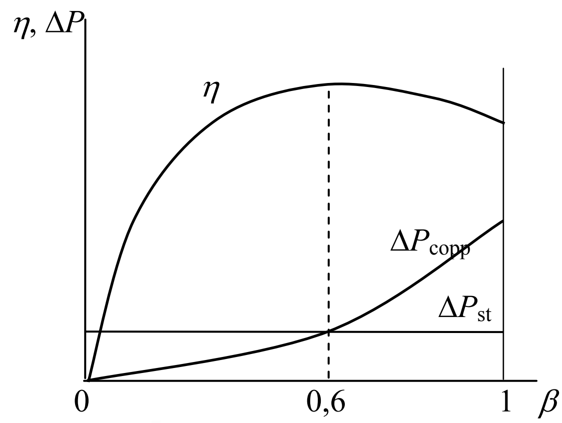


Figure 9.7: Dependeces ΔP_{st} and η from the load coefficient β

After substitution the expression for the COP will be

$$\eta = \frac{\beta \cdot S_L \cos\varphi_2}{\beta \cdot S_L \cos\varphi_2 + \Delta P_S + \Delta P_{CL} \beta^2} \; . \qquad (9.16)$$

Dependences ΔP_{st} and η from the load coefficient β are presented in figure 9.7. The dependence of η(β) has a maximum. Through selection of the parameters of the windings and magnetic core for power transformers it is chosen η_{max} when β = 0,6–0,7.

9.4 Three-phase transformers

For three-phase transformers equivalent schemes are shown for one phase and have the same form as for a single-phase transformer. The equivalent circuit parameters are determined from experiments of idle and short circuit.

Structurally three-phase transformers are performed rod (fig. 9.8). On each of the three rods is placed the primary and secondary windings of one phase. The resulting magnetomotive force of each phase shifted relative to each other by 120°, the sum of the vectors of magnetic flux equal to zero ($\dot{\Phi}_A + \dot{\Phi}_B + \dot{\Phi}_C = 0$). Phases of primary and secondary windings can be connected in star (Y) and triangle (Δ). Therefore, the vectors of the linear voltages $\dot{U}_1$ and $\dot{U}_2$ may differ in phase.

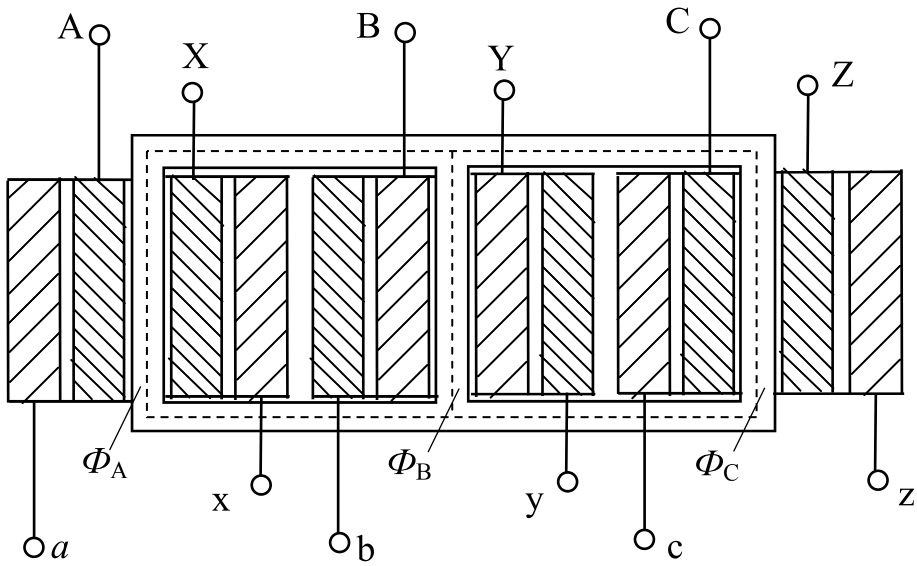


Figure 9.8: The device of a three-phase transformer

The phase shift is indicated by windings connection group. For example, in the scheme (see fig. 9.9, *a*) it is shown the connection group Y/Y–0, where 0 indicates the coincidence of the phase a secondary winding with a phase A of the primary one. Figure 9.9, *b* shows the connection scheme Y/Δ–11, here 11 indicates that the voltage vector $\dot{U}_{AB}$ of the primary winding is ahead of the phase $\dot{U}_{ab}$ of the secondary winding 30° and coincides with the position clockwise on the figure 11.

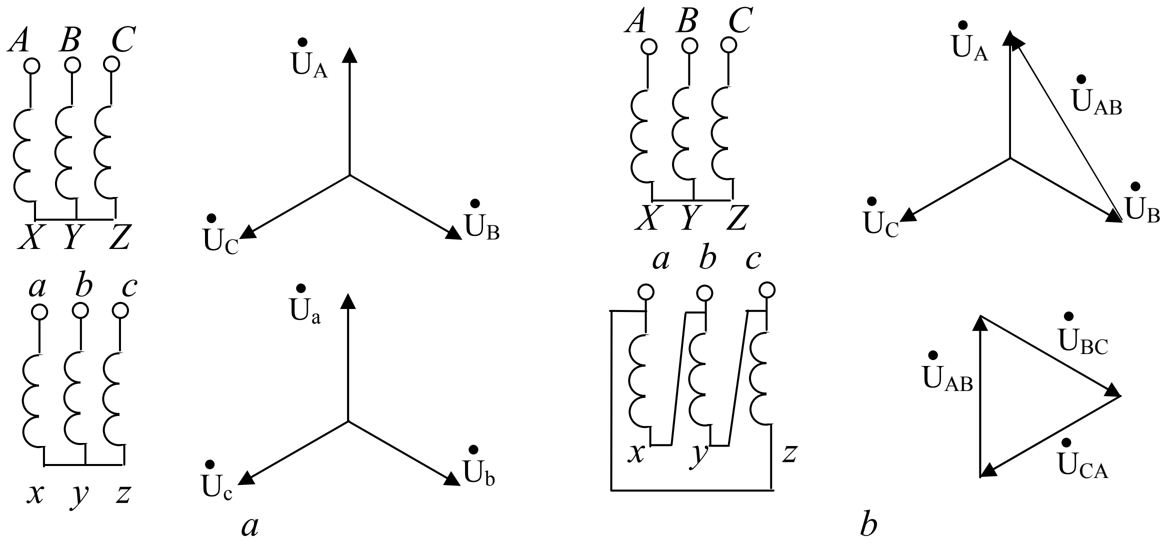


Figure 9.9: Schemes of connection of the windings of three phase transformers

9.5 Autotransformers

Conversion of the AC voltage can be performed by using a transformer. The autotransformer in structurally similar to an ordinary transformer: has a closed steel magnetic core, in which two windings are made of copper wire of various sizes. Unlike the transformer windings of autotransformer are electrically connected.

At the step-down autotransformer winding of secondary voltage is part of the primary winding voltage. At the step-up, on the contrary, the winding of primary voltage is part of the winding of secondary voltage. Thus, in the autotransformer in addition to the magnetic couple between the primary and secondary windings we have an electrical connection.

Electric schemes of step-down and step-up autotransformers are presented in figure 9.10. The voltage source applied to the connection terminals of the primary winding AX with the number of coils w_1, is balanced mainly EMF E_1 generated by alternating magnetic flux in the magnetic core. The secondary winding has a number of turns w_2. In it it is created EMF ЭДС $E_2 = E_1(w_1/w_2)$.

The transformation coefficient is equal to the ratio of the primary and secondary voltages:

$$n = U_1/U_2 = w_1/w_2 \,. \tag{9.17}$$

When connection to the connection terminals *ax* of load Z_L current in the general part of the winding *ax* I_{12} will be equal to the geometric sum of the currents of the primary and secondary circuits $\dot{I}_{12} = \dot{I}_1 + \dot{I}_2$.

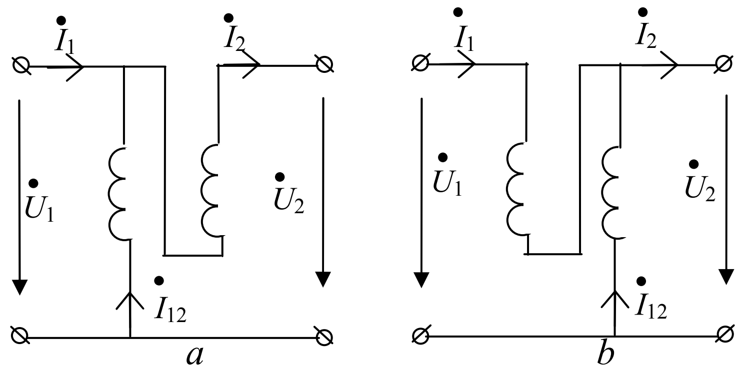


Figure 9.10: Schemes of autotransformers:
a – step-up, *b* – step-down

The power consumed by the autotransformer from the circuit, without taking into account loss will be equal to the power allocated in the load, i.e., $U_1 \cdot I_1 = U_2 \cdot I_2$, from where it follows $I_1/I_2 = U_1/U_2 = n$.

Meanwhile as in an ordinary transformer, the main magnetic flux Φ_{0m} remains at a constant voltage ется $\dot{U}_1$.

If you neglect to idle current, we can assume that the currents I_1 and I_2 are shifted in phase by 180° and their geometric sum is equal to the algebraic, i.e.

$$I_{12} = I_2 - I_1 = I_2\,[(1 - (1/n)] \,. \tag{9.18}$$

Electromagnetic processes in the autotransformer are nothing different from the processes in an ordinary transformer. The advantage of autotransformer before the transformer is more than a simple device, less consumption of copper, higher coefficient of performance, lower loss in the windings and magnet core steel. This is because in the autotransformer energy from the primary circuit to the secondary partially transmitted by the electrical connection.

However, the autotransformer compared to transformer has a very significant drawbacks; it has a low resistance of short circuit, which causes a large short-circuit current, and the electrical connection between the windings at high primary voltage is dangerous at person to touch the wires in the load circuit.

Electromagnetic processes in three-phase autotransformer are the same as in single-phase. Three phase transformers are used in electroenergetics for connection circuits of mixed voltages, such as 110 and 220 kilovolt, 220 and 500 kilovolt, etc. at the start of asynchronous three-phase motors in order to reduce starting currents.

The autotransformers of low voltage are performed on a small power (up to 7.5 kiloVA). They have, as a rule, winding with one wire section and can be used to increase and decrease voltage.

Laboratories are widely used autotransformers of low voltage of low power with continuous adjustment of the output voltage. At these autotransformers one connection terminal of load is made in the form of moving (sliding) contact.

9.6 Measuring transformers

Measuring transformers are used for measuring of voltages and currents. Measuring transformers of voltage are used to enable voltmeters, frequency meters, coil voltage wattmeters and counters. Measuring transformers of current are used for connection ammeters, relay, current windings of wattmeters and counters.

Figure 9.11 shows the scheme of inclusion for measuring instruments via measuring transformers in single-phase circuit.

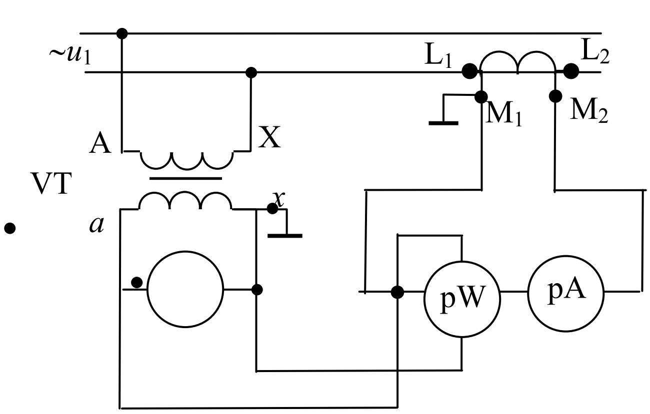


Figure 9.11: Inclusion schemes of the transformer of voltage and current

For security of touch to the devices one connection terminal of the secondary windings of transformers is earthed. The measured voltage U_1 on the reading of the voltmeter will be $U_1 = n_U \cdot U_2$, and the measured current I_1 according to the indications of the ammeter: $I_1 = n_I \cdot I_2$, where n_U and n_I – coefficients of transformation of the transformers of voltage and current, respectively.

The secondary winding have standard values U_2 and I_2: 100 V for. voltage transformers and 5 and 1 A for current transformers.

Therefore, the measurement range can be extended by adjusting the coefficients of transformation, i.e. the ratio of the number of turns.

Measuring transformer of voltage work in mode close to idle. Obviously, the accuracy of the measurement of voltage will be higher, the smaller the voltage drop on the transformer. Indeed, from the equivalent scheme (see fig. 9.4) it follows that the error

$$\delta_U = \frac{U_2 w_2 / w_1 - U_1}{U_1} 100$$

the smaller, the smaller RA and HC, which is achieved by reduction the number of turns at increase of the cut of steel of magnetic core.

In transformers of current, on the contrary, the reduction in error is achieved by reduction of the current I_{10}, for what it is used a magnetic core of annular shape from a material with low loss (small value of the coercive force H_C) and work in the unsaturated part of the magnetization curve.

As normal mode of work of the transformer of current is the short circuit mode, then for the switching in the secondary circuit are mounted devices, locking output connection terminals of the secondary winding.

Key findings

1. Alternating current of one voltage is converted into alternating current of another voltage of the same frequency by transformers.
2. There are three modes of work of transformer: idle, load and short circuit.
3. It is analyzed and calculated the modes of the transformer by means of equivalent schemes.
4. Three-phase transformers by equivalent scheme is represented as single-phase.
5. The autotransformer in structurally similar to an ordinary transformer, but its winding are electrically connected.

Control questions

1. Explain the purpose and principle of operation of transformer.
2. Why the magnetic core of the transformer is done of electrotechnical and not of ordinary steel, and consists of separate thin, isolated from each other sheets?
3. How is located the transformer windings on the core of the magnetic core?
4. What is called the transformation coefficient of the transformer and how to detect it?
5. For what purpose is it given the electrical equivalent circuit of the transformer?
6. What is the purpose of experiments of idle and short-circuit of transformer?
7. Which parameters of the transformer are called passport?
8. Compare vector diagrams of T - and Г-shaped equivalent schemes of the transformer and make on them the equations of electrical state.
9. How to connect windings of three-phase transformers?
10. What are the advantages and disadvantages of autotransformers in comparison with transformers?

10 DC MACHINES

Key concepts: geometric neutral, physical magnetic neutral, reaction of armature, process of commutation, circular fire on the collector, additional poles, the compensation winding, rated parameters, external characteristic, the idle characteristic, speed ability, mechanical characteristic.

The need of the use of DC machines (DCM) as generators or motors in sectors such as electrochemistry, electrical traction and lifting devices, in the electric drive with a wide range of speed regulation create conditions to a number of their features. In the construction industry DC machine are used in electric welding devices, in electric drive of tower cranes and other.

10.1 Device DC machine

Structurally, the generator and the direct current electric motor are the same and consist of two main parts: a stationary electromagnet – inductor, that generates a basic magnetic field of the machine, and a rotating armature, in the winding of which mechanical energy is converted into electrical (generator), or electrical – into mechanical (motor).

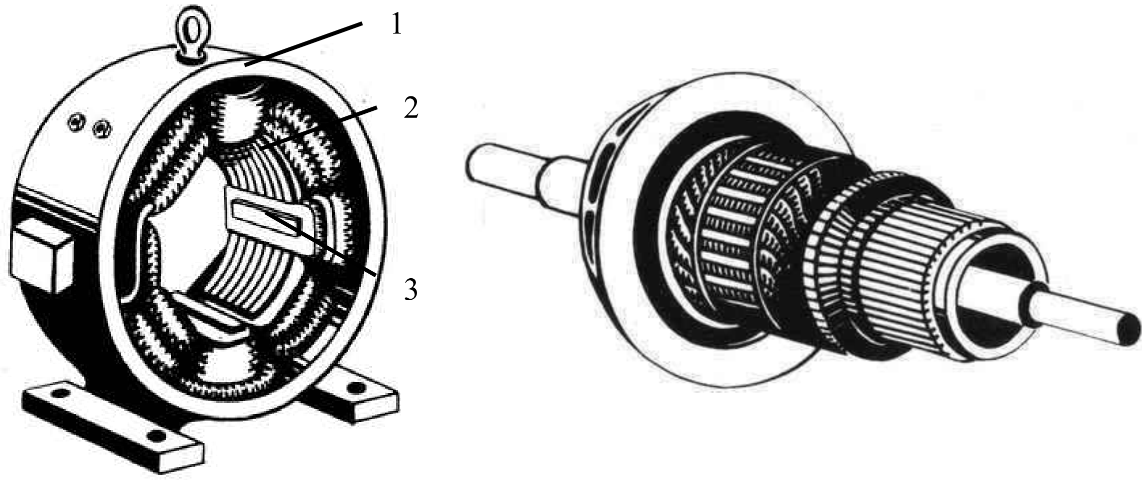
The fixed inductor is (fig. 10.1, *a*) steel cast base 1, main 2 and 3 additional poles. On the main poles windings are located, to which is brought a direct current is brought. It is created in a magnetic circuit of the machine main magnetic field (pole - air gap - armature - air gap - pole - base - pole).

The main poles are composed from lacquered sheets of electrotechnical steel of 0.5 mm to reduce loss from eddy currents arising due to pulsation of the magnetic field during rotation of the armature.

Additional poles are usually performed from forged steel, and their actuation winding is included in series with the armature winding. Additional magnetic field produced by them, is to improve the commutations.

The rotating part of DC machines - armature with windings and the collector (see fig. 10.1, *b*). The armature core is collected from lacquered plates of electrotechnical steel (0.5 mm) to reduce loss on eddy currents, that occur during the rotation of the armature in the magnetic field. On the outer surface of the armature it has grooves, into which a closed winding is placed.

The armature winding is made of copper isolated wire as sections. Section of winding are placed in slots in two layers (two-layer winding), isolated and fixed in them by wooden wedges, and part of the winding that gone out the butts of the armature from grooves, fixed by steel wire bands to prevent extraction of the winding from the grooves during the rotation of the armature. Sections of the winding are connected between each other with the collector plates and form a closed winding with a certain number of parallel branches. The number of branches is determined by the type of winding.

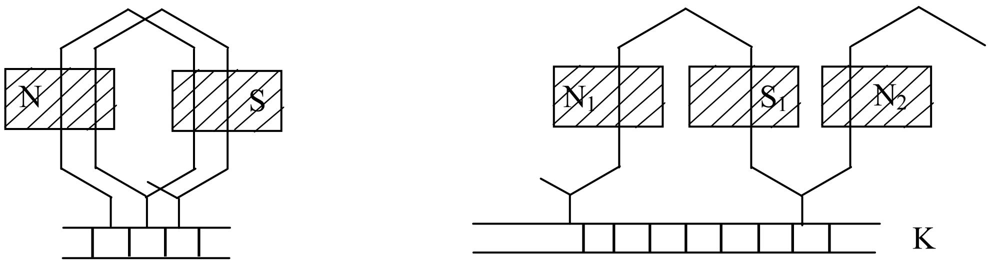


a *b*

Figure 10.1: Inductor (*a*) and armature (*b*) DC machine

There are parallel (serpentine) and serial (wave) windings. Serpentine (fig. 10.2, *a*) ones have the number of parallel branches are equal to the number of pairs of poles of the machine, and the wave (fig. 10.2, *b*) ones forms one pair of parallel branches.

The simplicity and reliability of serial winding cause their most spread in machines of general use.



a *b*

Figure 10.2: Types of winding of DCM

The collector (fig. 10.3) to which the armature winding is attached, consists of separate copper plates 1, isolated from each other and from the shaft by layings from micanite 2 (isolating material based on mica).

The collector is destined for rectification of alternating current of armature to the external circuit at the generators; at motors – for change of the direction of current in the conductors of the winding of the armature on its rotation. On the one hand to the collector plates sections of the armature winding are joined together. On collector stationary brushes slip mounted in special brush holders.

The DCM device in the assembled view is shown in figure 10.4. To the base 6 by bolts the main poles are anchored, consisting of the core 4 and the coil of winding of actuation 5. With butt sides to base they are anchored the side boards 7 with bearings that hold the shaft of the machine. Armature of the machine consists of a core 3, the winding 9 and the collector 1. On the shaft of the armature blast 8 is anchored, on the collector stationary brushes 2 are placed

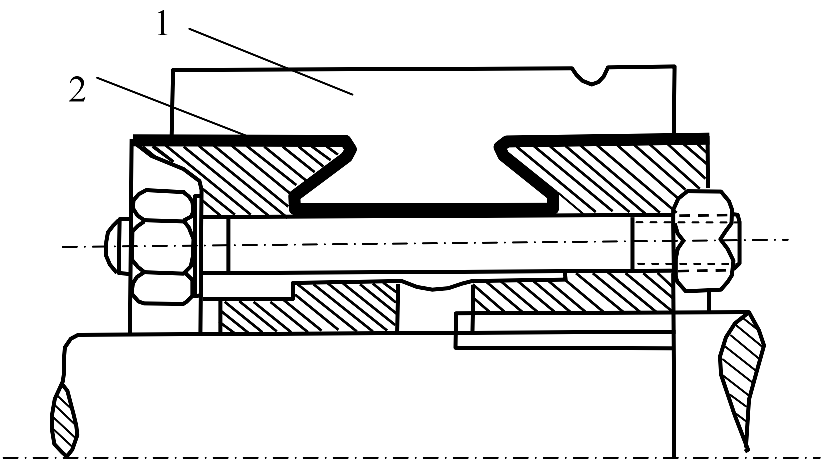


Figure 10.3: Collector device of DCM

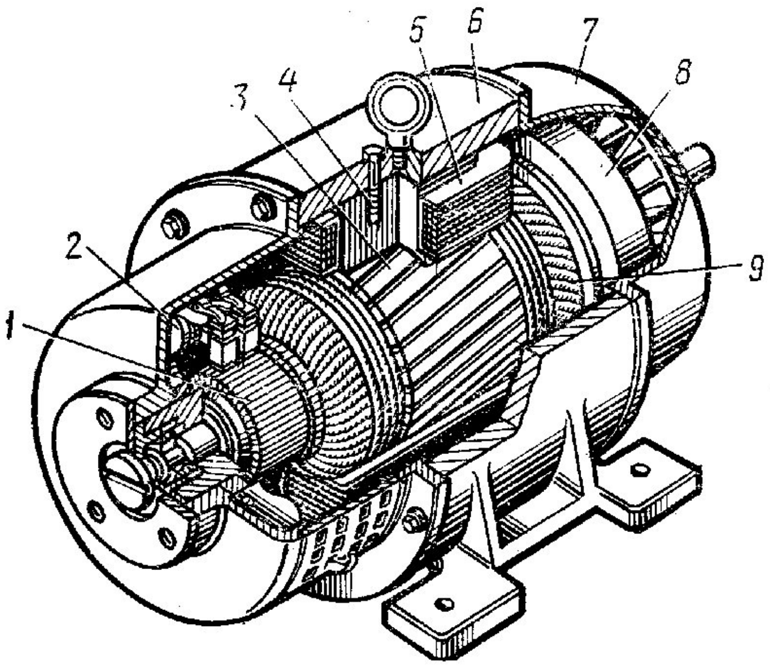


Figure 10.4: Device of DC machine

10.2 The principle of operation of DC generator

The DC generator converts mechanical energy brought from the primary motor, in electric, taking off from armature winding with help of collector and brushes. Consumers of electrical energy are attached to the brushes of the generator.

In the basis of the principle of operation of generator is the phenomenon of electromagnetic induction according to which on the movement of conductor in a magnetic field it is induced electromotive force in it. The magnitude and direction of the EMF is determined by the law of electromagnetic induction and principle of Lenz.

Consider the principled scheme of a direct current generator (fig. 10.5, *a*). In a magnetic field generated by the poles N and S, one turn of the armature winding rotates. The ends of the coil are connected with a simple collector in the form of two isolated from each other semirings, to which adjacent brushes a and b are connected with the load.

If the turn rotates at a constant speed and the magnetic field uniformly, then it will induce EMF and carry AC sinusoidal current. As the load is connected across the brushes to two semirings, then, despite the fact that the change of turn sides in places, current to them changes its direction, in the external circuit it will carry in one direction. Thus, the semirings (collector plates) carry out the conversion of alternating current of armature winding into a pulsating direct current in the external circuit of the generator (fig. 10.5, *b*).

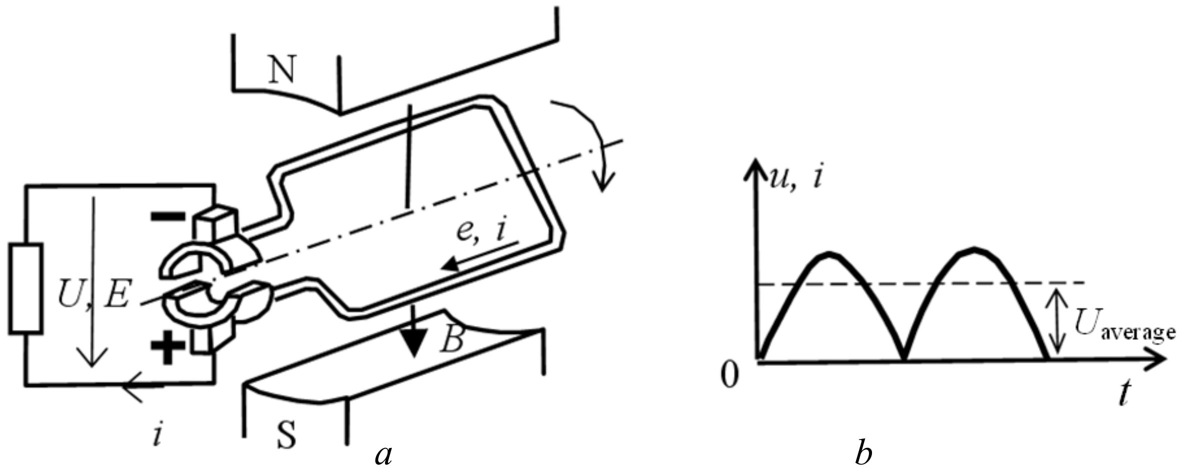


Figure 10.5: The principle of work of DC generator

To reduce the pulsation and receipt of greater EMF of the armature winding consists of many turns, which are attached to the respective number of collector plates.

The EMF of the armature winding of DC machine is determined by the machine design, the speed of rotation of the armature and value of the magnetic flux generated by the actuation system

$$E = \frac{N}{\alpha}\frac{pn}{60}\Phi = cn\Phi \text{, V} \qquad (10.1)$$

where: $c = \frac{N}{\alpha}\frac{pn}{60}$ – structural constant of the machine, depending on the type of armature winding and the number of pairs of poles;

N – the total number of armature conductors;

α – the number of pairs of parallel branches of armature winding;

p – the number of pairs of poles;

n – the speed of rotation, turns /minute;

Φ – the magnetic flux of the pair of poles of the machine, weber.

Thus, the EMF of the armature is directly proportional to the speed of the machine and the magnetic flux. Usually the speed of rotation of the armature is also constant, so the regulation of the magnitude of the EMF, and hence the voltage of the generator is produced by change of magnetic flux by regulation of the current in the windings of the actuation of the poles.

If the external circuit of generator is open (idle mode), the voltage at the connection terminals of the machine is equal to the EMF $U_0 = E$. If the generator is loaded, then on the armature winding current carries, and the voltage at the connection terminals of the machine becomes less then EMF on the magnitude of the voltage drop on the resistance in the armature circuit

$$U = E - I_a R_a \ ,$$

where $\boldsymbol{R}_\mathbf{a}$ – the total resistance of the armature circuit.

10.3 The reaction of armature

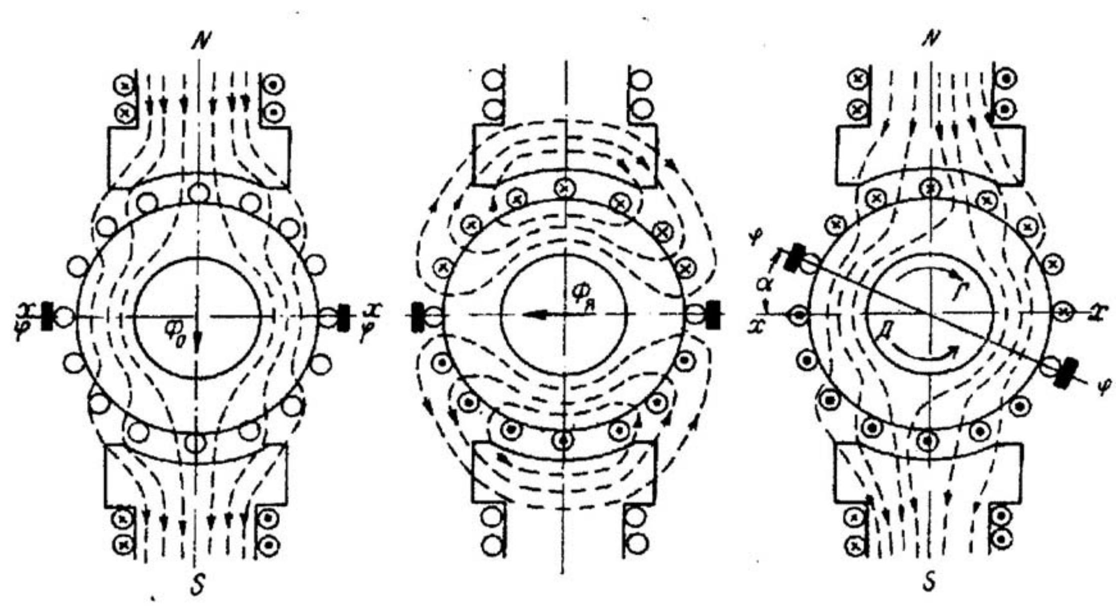
If the generator (motor) is running without load, the magnetic field is formed only by the current carrying in the actuation winding. This main magnetic flux of poles Φ_0 (fig. 10.6, *a*) is perpendicular to the geometrical neutral (x – x). In this case, the physical magnetic neutral (line φ – φ, passing through the point on the circumference of the armature, where the induction is equal to zero) coincides with the geometric neutral.

On the work of the generator on load on the armature winding current carries and an additional magnetic field of the armature Φ_a is created, fixed in space, the axis of which coincides with the axis of the brushes (fig. 10.6, *b*). The intensity of this field depends on the load current of the generator. The magnetic field of the armature is superimposed on the main magnetic field of poles, distorts and partially depresses it. This influence of the armature on the main thread of the poles is called the reaction of the armature (fig. 10.6, *c*).

Due to the saturation of the steel of poles strengthening of field is less, than weakening, so the resulting stream of poles decreases.

The distortion of the magnetic field leads to the fact that offset of the physical magnetic neutral happens at some angle α (at the generators - in the direction of rotation, at the motor – against rotation). This angle depends on the load of the generator (motor), i.e., the armature current.

When the machine is in operation in generator mode under the influence of the reaction of armature on diminishing edge of the pole resulting field is amplified and attenuated on the oncoming one. Work in the motor mode with the same direction of rotation is influenced by implification of the resulting field under the oncoming edge of pole and weakening under diminishing. In consequence, in the generator EMF is reduced and the voltage at its connection terminals, and in the engine it is reduced electromagnetic torque and speed changes. In addition, the reaction of the armature causes arcing under the brushes, which leads to the burning of collector plates.



a b c

Figure 10.6: Reaction of armature

Effective measure of reduction of the influence of the field of armature windings on field of the main poles is the use of ***the compensation winding***, the active conductors of which are placed in the grooves stamped on the surface of the polar bits. The compensation winding is included in series with the armature winding so that generated by these windings magnetic fluxes are directed oppositely and mutually compensated.

10.4 Commutation in DC machines

The process of switching of sections from one parallel branch to another by way of closing these sections by brushes and a set of events connected with the current change in the switchable sections, called commutation.

To explain the process of commutation let's refer to figure 10.7, *a* where the armature in one of the sections at different moments of time 1, 2 and 3 is shown. Indicated directions of current in the conductors of the armature can be concluded that by the time 3 considered section is moved from one parallel branch to another, and the current in the section changed its direction to reverse. During the time between time moments 1 and 3, when considered section together with the two collector plates and brush form a short circuit contour (fig. 10.7, *b*) the current in it changes from $+i_a$ to - i_a. On change of the current in a commutatable section EMF of self-induction $e_L = -L\frac{di_a}{dt}$ is induced. In the real machine brush usually overlaps two or three collector plates and the current direction change occurs simultaneously in several nearby sections. Therefore, in each of the simultaneously commutatable sections is induced EMF of mutual induction e_M. The amount of EMF of self-induction and mutual induction is called reactive EMF e_R.

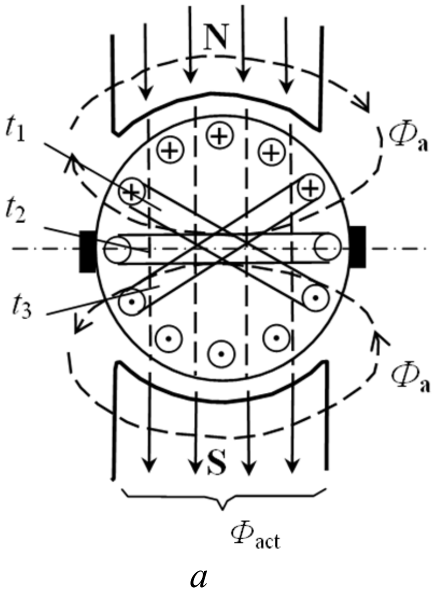
In addition to reactive EMF e_R in a commutatable sections by intersection of the resulting magnetic field (field of armature winding and the field of main poles) EMF of rotation e_{ROT} is also induced.

The resulting EMF of commutatable section $e_C = e_L + e_M + e_{ROT}$ may cause contour considerable commutation current in short circuit

$$i_C = \frac{e_C}{R_C} = \frac{e_L + e_M + e_{ROT}}{R_C}, \tag{10.2}$$

where $\boldsymbol{R_c}$ – the resistance of the commutating contour.

As each section has an inductance, the interruption of current in the transition of section from one parallel branch to another causes an increase in voltage in place of the break and formation of arcing between the brush and the collector plate. Arcing will be the stronger, the more commutation current i_C.



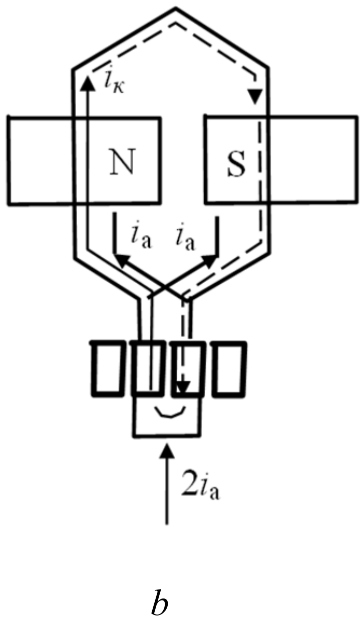


Figure 10.7: Commutation of sections of the armature winding:
a – the process of switching of sections; *b* – the positional relationship of the brushes and collector in the process of commutation

From (10.2) it follows that reduce of the current i_C, thereby improves the commutation of machine, you can: a) have increased the resistance of a circuit of commutatable section; b) have reduced reactive EMF e_R; c) have created in the circuit of commutatable sections EMF compensating e_R. The first of these methods of improvement of commutation is reduced to the choice of brushes with great resistance.

The most effective method of fighting with arcing is installation in machines of additional poles, creating in zone of commutation magnetic field directed toward the field of the armature. Meanwhile in commutatable section EMF of rotation is induced, which compensates reactive EMF and reduces EMF of commutatable section, thus improving the commutation of machine. The number of additional poles usually equal to the number of main (only machines with power from 300 W to 3.5 kiloW can be of two basic and one additional pole).

Additional poles are set on geometric neutrals, their winding are connected in series with the armature winding.

Picture of the magnetic flux and the polarity of the additional poles on work of the machine in mode of the generator G and the motor D subject to the direction of rotation is shown in figure 10.8. The magnetic circuit of extension poles must be unsaturated, because only under this induction in the gap under the additional pole will vary proportional to armature current, and the magnitude of the magnetizing force of additional poles – depending on load.

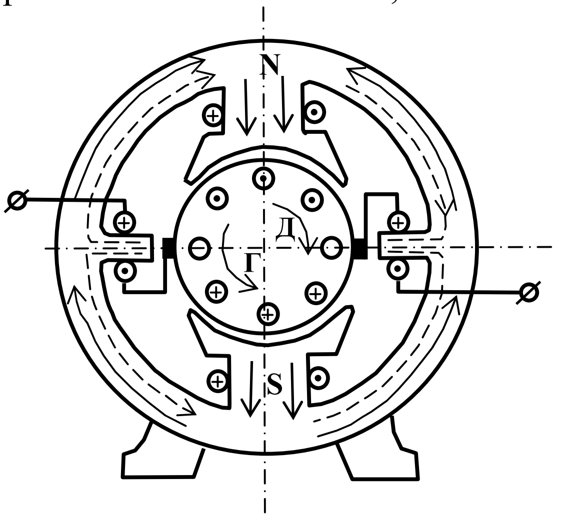


Figure 10.8: Circuit of inclusion of windings of additional poles

In irreversible machines of small power with a reduced number of additional poles improvement of commutation (reduction e_C) can be achieved by shift of the brushes from the geometrical neutral on the work of the machine in generator mode in the direction of rotation of the armature, in the mode of the motor – against rotation of the armature.

10.5 Energy loss and COP

The process of conversion of mechanical energy into electrical energy and vice versa, takes place in the DC machine is accompanied by energy loss. These losses are divided into electrical, magnetic, mechanical and additional.

Losses in the windings of the armature and actuation, and also loss in the brush contact are included in the electrical loss.

Loss in the armature winding and in the sequential winding of actuation is proportional to the square of the current and is equal to $I^2 \cdot R$ (R is the resistance of the armature circuit of the machine). The electrical losses in the brush contacts are determined on the basis of current armature circuit and voltage drop under the brushes of one polarity

$$\Delta P_B = 2\Delta U_B I_A \ . \tag{10.3}$$

The electrical loss in the resistance of the armature circuit, including brush contact, approximately 50% of all losses in the machine.

Energy loss in the winding actuation of machines with independent, parallel and mixed actuation is accepted to determine through the power absorbed by this winding $\Delta P_A = U_A \cdot I_A$. They are 0.5–7% of the rated power of the machine, with a smaller percentage of loss relates to more powerful machines.

The total electrical loss in the machine of DC

$$\Delta P_E = I_A^2\, R_A + I_A^2 R_{W.A.} + I_A^2 R_{ADD.P.} + \Delta P_B + \Delta P_{ADD}\,, \tag{10.4}$$

where $R_{W.A.}$, $R_{ADD.P.}$ – the resistance of the serial winding of actuation and winding of additional poles.

In machines of direct current magnetic flux is stationary in space and constant in time. Therefore, the rotational magnetization reversal is only subjected to steel of armature, causing its core is performed gathered from sheet material. Magnetic loss in the steel armature ΔP_s are less than 1–3% of the rated power of the machine. To magnetic ones also are included loss in the pole bits, the main poles and the yoke, due to the pulsation of the magnetic flux, reason of which is a dentated design of an armature.

Mechanical loss ΔP_{mech}, to which losses are included from friction in the bearings, ventilation losses and the friction of the brushes on the collector, depending on the frequency of rotation of the armature, and for machines with a power of 10–500 kW is 0.5–2% of P_{rat} (lower percentage concerns to more powerful machines).

In machines of direct current there are other losses, grouped additional loss ΔP_{add}. It is believed that they are equal 0,01 P_{rat}.

Knowing the sum of all losses in the machine

$$\Delta P_{\Sigma} = \Delta P_e + \Delta P_s + \Delta P_{mech} + \Delta P_{add}\ , \tag{10.5}$$

you can determine its COP (in percent):

for generators $$\eta = \left(1 - \frac{\Delta P_{\Sigma}}{P_2 + \Delta P_{\Sigma}}\right)100\ , \tag{10.6}$$

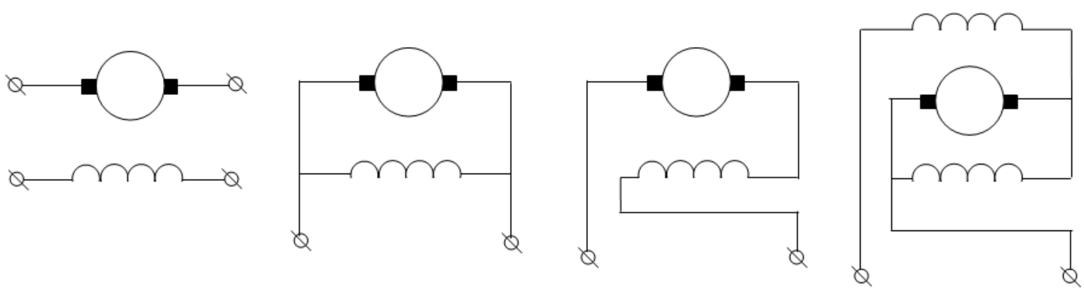
for motors $$\eta = \left(1 - \frac{\Delta P_{\Sigma}}{P_1}\right)100\ , \tag{10.7}$$

where: P_1 – the power delivered to the motor;
P_2 – the useful power delivered by the generator.

Machines of power up to 100 kW have COP 75–90%, power 500–1000 kW – 91–95%.

10.6 Inclusion schemes of the windings of the actuation

Depending on the scheme of the inclusion distinguished DCM with independent, parallel, serial, and mixed actuation are distinguished



a *b* *c* *d*

Figure 10.9: Schemes of inclusion of the winding of the actuation of machine: *a* – with independent actuation; *b* – parallel actuation; *c* – sequential actuation; g – mixed actuation

In machines with independent actuation (fig. 10.9, *a*) the actuation winding is powered from an external source of electric current, it is not directly associated with the armature circuits of the machine.

In DCM of parallel actuation winding of actuation connects in parallel with circuit of armature (fig. 10.9, *b*). If necessary, the actuation winding of this machine can be switched on and under the scheme with independent actuation.

In machines of the sequential actuation winding of actuation connects in series with circuit of anchors (fig. 10.9, *c*). It is done by wires of large section with a small number of turns and small resistance.

On the main poles of machines of mixed actuation (fig. 10.9, *d*) there are two coils, one of which connects to circuit of anchors in series, and the second – in parallel.

In DCM of small power (tens – hundreds watts), the magnetic field of the actuation is generated by the permanent magnets.

Diversity of schemes of inclusion of windings of actuation determines the difference of the characteristics and properties of the generators and motors of DC.

10.7 Rated parameters and characteristics of DC machines

The nominal parameters of electric machines are the parameters that characterize the rated mode of work of the machine, i.e., the mode of work under the conditions for which it is intended.

Under a rated power of DCM it is understood:

- in generator mode – electrical power is delivered to the external circuit;
- in the motor mode – useful mechanical power is on the shaft.

On the rated values of the parameters, you can choose one or the other DCM, to determine the rationality of its usage in a particular electrical installation, to ensure proper maintenance for its reliable work within the prescribed period.

Properties and characteristics of electrical machines it is customary to analyze using the graphs – characteristics, which are in collections of technical data or directories, or experimentally collected (individual characteristics can be calculated).

The greatest practical interest for generators represents the dependence of the voltage on its connection terminals U from the load current I_a at constant speed ω and the current of winding of actuation I_{act}; this dependence is called the external characteristic $U = f(I_a)$. Feature of idle is often used, representing the dependence of the voltage U from the actuation current $I_{W.ACT}$ at a constant speed and open the external circuit ($I_a = 0$). As the voltage on the generator is equal to its EMF E, then the characteristic of idle is the dependence $E = f(I_{act})$ when ω_{nom} = const and $I_a = 0$.

For DC motors, the greatest interest is the dependence of the rotation speed ω from the current I_a or torque M at a constant voltage of supply. The dependence $\omega = f(I_a)$ is called the high-speed characteristic, and the dependence $\omega = f(M)$ – mechanical characteristics.

10.8 DC generators

10.8.1 Generator with independent actuation. The electric circuit of the generator is shown in figure 10.10. The actuation winding is supplied from the auxiliary DC source. The exciter power is usually 3-4% of power of generator. The current in the actuation winding is determined by the voltage source and the resistances of the actuation circuit

$$I_{act} = \frac{U_{act}}{R_{act} + R_R} \tag{10.8}$$

and does not depend on the load generator. Changing the resistance of the adjusting resistor R_R, it is possible to smoothly change the actuation current, and therefore, the magnetic flux of the machine, thus altering the EMF and the voltage of the generator, as

$$U = E - I_a R_a = cn\Phi - I_a R_a .$$

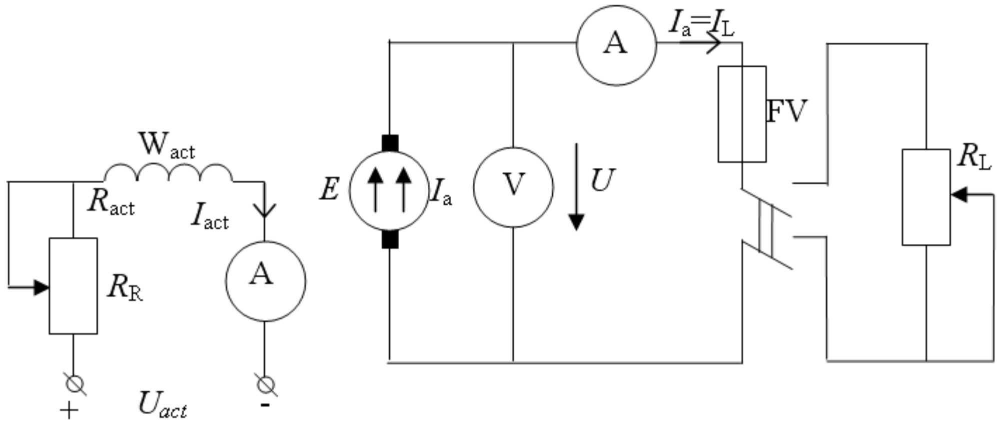


Figure 10.10: Scheme of generator with separate actuation

In idle mode the generator $I_a = 0$ and $U_0 = E = cn\Phi = f(I_{act})$. This dependence of EMF from current of actuation is called the ***characteristic of idle***. It is shown on figure 10.11, *a*.

Feature of idle begins with value of voltages E_{res}, caused when $I_a = 0$ by flux of residual magnetism of the poles. The view of this characteristics is determined by the magnetization curve of the magnetic circuit of the machine.

A characteristic feature of the generators with independent actuation is the possibility of smooth adjustment of the EMF, and hence the voltage at the connection terminals of the machine in a wide range from 0 to U_{nom}.

In the load mode, the voltage at the connection terminals of the armature is reduced mainly because of the voltage drop in the resistance of the armature circuit, as $U = E - I_a{\cdot}R_a$. The influence of the reaction anchor within the rated load of the generator is low. Therefore, the decrease in voltage at the connection terminals of the machine by increasing the armature current from 0 to $I_{a.nom}$ little and is

$$\Delta U = \frac{U_0 - U_{nom}}{U_{nom}} = 5 \div 10\%. \tag{10.9}$$

The dependence U = f(I_a) is called the ***external characteristic of the generator*** and has the form shown in figure 10.11, *b*. The voltage of generator can be maintained constant, slightly changing the actuation current by adjustment rheostat R_R.

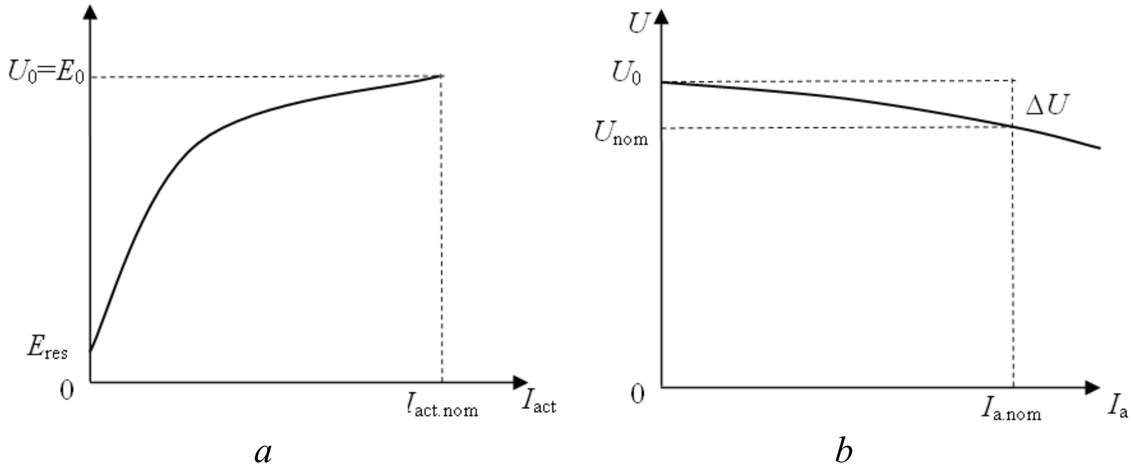


Figure 10.11: Characteristics of the generator with independent actuation: *a* – characterization of idle; *b* – external characteristics

The lack of a constant current generator with independent actuation is necessary to have an auxiliary power source for the actuation circuit. When overload of generator of in excess of the rated power voltage at the connection terminals of the machine decreases more sharply because of the increasing demagnetizing action of the reaction of armature. The armature current increases and during short circuit reaches a large value $I_{sh.c}$ = (15–20)/I_{nom}. Therefore, in order to prevent damage of the generator must be installed protection, which disables it during a short circuit (for example, fuse *FV* on figure 10.10).

10.8.2 Generator with parallel actuation. The electric circuit of the generator is shown in figure 10.12. The actuation winding connects in parallel to the connection terminals of the armature, is calculated on a small current I_{act} = (0.01–0.03)·I_{nom} and has a large number of turns of wire of small cut. The current in the actuation winding is determined by the voltage at the connection terminals of the armature and by resistance of the actuation circuit.

$$I_{act} = \frac{U}{R_{act}} = \frac{E - I_a R_a}{R_{act.} + R_R} \ . \qquad (10.10)$$

Feature of idle E = f(I_{act}) (fig. 10.13, *a*) is similar to the characteristic of the generator with independent actuation. Generator stably sings in that case, if $E > I_{act} \cdot R_{act}$, i.e. if the characteristic of idle E = f(I_{act}) is above characteristics of the actuation circuit (straight line $U = I_{act} \cdot R_{act}$). The point of intersection of these characteristics determines the final EMF, and hence the voltage, up to which it is actuated a generator. If $E \leq I_{act} \cdot R_{act}$, self-excitation of the machine does not happen. The resistance of the actuation circuit that determines the beginning of the self-excitation (the threshold of self-excitation), is called ***critical resistance***. In this case, the voltage at the generator connection terminals is unstable.

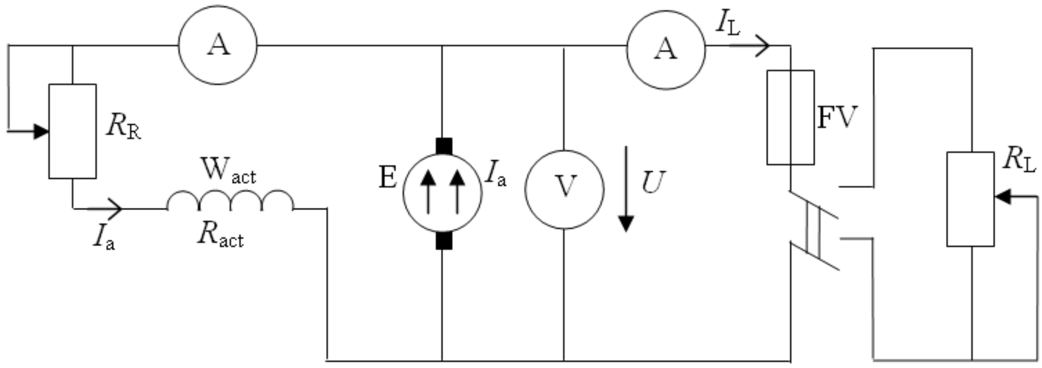


Figure 10.12: Scheme of generator with parallel actuation

This feature reduces the adjustment properties of the generator. The generators of the normal performance stable voltage at idle is (0.6–0.7)·U_{nom}. To extend the range of voltage regulation it is used a special poles with rich absorbent sections that provides a stable voltage from 0.2·U_{nom}.

In the load mode, the voltage at the connection terminals of the machine is reduced increasingly than at the generator with independent actuation. This is because the voltage decreases not only because of the increased voltage drop in the armature circuit ($U = E - I_a \cdot R_a$) and the reaction of armature, but also due to the decrease of the actuation current, as

$$I_{act} = \frac{U}{R_{act}} = \frac{E - I_a R_a}{R_{act}}. \tag{10.11}$$

The influence of the reaction anchor and the reduction of the actuation current is most sharply shown in the increase of load in excess of rated. Change of the load resistance below a certain value leads to a decrease of the load current due to a sharp decrease of the EMF of generator.

The maximum possible current of the generator is called the critical current, and is I_{cr} = (2–2.5)·I_{nom}. The short circuit current of generators with parallel actuation is determined by the residual EMF and armature resistance

$$I_{sh.c.} = \frac{E_{res}}{R_a} \ . \tag{10.12}$$

Machines of high power short-circuit current is a few more of the nominal current. External characteristics of the generator with parallel actuation is shown in figure 10.13, *b*. The decrease in voltage at the connection terminals of the armature when the increase in generator load to rated one is

$$\Delta U = \frac{U_0 - U_{nom}}{U_{nom}} 100 = 10 \div 15\% \ . \tag{10.13}$$

The voltage generator can be maintained constant by changing the actuation current by adjustment rheostat R_R.

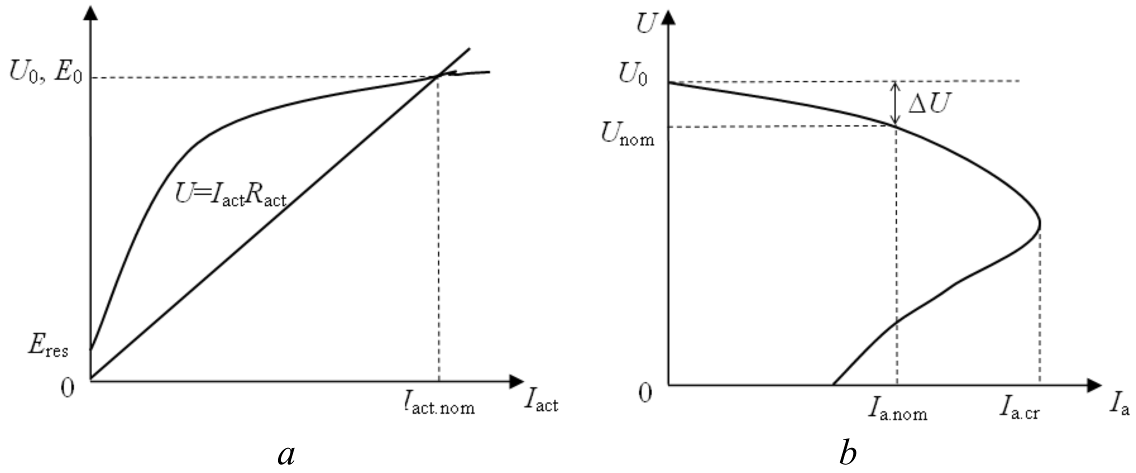


Figure 10.13: Characteristics of the generator with parallel actuation:
a – characteristic of idle; *b* – external characteristic

Generators with parallel actuation is used in cases when it is necessary to maintain the voltage at the load is constant or voltage regulation within certain limits.

10.8.3 Generators with serial actuation. The electric scheme of the generator is shown in figure 10.14, *a*. The actuation winding of the generator is included in series with the armature, is calculated on the armature current, as $I_{act} = I_a = I_{nom}$ and has a few turns of big cut wire. Feature of idle of generator with serial actuation can be taken off only when the supply of actuation windings from an external source and has the same appearance as generator with independent actuation.

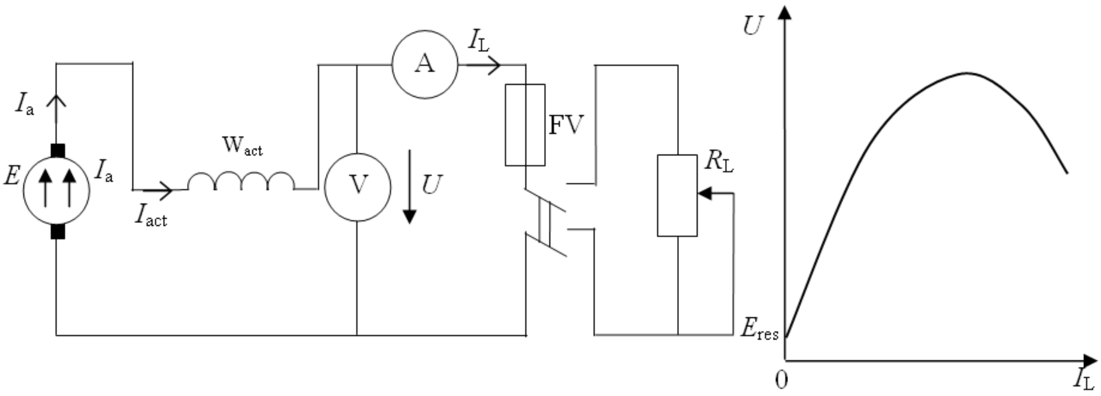
The self-excitation of the generator occurs when armature circuit is closed. The armature current, carrying through the actuation winding, increases the magnetic field of the poles, and therefore, the EMF of the machine and the voltage at the connection terminals of the armature, so as

$$U = E - I_a(R_a + R_{act})\,. \tag{10.14}$$

With the increase of the load armature current and voltage increase. After saturation of the magnetic circuit of the machine the growth of the voltage is stopped, and at the further increase of the load current, the voltage starts to decrease because of the increased drop it in the resistances of the armature and field of winding and demagnetizing action of the reaction of armature.

External characteristics of the generator with serial actuation is shown in figure 10.14, *b*. The generator can be used to supply a constant load.

Property of serial winding of actuation to increase the generator voltage with increase of load is used to compensate for voltage reduction observed in generators with parallel actuation.



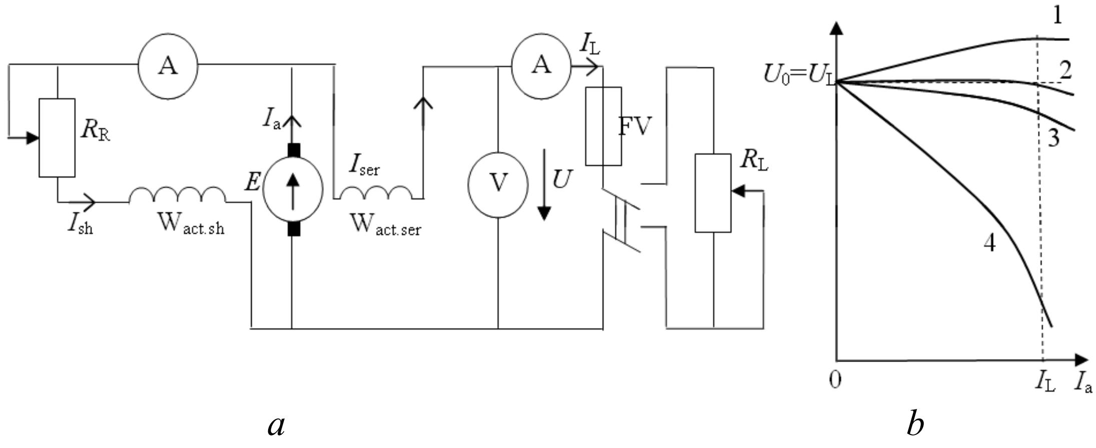
a *b*

Figure 10.14: Generator with serial actuation:
a – electric scheme; *b* – external characteristic

10.8.4 Generator with mixed actuation. The electric scheme of the generator is shown in figure 10.15, *a*. The generator has two windings of actuation: parallel $W_{act.sh}$ (shunt) included in parallel to the anchor and serial $W_{act.ser}$ (series). Parallel winding is calculated on a small current $I_{a.sh.} = (0{,}01–0{,}03)\cdot I_L$, has a large number of turns of wire of small cut. Sequential winding is calculated based on the load current $I_L = I_{ser} = I_a - I_{sh}$ and has a few turns of big cut wire. Because of the parallel winding of the generator with mixed actuation can self-excite and on open circuit load. Characteristic of idle has the same form as that of the generator with parallel actuation.

When turning on the load current appears in serial winding of actuation, which creates an additional magnetic field $\Phi_s = f(I_L)$. If this winding is connected so that the magnetic fluxes of the two field windings are formed, i.e., the magnetic flux of the poles $\Phi = \Phi_{sh} + \Phi_{ser}$, then the EMF of the machine by increase of the load will increase, as the stream of serial windings increases. This allows you to compensate for the decrease in voltage at the connection terminals of the machine from the reaction of armature and the voltage drop in the armature resistance and serial winding of actuation, so as

$$U = c \cdot n \cdot (\Phi_{sh} + \Phi_{ser}) - I_a \cdot (R_a + R_{a.ser}). \qquad (10.15)$$



a *b*

Figure 10.15: Generator with mixed actuation:
a – electric scheme; *b* – external characteristic

Therefore, the external characteristic of the generator (curve 2, fig. 10.15, *b*) goes above characteristic of the generator with parallel excitation (curve 1). By adjusting R_R it is possible to obtain an increase in voltage with increase of load (curve 3).

The generator with the agreed inclusion of serial and parallel windings of the actuation is used in cases when it is necessary to regulate the voltage of consumer (changing the current I_{sh} by resistance R_R), or when you want to automatically maintain a constant voltage under varying load.

In some cases, for example, in welding units, it is necessary that at increase of the load voltage was decreased more sharply than at the generator with parallel actuation. To obtain such falling characteristics (curve 4, fig. 10.15, *b*) serial winding is included oppositely so that the resulting magnetic flux with increase of load is decreased $\Phi = \Phi_{sh} - \Phi_{ser}$. Then the voltage

$$U = c \cdot n \cdot (\Phi_{sh} - \Phi_{ser}) - I_A \cdot (R_a + R_{ser}) \tag{10.16}$$

will also drop with increase of load current.

10.9 DC electric motors

10.9.1 DC motor with independent actuation. Circuit of DC motor of independent actuation is presented in figure 10.16. The armature of the motor M and its actuation winding W.ACT usually are independent from each other voltage sources U and U_{act} that allows you to individually adjust the voltage on the armature of the motor and the actuation winding and run them on different rated voltage. Only in the presence of a DC circuit the actuation winding supplies from the same voltage source as the armature of the motor. But in this case, the actuation current I_{act} does not depend on the current I_a of the motor armature.

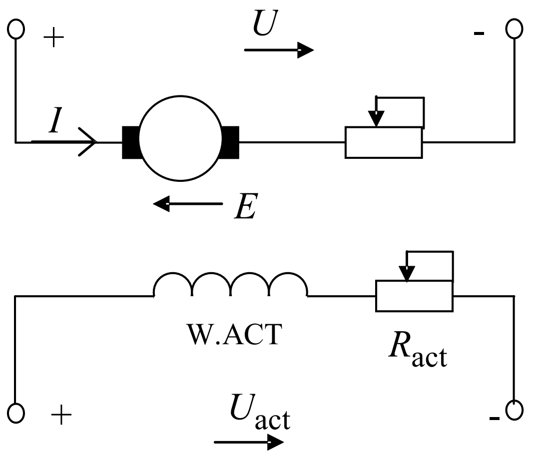


Figure 10.16: Circuit of inclusion of DC motor of independent actuation

The directions of the current I and the EMF of rotation of the motor E, shown in figure 10.16, correspond to the motor mode of work, when electrical energy is consumed by the motor from the circuit (from a voltage source U) and is converted into mechanical energy. On the shaft of the motor occurs torque electromagnetic torque M_{elm}. Useful torque M on the shaft of the motor is less electromagnetic one on the value of the opposing moment generated in the vehicle by the forces of friction and is equal to the moment M_{im} in idle mode, i.e. $M = M_{elm} - M_{im}$.

From mechanics we know that the mechanical power of motor P can be expressed through the torque and angular speed as

$$P = \omega \cdot M\ , \qquad (10.17)$$

where: $\boldsymbol{\omega} = \frac{2\pi \cdot n}{60}$ – the angular speed of the rotation of the armature, rad/s;
n – frequency of rotation, turn/minute.

Then the useful torque of the motor *M* (newton·metre), expressed through the useful power *P*, kiloWatt, will be determined as follows

$$M = \frac{P}{\omega}\ . \qquad (10.18)$$

The relationship between *M* and ω of the motor is determined by its mechanical characteristic. The analytical expression of the mechanical characteristics of the motor can be obtained from the equation of equilibrium of voltages developed for anchor circuit of sheme (fig. 10.16). During steady mode of work of the motor applied voltage *U*, is balanced by the voltage drop in the armature circuit of the *I·R* and induced in the armature EMF of rotation *E*, i.e.

$$U = I \cdot R + E, \qquad (10.19)$$

where ***I*** – the current in the armature circuit of the motor, A;
R – the total resistance of the armature circuit, Ohm, including external resistor R_R and the internal resistance of the motor armature R_a.

The EMF of rotation is determined by the speed of rotation of the armature and the value of magnetic flux

$$E = k \cdot \Phi \cdot \omega, \qquad (10.20)$$

where: ***k*** – the coefficient depending on the constructive data of motor, $\boldsymbol{k = p \cdot N/2\pi \cdot a}$ (***p*** – the number of motor pole pairs; ***N*** – the number of active conductors of armature winding; ***a*** – the number of pairs of parallel branches of armature winding);
Φ and ***ω*** – accordingly, magnetic flux, weber, and the angular speed of the motor, radian /second.

If (10.19) instead of E to substitute its value from (10.20), we get the equation for the speed of the motor

$$\omega = \frac{U - IR}{k\Phi}, \qquad (10.21)$$

which represents the dependence of the speed of the motor from armature current. This dependence ω = *f(I)* is called the ***electromechanical characteristic of the motor***.

To obtain the equations of the mechanical characteristic it is necessary to find the dependence of speed on motor torque. The moment developed by the motor, is connected with the armature current and the magnetic flux dependence

$$M = k \cdot \Phi \cdot I. \qquad (10.22)$$

We substitute in (10.21) the value of current *I,* is found from (10.22) and we obtain the expression for the ***mechanical characteristic of the motor***:

$$\omega = \frac{U}{k\Phi} - \frac{M \cdot R}{k^2 \cdot \Phi^2}, \qquad (10.23)$$

or

$$\omega = \frac{U}{c} - \frac{M \cdot R}{c^2}, \tag{10.24}$$

where $$c = k \cdot \Phi. \tag{10.25}$$

Mechanical characteristic of the motor of independent actuation at constant parameters U, Φ and R is a straight line. Changing a particular parameter of mechanical characteristic, it is possible to obtain different speeds of the motor, i.e. to adjust the speed of rotation.

Consider the impact resistance of the armature circuit on the mechanical characteristics of the motor.

Figure 10.17 presents the mechanical characteristics of the motor with independent actuation for different resistances armature circuit.

As can be seen from (10.17), when $M = 0$ all specifications pass through a single point lying on the axis of ordinates. The angular speed at this point has a value that does not depend on the resistance of the anchor circuit. This speed is called the ideal speed of idle ω_0 and is determined by the expression

$$\omega_0 = \frac{U}{k\Phi}. \tag{10.26}$$

At speed of ideal idle, when the current in the armature circuit RA is equal to zero, EMF of armature directed towards the applied voltage, equal in absolute value. If the motor up to the application of load worked with the angular speed ω_0, then on occurrence on its shaft resistance moment angular velocity will decrease. The result of it will be a decrease in EMF of rotation in accordance with (10.20) and the increase in armature current in accordance with (10.19) and motor moment on (10.22). The angular speed will decrease as long as the motor moment is equal to the moment of resistance. The difference between the values of the steady speed of the electrical drive before and after apposition of a given static load is referred to as ***static drop of speed***.

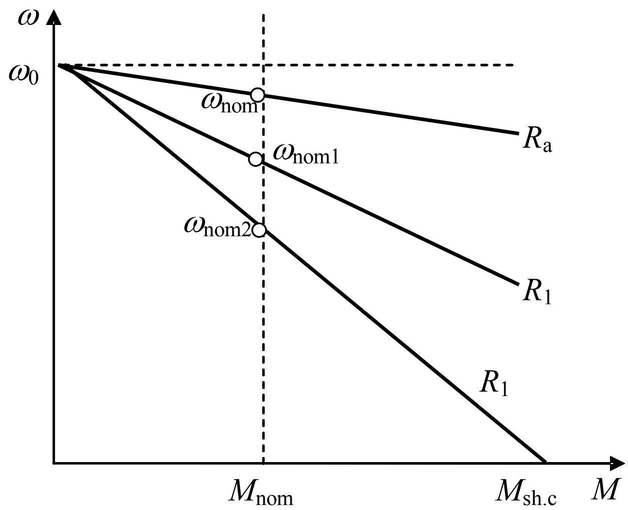


Figure 10.17: Mechanical characteristics of DC motor of independent actuation

The second member of (10.24) characterizes a static drop of angular speed (differential) relative to the angular speed of the ideal idle:

$$\Delta\omega = \frac{M \cdot R}{k^2 \cdot \Phi^2}. \tag{10.27}$$

Thus, the equation for the speed of the motor can be written as follows:

$$\omega = \omega_0 - \Delta\omega. \tag{10.28}$$

The top characteristic on figure 10.17 is called natural. ***Natural characteristic*** is a characteristic of the motor, which is obtained in the absence of external resistors in armature circuit and rated values of voltage and magnetic flux of motor The stiffness of the natural characteristic depends on the internal resistance of the armature circuit of the motor R_a. The internal resistance of armature circuit includes its own resistance of anchor winding, winding resistance of the additional poles, the compensation winding and brushes. Accordingly, the difference in speed for the natural characteristic

$$\Delta\omega = \frac{M \cdot R_a}{k^2 \cdot \Phi_{rat}^2}.$$

(10.27) is determined the static drop of speed for any of the characteristics of the motor with independent actuation, is presented in figure 10.17. For example, when additionally included rheostat, having a resistance R_R, static drop rate will be determined from the relation

$$\Delta\omega = \frac{M \cdot (R_a + R_R)}{k^2 \cdot \Phi^2}. \tag{10.29}$$

Dividing (10.28) for the ω_0, will get static decrease of speed in relative units:

$$\Delta\omega^* = \frac{\Delta\omega}{\omega_0} = \frac{\omega_0 - \omega}{\omega_0}.$$

If the armature circuit of the motor an additional resistor (rheostat) is included, the mechanical characteristics obtained in this way are called artificial or ***rheostat characteristics***. These characteristics intersect in a single point ω_0. Rheostat characteristics are as linear as a natural characteristic, but have much greater inclination to the axis of moments, i.e., have lower rigidity. The more put included in the circuit of the armature resistance of resistor, the steeper is the characteristic, the lower its rigidity.

From the equation of the electric equilibrium of the armature circuit of the motor (10.19) it follows that at the time of start, on $\omega = 0$, the EMF rotation $E = 0$, and the starting current of the motor $I_{st} = \frac{U}{R_a}$ in 10–30 times higher than the rated value. Therefore, a direct start of the motor, i.e., the direct connection of the anchor to the circuit voltage, is unacceptable. To limit large starting current of armature, before the start sequentially with armature it is included starting rheostat R_{st} with little resistance.

10.9.2 Motor with serial actuation. The actuation winding of the motor is included serially with the armature (see fig. 10.18, *a*) because the load current is the armature current and actuation current ($I = I_a = I_{act}$). This moment significantly affects on the properties and characteristics of the motor, because the change of the load moment is inevitably accompanied by a change in magnetic flux of the stator.

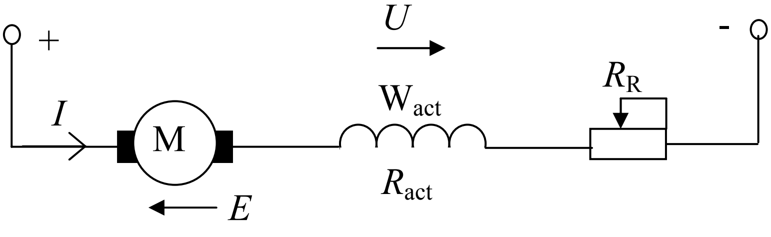


Figure 10.18: Circuit of inclusion of DC motor of sequential actuation

For motor of serial actuation equation of electromechanical characteristic, as well as for the motor of independent of the actuation has the form:

$$\omega = \frac{U - I \cdot R}{k \cdot \Phi}, \tag{10.30}$$

where $\boldsymbol{R} = R_a + R_{act} + R_R$ – the total resistance of the armature circuit.

Unlike motor of separate actuation here the magnetic flux Φ is a function of the armature current I. This dependence (see fig. 10.19) is called the magnetization curve. As there is no exact analytical expression for the magnetization curve, it is difficult to give an exact analytical expression for the mechanical characteristics of the motor of serial actuation. If we assume a linear relationship between the flux and the armature current, i.e., assume $\Phi = \alpha \cdot I$, the motor moment can be expressed as follows

$$M = k \cdot \Phi \cdot I = \alpha \cdot k \cdot I^2. \tag{10.31}$$

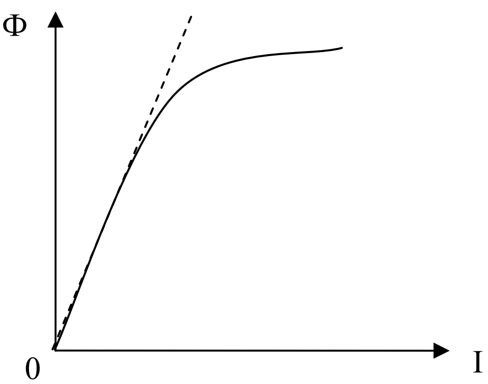


Figure 10.19: Magnetization curve of a DC motor of serial actuation

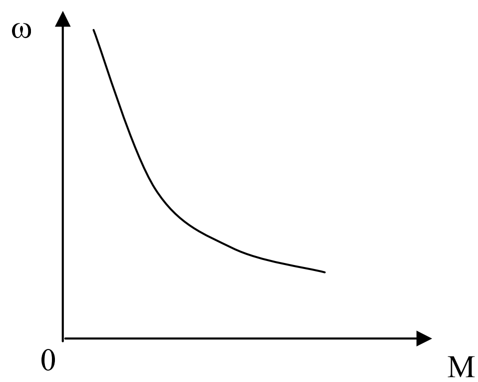


Figure 10.20: Natural mechanical characteristics of the DC motor of serial actuation

Substituting in (10.30) the current value (10.31), we obtain the expression for the mechanical characteristics:

$$\omega = \frac{U}{\alpha k \sqrt{\dfrac{M}{\alpha k}}} - \frac{R}{\alpha k} = \frac{A}{\sqrt{M}} - B. \tag{10.32}$$

It follows that when unsaturated magnetic circuit of the motor mechanical characteristic is represented by curve (fig. 10.20), for which the y-axis is an asymptote. A feature of the mechanical characteristic of the motor of serial actuation is a large slope in the region of small values of moment. A significant increase in the angular speed at low loads caused by a corresponding decrease of the magnetic flux.

Equation (10.32) gives only a general idea about the mechanical characteristic of the motor of serial actuation. In the calculations by this equation it cannot be used, because machines with unsaturated magnetic system does not build. Due to the fact that the actual mechanical characteristics are very different from the curve expressed by the equation (10.32), construction of characteristics used graph-analytical methods. Typically, the construction of artificial characteristics are based on the data of directories, where there are natural characteristics: ω = $f(I)$ and $M = f(I)$. For a series of motors of a particular type these characteristics are given in relative units: $\omega^* = f(I^*)$ and $M^* = f(I^*)$. Such universal characteristics are shown in figure 10.21.

The catalogue also provides the dependence of the moment on the shaft of the motor from current. When mechanical characteristics are being constructed the dependence of the angular speed from the electromagnetic torque is assumed. It is almost acceptable because of the small difference between the electromagnetic torque and torque on a shaft.

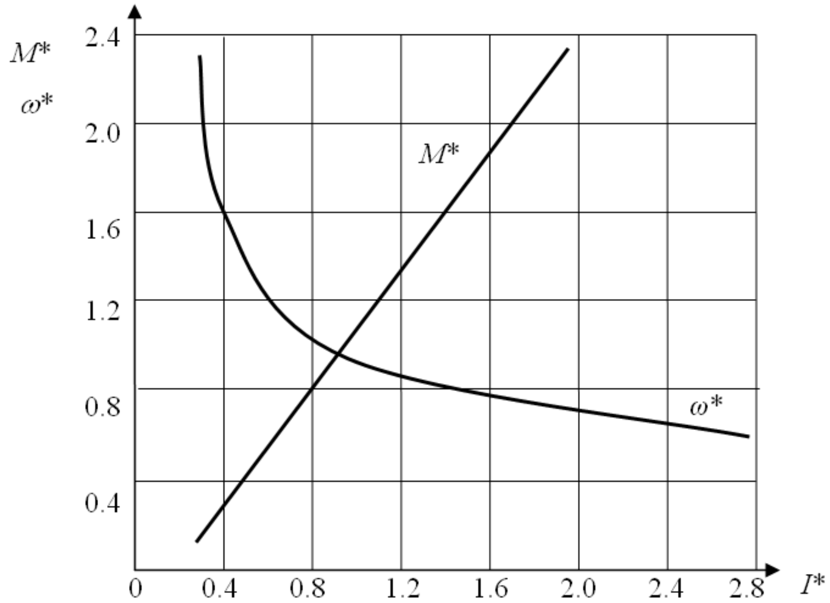


Figure 10.21: Dependence of the moment and angular speed from the armature current of a DC motor of serial actuation

Artificial characteristics can be obtained in the following way. From (10.30) when $R_R = 0$, we obtain the equation of natural feature:

$$\omega_{ac} = \frac{U - I \cdot R_m}{k \cdot \Phi},$$

where $\boldsymbol{R_m = R_a + R_{act}}$

$$\omega_{ac} = \frac{U}{k \cdot \Phi}\left(1 - \frac{I \cdot R_m}{U}\right). \qquad (10.33)$$

When inclusion of the motor of an additional resistor R_R in the anchor curcuit, the motor will run on the device characteristics, for which

$$\omega = \frac{U}{k \cdot \Phi}\left(1 - \frac{I(R_m + R_R)}{U}\right). \qquad (10.34)$$

Have divided (10.34) on (10.33) we get:

$$\frac{\omega}{\omega_{ac}} = \frac{U - I(R_m + R_R)}{U - I \cdot R_m},$$

from where

$$\omega = \omega_{ac} \frac{U - I(R_m + R_R)}{U - I \cdot R_m}. \qquad (10.35)$$

Or in relative units

$$\omega^* = \omega_{ac} * \frac{1 - I * \cdot R *}{1 - I * \cdot R_m *}\ , \tag{10.36}$$

where $R^* = \dfrac{R_m + R_R}{R_{rat}}$ – the total resistance of the armature circuit in relative units;

$R_\partial * = \dfrac{R_m}{R_{rat}}$; $\omega * = \dfrac{\omega}{\omega_{rat}}$; $\omega_e * = \dfrac{\omega_e}{\omega_{rat}}$; $I^* = \dfrac{I}{I_{rat}}$.

Order of construction of rheostat characteristics is that, asking some random values of current I_1*, according to the natural characteristics they find ω_{e1}*. Then on (10.36) at a certain $R^* = R_1^*$, for which it is built a rheostat feature, and the same I_1* it is determined the desired value ω_1*. Similarly for other values of I* it is determined the desired value of the speed ω_2*, ω_3* and so on. Figure 10.22 shows the natural characteristic of the motor of serial actuation $R_д$* and rheostat R_1*. With increase of resistance of the armature circuit the motor speed at the same time is reduced and the characteristic shift downward. The rigidity of characteristic decreases with increase of additional resistance in the armature circuit. A feature of the mechanical characteristics of the motor is the inability to obtain the ideal mode of idle.

When the load is below 15–20 % of the rated work of the motor is practically inadmissible due to excessive speed of the armature.

The motors of serial actuation are widely used in pulling unit (crane motors) and transport (traction engines), where a large torque is required (especially at start).

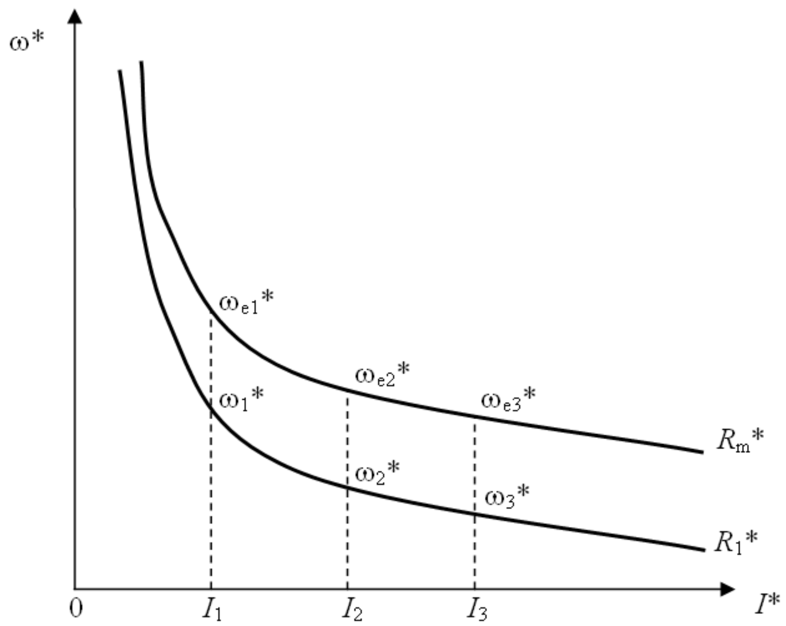


Figure 10.22: Electromechanical characteristics of the motor of DC of serial actuation

10.9.4 Motor of mixed actuation (fig. 10.23) has two windings: independent W_{act2} and serial W_{act1}, so its mechanical characteristics are intermediate between the corresponding characteristics of the motors of independent and serial actuation. Mechanical characteristic of the considered motor due to a change in magnetic flux when the load changes has no analytical expression, so in the calculations usually use natural universal characteristics of moment and speed from the armature current, which is given in the catalogue. Such characteristics in relative units are presented in figure 10.24.

Unlike motor of serial actuation motor of the mixed actuation has end value of the speed of ideal idle. This speed is determined only by the magnetic flux generated by magnetomotive force of independent winding, and equal:

$$\omega_0 = \frac{U}{k \cdot \Phi_0},$$

where $\boldsymbol{\Phi_0}$ – the magnetic flux created by the actuation current of independent winding.

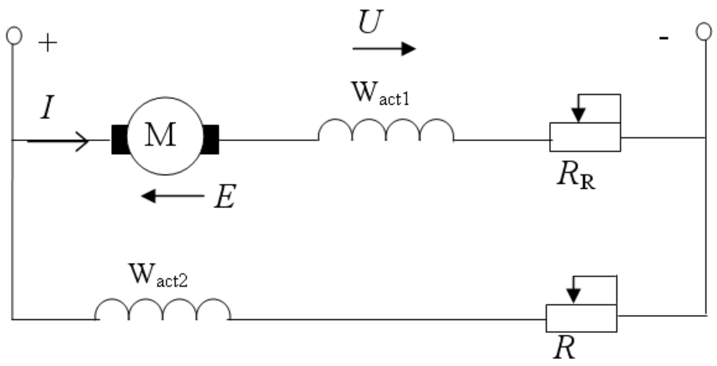


Figure 10.23: Circuit of a DC motor of mixed actuation.

The ratio of the magnetomotive forces of independent and serial windings are different for motors of different series. The most common is the ratio, which at rated current gives equal magnetomotive forces of the both windings of actuation. The motor speed of mixed excitation at low loads changes significantly, and then when the load increases it slowly decreases almost in a straight line, as the motor with independent actuation. This is due to the fact that at high loads saturation of the machine comes, and although magnetomotive force of serial winding increases, the magnetic flux does almost not change.

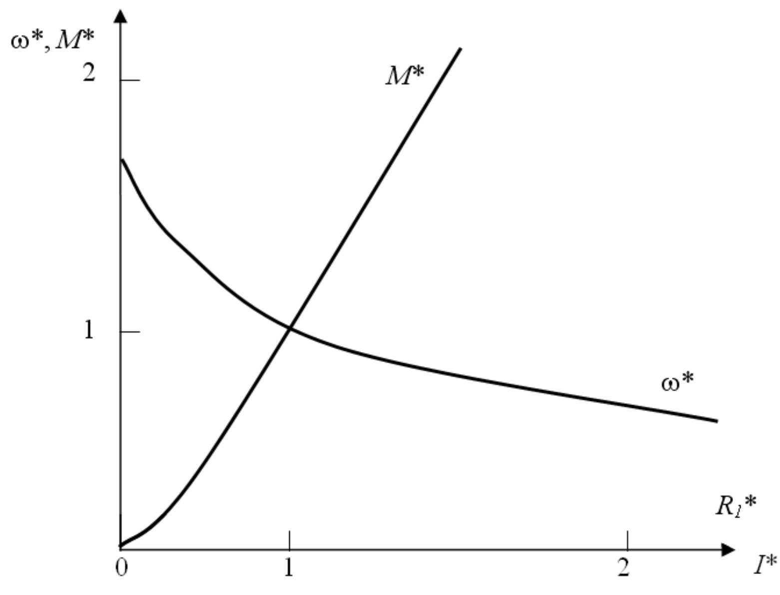


Figure 10.24: Dependence of the moment and angular speed from the current of armature for a DC motor of the mixed actuation

For calculation of the rheostat characteristics it order of construction of characteristics of the motor of serial actuation can be applied.

Key findings

1. DC electric machine can be operated in the generator mode, and in electric mode. The DC generator converts mechanical energy brought from the primary motor, into electrical energy. In the direct current electric motor electrical energy converts into rotational movement of the motor armature.
2. The reaction of armature shows the distortion of the main magnetic flux. The generation reduces EMF and electromagnetic torque is reduced in the engine.
3. To combat with arcing during commutation of sections of the armature winding additional poles are used, creating in a zone of commutation magnetic field directed toward the field of armature.
4. The main characteristics of DC machines are: for generator - external characteristic $U = f(I_a)$, for electrical motor – speed $\omega = f(I_a)$ and mechanical $\omega = f(M)$ characteristics.
5. To include DC machines in the electrical circuit schemes of independent, parallel, serial, and mixed actuation are used.

Control questions

1. Call the major parts of DC machine and explain their design.
2. Explain the principle of operation of DC generator and the destination of collector of the generator and the motor.
3. How do you change EMF of generator?
4. Explain the influence of the reaction of armature on the value of EMF of DC machine.
5. Explain the essence of commutation of DC machine.
6. How to reduce the harmful effect of the reaction of armature on the work of DC machine?
7. Classification of DC generators according to the method of actuation.
8. What are the characteristics of the generator with independent actuation?
9. Explain the view of external characteristic and the possibility of voltage regulation of generators with independent actuation.
10. Explain the principle and conditions of self-excitation of DC generators.
11. What are the characteristics of the electric circuit of the generator with parallel actuation?

12. What is final voltage determined by?

13. What are the features of the external characteristic of the generator with parallel actuation?

14. What are the advantages and disadvantages of the generator with serial actuation?

15. What view has the external characteristic of the generator with mixed actuation?

16. The disadvantages of the common generator with independent actuation as power amplifier.

17. Explain the principle of action of rotary amplifier of power with the diametrical field.

18. Explain the principle of action of DC motor.

19. Write the equations of the anti-EMF and the armature current of the motor.

20. Derive an equation of torque of the motor. How to change the direction of rotation of the motor armature?

21. Derive an equation for the speed of the engine and explain the possibility of its regulation.

22. What is the purpose of starting rheostat and how the magnitude of its resistance is selected?

23. Scheme features of engine with parallel actuation.

24. How is the motor speed regulated? Why breakage of circuit of actuation is dangerous for the motor?

25. How the speed of the motor is controlled by change of the voltage?

26. How the speed of the motor with serial actuation changes when the load changes on its shaft? Why work with a small load for the engine is inadmissible?

27. How the speed of the motor with serial actuation is regulated?

28. How to change torque and the motor speed with mixed actuation with increase of load?

29. Which of loss in machine of DC depends on the load? What loss are permanent?

11 ASYNCHRONOUS ELECTRIC MACHINES

Key concepts: a stator, a rotor, a rotating magnetic field, slip, motor (generative, opposition circuit) mode of work of asynchronous machine (AM), the transformation coefficient of the EMF of asynchronous motor (AMT), electrical balance of circuit of the stator (rotor), equivalent scheme of AMT, the full vector diagram of AMT, characteristic of idle, speed characteristic, mechanical characteristic.

Asynchronous machine mainly is used as motors. According to experts estimation from the total number of electric motors being in exploitation, on asynchronous motors it is fitted for 90–95%. Simplicity, high reliability in work, small overall size and low cost of three-phase AMT power exceeding 0.5 kilowatt cause their wide use in electric drives of metal-cutting lathes, lift-and-carry mechanisms, forge-and-press machines, pumps, blasts, compressors and other.

Asynchronous motors with power up to 0.5 kilowatt are implemented one- and two-phase. They are used in automation systems, household devices, in electrified instrument and other.

11.1 The device and principle of action of three-phase asynchronous machine

11.1.1 The device of asynchronous machine. Asynchronous machine, like any electrical machine, reversible and can work in the motor and generative modes. In most cases, the asynchronous machine are used as motors.

The main structural elements of the asynchronous motor means are a ***static stator and a rotating rotor*** (see fig. 11.1, and fig. 11.2). Between the stator and rotor is an air gap. The stator consists of the base 1, which is simultaneously the motor box, and fixed in it magnetic core 2 and winding 3.

The magnetic core of the stator, which represents by itself the major part of the magnetic circuit of the machine, collect from stamped isolated from each other electrical steel sheets with a thickness of 0.35–0.5 mm On the inner cylindrical surface of the magnetic core has grooves into which go the conductors of the stator winding. To the base are attached to the two side board 4 with a through central holes for the bearings of the rotor shaft.

The rotor of an asynchronous motor 5 (fig. 11.2) consists of a pack of magnetic core collected from stamped sheets of electrical steel and windings. Got on shaft 6 package of the magnetic core has the form of a cylinder, on the outer surface of which it was implemented the grooves in which is placed a winding.

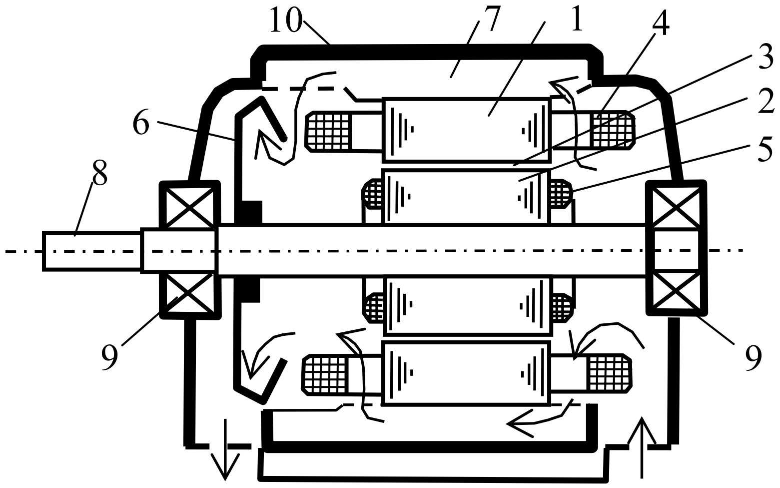


Figure 11.1: The device of asynchronous motor: 1 – stator core;
2 – core of the rotor; 3 – air gap; 4 – winding of stator; 5 – rotor winding;
6 – blast; 7 – ventilation canals (arrows show the direction of movement
of the cooling air on channels); 8 – shaft; 9 – bearings; 10 – base

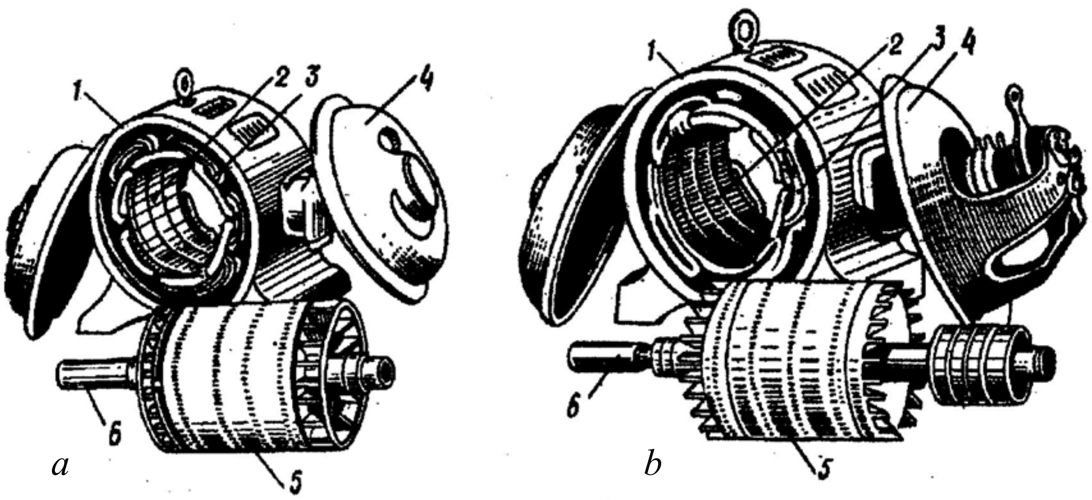


Figure 11.2: Structural elements of : a – the with short-circuit rotor;
b – with phase rotor

Depending on the type of winding, the rotor of AMT may be short-circuit a or phase. In the grooves of the short-circuit rotors copper rods were packed up, connecting from butts by short-circuit rings; this winding has the appearance of a "squirrel cage" (fig. 11.3*, a*). A simplified visual representation of asynchronous motor with short-circuit rotor shown in figure 11.3, *a*.

In the grooves of the phase rotor stack conductors of three-phase winding sections, usually connected in a star (fig. 11.3, *b*). The free terminals of the phase windings of the rotor (denoted P_1, P_2, P_3) join to three (on number of phases) isolated from each other the contact rings. On rings it is put in reinforced in the brush holders brushes, with which the circuit of rotating rotor winding is connected with the adjusting or starting rheostat. A conditional visual representation of asynchronous motor with phase rotor is shown in figure 11.3, *b*.

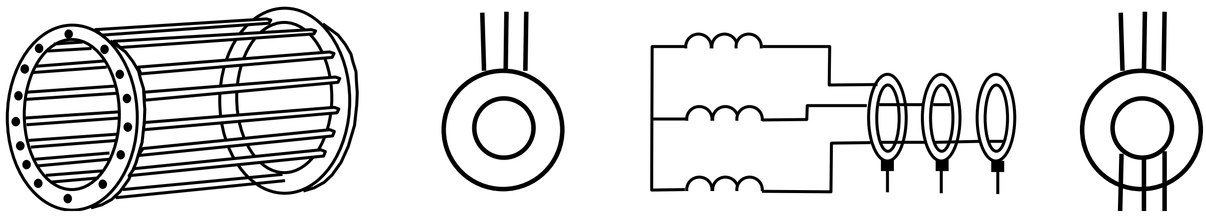


Figure 11.3: Windings of rotor of AMT: a - short-circuit; b – with phase rotor.

11.1.2 The principle of work of AM bases on the use of a rotating magnetic field. When the network connection of three-phase winding of the stator it is created a rotating magnetic field, the angular speed which is determined by the frequency *f* and the number of pairs of poles of the winding *p*

$$\omega = \frac{2\pi \cdot f}{p}\text{, radian /second.}$$

Crossing the conductors of the winding of stator and rotor, this field according to the law of electromagnetic induction induces an EMF in the windings. On closed winding of the rotor in its circuit current carries, which in result from the interaction with the resulting magnetic field creates an electromagnetic torque on the shaft. If this moment exceeds the resistance moment on the motor shaft, the shaft starts to rotate and drives the operating mechanism. Usually the angular speed of the rotor ω_2 is not equal to the angular speed of the magnetic field ω_1, called synchronous. Hence the name of the asynchronous motor, i.e., non-synchronous.

The work of the asynchronous machine is characterized by the ***slip s***, which represents ***the relative difference of the angular speeds of the field ω_1 and ω_2 rotor***:

$$s = \frac{\omega_1 - \omega_2}{\omega_1} \quad . \tag{11.1}$$

The value and sign of the slip dependent from angular speed of the rotor relative to the magnetic field, determine the mode of operation of the asynchronous machine. Thus, in the ideal mode of idling rotor and the magnetic field rotate with the same frequency in the same direction, i.e., the rotor is stationary relative to the rotating magnetic field, and the slip *s* is equal to zero. The EMF in the rotor winding is not induced, rotor current and electromagnetic torque of the machine is equal to zero. When you start the first time the rotor is stationary: $\omega_2 = 0$, $s = 1$, thus, the slip in the motor mode changes from $s = 1$ at startup to $s = 0$ in the ideal mode of idle.

When the rotor rotation with speed $\omega_2 > \omega_1$ in the direction of rotation of the magnetic field, the slip becomes negative. The machine goes in the generative mode and develops braking torque. During rotation of the rotor in the direction opposite to the direction of rotation of the magnetic field (s > 1), the asynchronous machine enters in a mode of opposition circuit and also develops the braking torque. Thus, depending on the slip they distinguish ***motor*** ($s = 1 \div 0$), ***generative*** ($s = 0 \div -\infty$) ***modes*** and ***the mode of opposition circuit*** ($s = 1 \div +\infty$). The generative modes and the opposition circuit are used for braking of asynchronous motors.

In modern AMT, depending on their type, at rated load slip is $s_{\text{nom}} = 0{,}015 \div 0{,}07$.

11.1.3 The circuit connections of the windings of the stator. The beginning of phase windings are denoted C_1, C_2, C_3; ends – C_4, C_5, C_6. To enable the motor in the circuit the stator winding are connected in a star or triangle. A question about the connection scheme decides depending on line voltage of circuit and nominal phase voltage of windings od stator. The instructions about it are given in the ticket of the motor. On the connection scheme in the star (see fig. 11.4, *a*) all three ends of the phase windings C_4, C_5, C_6 connect in the zero point, at the connection scheme in the triangle (see fig. 11.4, *b*) interconnect between each other in pairs beginning and ends of adjacent phases: C1 – C6, C2 – C4, C3 – C5 The network joins in the first case to the three beginnings C1, C2, C3, the second - to the total points of C1 – C6, C2 – C4, C3 – C5.

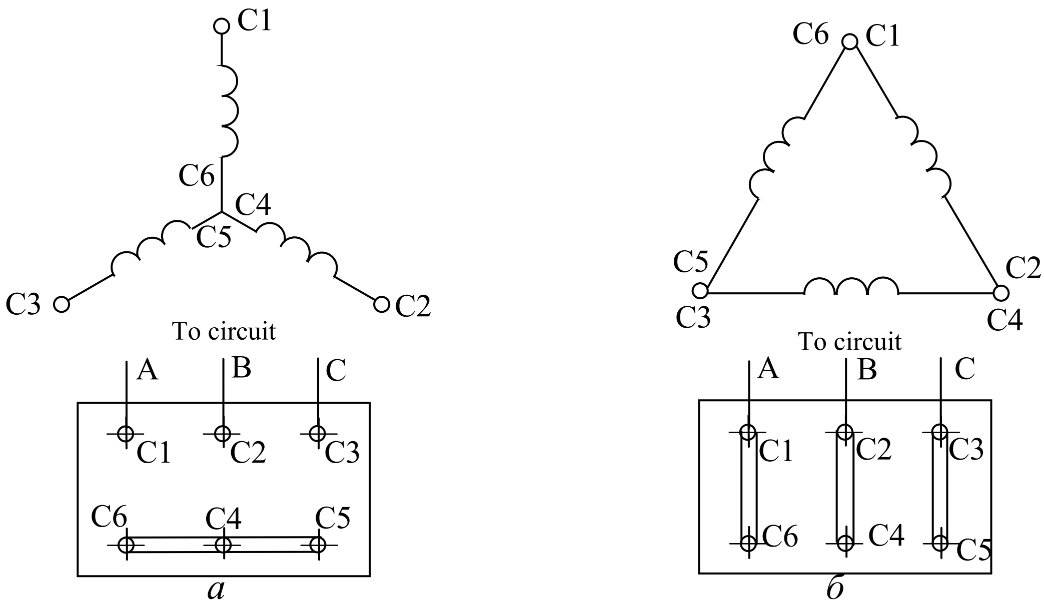


Figure 11.4: Scheme of connection of the phase windings of AMT:
a – star; *b* – triangle

The concept of "beginning" and "end" of phase windings are conditional, however, they are necessary for the proper connection of the windings in the scheme. For one of the phase windings it can be arbitrarily choose its beginning (C1), but then for the other two for the beginning it is necessary to accept such contacts, going from which winding wind up in the same that the first one directions. In the assembled motor to install correspondence of contacts almost impossible. In the absence of signs of the contacts of the stator for their markup, you can perform the following measurements. If the two windings of stator connect serially, then it is possible two versions of inclusion – accordant connection, when it is connected the beginning and the end (fig. 11.5, *a*), and opposite, when the windings are connected by their ends or beginnings (fig. 11.5, *b*). The resulting magnetic flux of both windings $f = F1 + F2$ will be differently oriented relative to the third winding. In the first case it will be pierced by the resulting stream, which induct in it EMF. The voltmeter on its connection terminals will measure the voltage U_2 is equal to about half of the brought U_1. In the second case, the resulting stream will close in the plane of the third winding – the voltmeter shows zero.

Thus, on the readings of the voltmeter it is settled the same connection termminals of the first two windings. After marking one of them changes place with the third and the experience is repeated to determine the same connection terminals of the third winding. During the experiment it is necessary to use low voltage ($U \approx 0.3 \cdot U_{nom}$).

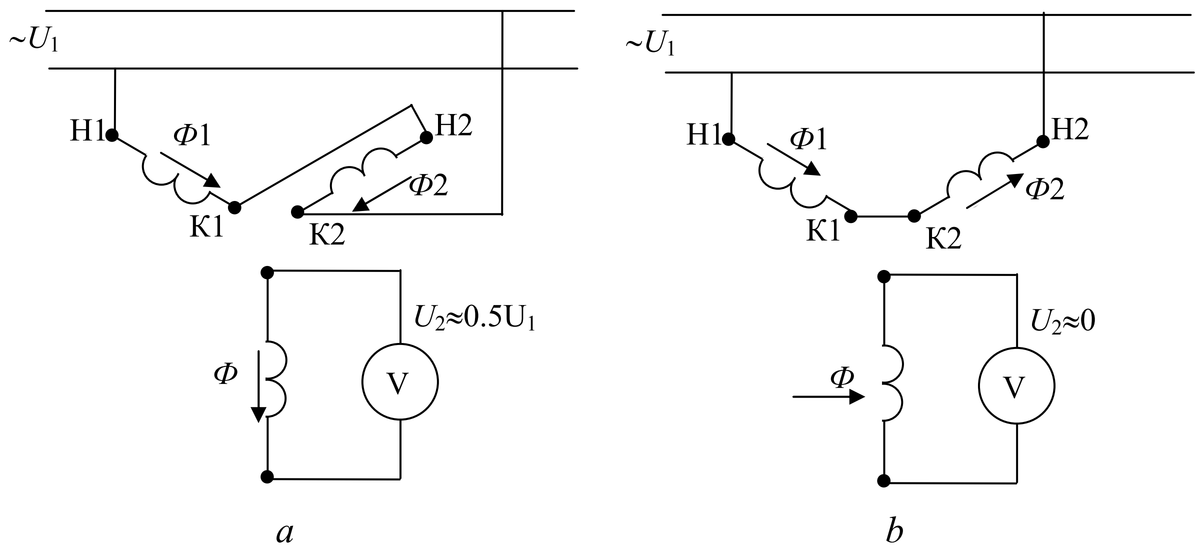


Figure 11.5: Definition scheme of beginnings and ends of the windings of the stator

11.2 The EMF of the stator and rotor

A rotating magnetic field in each of the windings of the stator and rotor induct variable EMF. Due to the distribution of each winding in the grooves it is provided almost sinusoidal shape of the curves EMF in the windings, but in the result of the EMF of the individual sections winding have different phases and they should be sum up by vector. Therefore, the total EMF phase winding is less than the arithmetic sum of the EMF separate sections. To account for this circumstance in the formula EMF of the motor it is introduced winding coefficient k, which is assumed to be less than or equal to one. Thus, EMF of phase stator winding is determined by the formula

$$E_1 = 4{,}44 \cdot f_1 \cdot w_1 \cdot k_1 \cdot \Phi \,, \tag{11.2}$$

where: $\boldsymbol{f_1}$ – the frequency of the stator current;
$\boldsymbol{w_1}$ – the number of turns of the phase;
$\boldsymbol{k_1}$ – the winding coefficient of the stator ($k_1 = 0.9 \div 0.95$);
$\boldsymbol{\Phi}$ – the magnetic flux of the rotating field.

The same formula is determined the EMF of phase of rotor

$$E_2 = 4{,}44 \cdot f_2 \cdot w_2 \cdot k_2 \cdot \Phi \,, \tag{11.3}$$

where: $\boldsymbol{f_2}$ – frequency of rotor current;
$\boldsymbol{w_2}$ – the number of turns of the phase of the rotor;
$\boldsymbol{k_2}$ – the winding coefficient of the rotor. In the case of short-circuit rotor $k_2 = 1$.

Magnetic field, rotating in space with the synchronous speed n_1, relative to the rotating rotor has speed

$$n_s = n_1 - n_2. \tag{11.4}$$

For each turn of the field relative to the rotor phase its EMF is changed to p 360 electrical degrees, which corresponds to p full periods of EMF. The number of periods per second, i.e., the frequency of the EMF of the rotor, is equal to

$$f_2 = \frac{n_s \cdot p}{60} \qquad \text{or} \qquad f_2 = f_1 \cdot s \,, \tag{11.5}$$

i.e. the frequency of the EMF of the rotor, and hence the magnitude of the EMF E2 (11.3) is proportional to the slip. The highest frequency EMF rotor will be in the initial start, when $s = 1$. Meanwhile It is equal to the frequency of supplying circuit f1. With the acceleration of the motor slip decreases and the frequency of rotor current reduces. In the working mode of the motor, supplied by a current of frequency $f_1 = 50$ Hz, the frequency of the rotor current is 1–2 Hz. When $s =1$, $f_2 = f_1$, then the EMF of the stationary rotor has the form

$$E_{2st} = 4{,}44 \cdot \mathrm{f}_1 \cdot w_2 \cdot \mathrm{k}_2 \cdot \Phi \,. \tag{11.6}$$

Have substituted E_{2st} in the formula (11.3), we obtain the electromotive force of the rotating rotor:

$$E_2 = E_{2st} \cdot s \,. \tag{11.7}$$

Thus, the EMF E_2 as its frequency, with the acceleration of the rotor is reduced. The ratio of the EMF of the stator E_1 (11.2) to EMF of stationary rotor E_{2H} (11.6) is called the ***transformation coefficient of the EMF of asynchronous motor***, which is equal to

$$k_e = \frac{E_1}{E_{2st}} = \frac{w_1 \cdot k_1}{w_2 \cdot k_2} \,. \tag{11.8}$$

The EMF of the stator and rotor E_1 and E_2 are created by the main rotating flow. This thread is closed through the air gap between the stator and the rotor. In addition to the main stream, ***each of the windings creates a thread, coherent only with itself***. This thread is called ***leakage flux***. The latter one is closed through the slots and around the frontal parts of the windings. Considering that leakage fluxes go mainly through the air, can them consider proportional to the currents in the windings. Leakage fluxes induce in the windings of stator and rotor leakage voltage E_{p1} and E_{p2}. These EMF can be considered as the inductive voltage drop in the windings:

$$E_{p1} = -I_1 \cdot X_1 = - I_1 \cdot \omega_1 \cdot L_1 \ ; \quad (11.9)$$

$$E_{p2} = -I_2 \cdot X_2 = - I_2 \cdot \omega_2 \cdot L_2 \ , \quad (11.10)$$

where: $\boldsymbol{I_1}$ and $\boldsymbol{I_2}$ – the currents of the stator and rotor;
$\boldsymbol{X_1}$ and $\boldsymbol{X_2}$ – the inductive resistance of stator and rotor windings;
$\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ – the leakage inductances of stator and rotor.

Since we assume a proportional dependency between leakage fluxes and currents, creating them, the leakage inductance L_1 and L_2 are constant values. They depend on the design features of the windings, form of the grooves of the stator and rotor. Since the frequency of the rotor is not constant, but depends on the slip, dependent from slip are found inductive reactance of the rotor

$$X_2 = \omega_2 \cdot L_2 = 2\pi \cdot f_2 \cdot L_2 = 2\pi \cdot f_1 \cdot L_2 \cdot s,$$

or
$$X_2 = X_{2st} \cdot s \ , \quad (11.11)$$

where $\boldsymbol{X_{2st}} = \boldsymbol{2\pi \cdot f_1 \cdot L_2}$ – the inductive reactance of the winding of the stationary rotor индуктивное сопротивление обмотки неподвижного ротора (at s =1).

11.3 The equations of the electric equilibrium of the stator and rotor

The EMF of each phase of the stator winding are balanced by the applied voltage of circuit U_1. In addition, by voltage circuit it is covered a voltage drop in the active resistance of the stator winding R_1, created by the stator current I_1 and is equal to $I_1 \cdot R_1$. On this basis we can write the equation of voltages or, in other words, ***the equation of the electric equilibrium of the stator***:

$$\overline{U}_1 + \overline{E}_1 + \overline{E}_{P1} - \overline{I}_1 \cdot R_1 = 0 \ ,$$

or
$$\overline{U}_1 = -\overline{E}_1 - \overline{E}_{P1} + \overline{I}_1 \cdot R_1 \ . \quad (11.12)$$

On phase E_1 lags from the rotating flow of a quarter period or at a 90° angle. Leakage voltage E_{p1} lags at a 90° angle from the current I_1. The active voltage drop $I_1 \cdot R_1$ coincides with the current phase.

As $E_{p1} = I_1 \cdot X_1$ then

$$\overline{U}_1 = -\overline{E}_1 + \overline{I}_1 \cdot R_1 + \overline{I}_1 \cdot X_1 \ , \quad (11.13)$$

or
$$\overline{U}_1 = -\overline{E}_1 + \overline{I}_1 \cdot Z_1 \ , \quad (11.14)$$

where $\boldsymbol{Z_1} = \sqrt{\boldsymbol{R_1^2} + \boldsymbol{X_1^2}}$ – impedance of phase of stator.

In the complex form of the equation of the electric equilibrium of the stator has the form

$$\dot{U}_1 = -\dot{E}_1 + \dot{I}_1 (R_1 + jX_1) = -\dot{E}_1 + \dot{I}_1 \cdot \underline{Z}_1 \ . \quad (11.15)$$

As the resistance of the phase windings of stator Z_1 usually small, we can neglect the voltage drop $I_1 \cdot Z_1$, then

$$U_1 \approx E_1 \equiv \Phi . \qquad . \tag{11.16}$$

Therefore, when a constant voltage of circuit rotating magnetic flux is almost constant, independent from load. This is only valid for small values of currents of the stator.

The rotating magnetic flux induct in the phases of the rotor EMF E_2, which causes the current I_2. Rotor current creates a magnetic flux, an leakage voltage $E_{p2} = - I_2 \cdot X_2$ and the voltage drop in the active resistance of the rotor $I_2 \cdot R_2$.

Thus, ***the equation of the electric equilibrium*** of the rotor has the form

$$\overline{E}_2 + \overline{E}_{p2} = \overline{I}_2 \cdot R_2 ,$$

or

$$\overline{E}_2 = \overline{I}_2 \cdot R_2 + \overline{I}_2 \cdot X_2 = \overline{I}_2 \cdot Z_2 , \tag{11.17}$$

where $\boldsymbol{Z_2} = \sqrt{\boldsymbol{R_2^2} + \boldsymbol{X_2^2}}$ – impedance of phase rotor.

In a complex writing

$$\dot{E}_2 = \dot{I}_2 \cdot (R_2 + jX_2) = \dot{I}_2 \cdot \underline{Z}_2 . \tag{11.18}$$

This equation corresponds to the vector diagram of the rotor shown on figure 11.6.

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} . \tag{11.19}$$

The values E_2 and X_2 depend on the slip. Considering that $E_2 = E_{2st} \cdot s$ and $X_2 = X_{2st} \cdot s$, get

$$I_2 = \frac{E_{2st} \cdot s}{\sqrt{R_2^2 + X_2^2 \cdot s^2}} = \frac{E_{2st}}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_{2st}^2}} . \tag{11.20}$$

In this formula variable, dependent on the load variable is the slip s. The formula shows that the rotor current will be maximum when the maximum slip, i.e., at the initial start of the motor, when s =1. With decrease of slip rotor current decreases.

Typically, $R_2 < X_{2st}$ therefore a member of the R_2/s under the root in the denominator of the formula of current has a significant impact on the amount of current only at small slipes.

The phase of the current relative to the EMF E_2 also depends on the slip

$$tg\psi_2 = \frac{X_2}{R_2} = \frac{X_{2S} \cdot s}{R_2} . \tag{11.21}$$

At low sliding angle ψ_2 is small. On the basis of (11.20) the electrical circuit of the rotor can be represented by a circuit consisting of active R_2/s and inductive X_{2st} (instead of the real R_2 and X_2) resistance to which applied voltage U, is equal to the EMF of the stationary rotor E_{2st} (fig. 11.7, *a*). Vector diagram of this scheme rotor shown in figure 11.7, *b*.

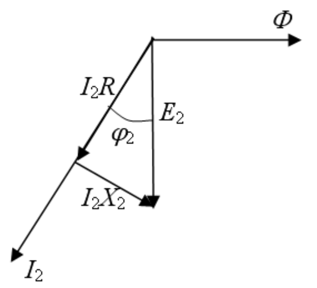
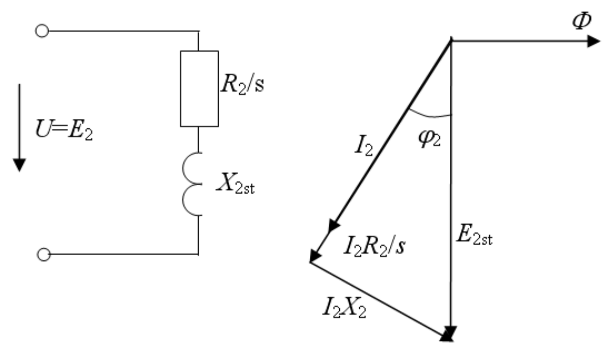


Figure 11.6: Vector diagram for equation (11.18)



a *b*

Figure 11.7: Circuit rotor (*a*) and vector chart (*b*) in the case of small slipes

11.4 Equation of magnetizing forces and currents

The rotating magnetic flux of the motor in working mode is created by the joint action of the magnetizing forces of the stator and rotor, which depend on the number of phases, number of turns, the magnitudes of the currents in the windings, as well as the distribution of the windings in the grooves.

Magnetizing force of the stator is proportional to $m_1 \cdot I_1 \cdot w_1 \cdot k_1$, and rotor – $m_2 \cdot I_2 \cdot w_2 \cdot k_2$, where m_1 – the number of stator phases, m_2 – the number of phases of the rotor, k_1, k_2 – correction coefficients.

Magnetizing force of the stator rotates in space with the synchronous speed determined by the frequency of the supply current and the number of pairs of poles, $n_1 = \dfrac{f_1 \cdot 60}{p}$. Rotor current has a frequency f_2. Its magnetization is rotated relative to the rotor speed $n_s = \dfrac{f_2 \cdot 60}{p}$.

The rotor rotates in space with the speed n_2. Therefore, magnetizing force of the rotor in the space rotate with speed $n_2 + n_s = n_1$, i.e., synchronous speed.

Thus, the magnetizing forces of the stator and rotor in space rotate with the same synchronous speed. In relation to each other they appear to be stationary. This circumstance allows you to sum up them geometrically independently of the stationary rotor or rotating. The geometric sum of the magnetizing forces of the stator and rotor determines the resultant magnetizing force, which creates a rotating flow in the working mode. It is equal to

$$m_1 \cdot \dot{I}_0 \cdot w_1 \cdot k_1 = m_1 \cdot \dot{I}_1 \cdot w_1 \cdot k_1 + m_2 \cdot \dot{I}_2 \cdot w_2 \cdot k_2 . \tag{11.22}$$

This equation is called ***the equation of magnetizing forces of asynchronous motor.***

On the basis of equation of magnetizing force of the motor stator current can be divided into two parts

$$\dot{I}_1 = \dot{I}_0 - \dot{I}_2 \frac{m_2 \cdot w_2 \cdot k_2}{m_1 \cdot w_1 \cdot k_1} .$$

Have denoted $$I_2 \frac{m_2 \cdot w_2 \cdot k_2}{m_1 \cdot w_1 \cdot k_1} = I_2 \cdot \frac{1}{k_i} = I_2^{'} , \tag{11.23}$$

get $$\dot{I}_1 = \dot{I}_0 + (-\dot{I}_2^{'}) , \tag{11.24}$$

where: $\boldsymbol{I_0}$ – magnetizing current of stator;

$\boldsymbol{I'_2}$ – reduced current of the rotor, i.e. the part of the stator current, which balances the demagnetizing effect of the rotor;

$\boldsymbol{k}_{\mathbf{i}} = \dfrac{\boldsymbol{m_1 \cdot w_1 \cdot k_1}}{\boldsymbol{m_2 \cdot w_2 \cdot k_2}}$ – the coefficient of transformation of the currents of AMT.

From equation (11.16) it follows that the rotating magnetic flux is mainly determined by the circuit voltage. Consequently, from the voltage it depends the magnetizing component of the stator current I_0. If U_1 = const, and $I_0 \approx$ const, i.e., we can assume that the current I_0 practically does not depend on motor load. When the load changes on the shaft the stator current is changed due to its component – *I'2*. Electromagnetic phenomena by which accompanied the transition of the asynchronous motor from one load to another carry the same as at the transformer on change its secondary load.

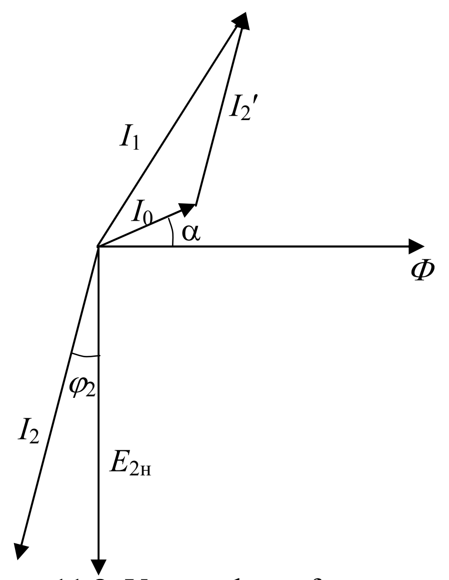


Figure 11.8: Vector chart of currents

According to equation (11.24), we can construct the vector diagram of the currents (fig. 11.8). Its build is more convenient to start with a vector of rotating flow *Φ*. Current I_0 due to loss in the magnetic core on eddy currents and hysteresis is ahead of the flow *Φ* to the loss angle *α*. For a given flow of current I_0 and the angle α are set on the magnetic characteristics of the magnetic core. The magnitude and phase $-\dot{I}_2^{'}$ can be determined by the magnitude and phase of the rotor current $\dot{I}_2$ using expressions (11.20), (11.21), (11.23). The sum $\dot{I}_0 + (-\dot{I}_2^{'})$ gives the stator current $\dot{I}_1$. The share of the magnetizing current $-I_2'$ in the stator of AMT compared with the share of idle current of the transformer in primary current is much greater, as the magnetic core of the motor has an air gap.

11.5 The equivalent scheme of an asynchronous motor

Convert the equation of the stress of the rotor (11.18), have divided both parts on the slip

$$\frac{\dot{E}_2}{s} = \dot{I}_2 \frac{R_2}{s} + \dot{I}_2 \cdot j \frac{X_2}{s} . \tag{11.25}$$

Taking into account, that $\frac{\dot{E}_2}{s} = \dot{E}_{2S}$, but $\frac{X_2}{s} = X_{2S}$, it's possible to write

$$\dot{E}_{2S} = \dot{I}_2 \frac{R_2}{s} + \dot{I}_2 \cdot jX_{2S} \quad . \tag{11.26}$$

Taking into account that $I_2 = I_2^{'} k_i$, where $I_2^{'}$ – reduced current rotor, i.e. the part of the stator current, which balances the demagnetizing effect of the rotor current; $k_i = \frac{m_1 \cdot w_1 \cdot k_1}{m_2 \cdot w_2 \cdot k_2}$ – the coefficient of transformation of the currents of an asynchronous motor, multiplying (11.26) in the coefficient of transformation of EMF k_e (11.8), we obtain

$$\dot{E}_{2st} \cdot k_e = \dot{I}_2^{'} \frac{R_2 \cdot k_e \cdot k_i}{s} + \dot{I}_2^{'} \cdot jx_{2st} \cdot k_e \cdot k_i \, ,$$

or

$$\dot{E}_2^{'} = \dot{I}_2^{'} \frac{R_2^{'}}{s} + \dot{I}_2^{'} \cdot jX_{2st}^{'} \, , \tag{11.27}$$

where: $\boldsymbol{E_2^{'} = E_{2st} \cdot k_e}$ – reduced to the stator EMF of stationary rotor, it is equal to the EMF of the stator E_1;

$\boldsymbol{R_2^{'} = R_2 \cdot k = R_2 \cdot k_e \cdot k_i}$ – reduced to stator active resistance of rotor winding;

$\boldsymbol{X_{2st}^{'} = X_{2st} \cdot k = X_{2st} \cdot k_e \cdot k_i}$ – reduced inductive resistance of the stationary rotor;

$\boldsymbol{k = k_e \cdot k_i = \frac{m_1 \cdot w_1^2 \cdot k_1^2 \cdot k^2}{m_2 \cdot w_2^2 \cdot k^2}}$ – the coefficient of reduction of the resistance of the rotor to the stator.

Such replacement of the actual magnitude of the rotor reduced does not change power relations in it, but you can go from electromagnetic coupling between the circuits of the rotor and stator to an electrical connection between them.

On the basis of the equations of stator voltages (11.14) and rotor (11.27) and equation currents (11.24) ***AMT can be represented by the electrical equivalent scheme*** shown in figure 11.9.

This circuit corresponds to the full vector diagram of an asynchronous motor, which is shown in figure 11.10. And the equivalent scheme and vector diagram satisfy the equations of voltages and currents. Branch of scheme with current I_0 is called the magnetization branch. Resistances R_0 and X_0 are determined by the magnetic properties of the magnetic core. The voltage drop caused by the current I_0 is equal and opposite to the phase of the stator EMF E_1 and reduced EMF of rotor E'_2:

$$\dot{I}_0 \cdot R_0 + \dot{I}_0 \cdot jX_0 = -\dot{E}_1 = -\dot{E}_2^{'} \quad . \tag{11.28}$$

The equivalent scheme parameters can be set by calculation or experimental data. For AMT, and as for transformers, are conducted experiments of idle and short circuit.

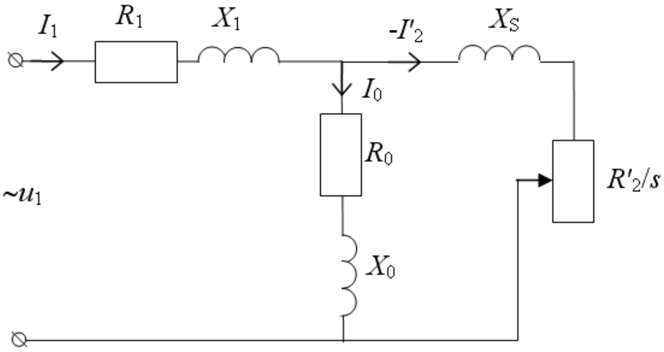


Figure 11.9: Equivalent scheme of AMT

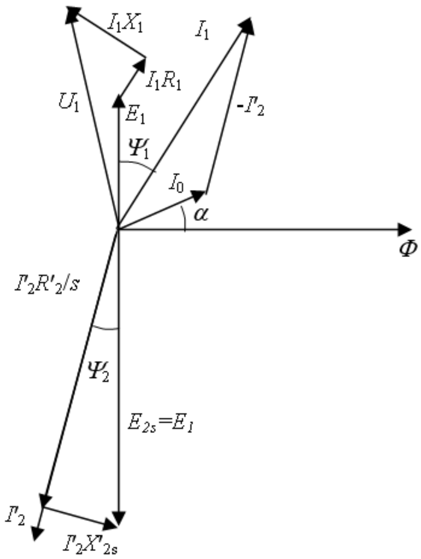


Figure 11.10: Complete vector diagram of AMT

11.6 Loss and COP of asynchronous electrical motor

In AMT the electrical energy is converted into mechanical. In the process of such conversion part of it it losts in the motor. The power P_1 consumed by the stator winding from the circuit, is spent on loss in the stator and loss in the rotor of the electric motor.

Loss in the stator are formed of the power of loss in the stator core on hysteresis and eddy current (loss in steel of stator) P_s and the power of electric loss in the stator winding $P_{e.s}$ with its heat by current

$$P_{e.s.} = m_1 \cdot I_1^2 \cdot R_1 , \tag{11.29}$$

where: $\boldsymbol{m_1}$ – the number of phases of the stator;
$\boldsymbol{I_1}$ – the stator current;
$\boldsymbol{R_1}$ – active resistance of phase of stator.

The remaining part $P_{E.P.}$ is called an electromagnetic power of motor. It has a rotating field. Some its part composes the electrical loss in the rotor $P_{E.R.}$:

$$P_{E.R.} = m_2 \cdot I_2^2 \cdot R_2 , \tag{11.30}$$

where: $\boldsymbol{I_2}$ – rotor current;
$\boldsymbol{R_2}$ – the active resistance of phase of the rotor;
$\boldsymbol{m_2}$ – the number of phases of the rotor.

At motors with contact rings $m_2 = m_1$. The short-circuit motor rotor is multiphase. Its number of phases equal to the number of rod of cells per one pair of poles. If the number of grooves of the rotor Z_2, the number of phases $m_2 = Z_2/p$. Loss in the steel of the rotor because of the low frequency rotor current can be neglected. The remaining part of the electromagnetic power is converted into mechanical P'_2. If you subtract from it the power of the mechanical loss P_{mech}, due to friction in the bearings and on the air, and the power of incremental loss P_{inc}, associated with the pulsation of the magnetic flux in the tooth zone of magnetic core and with leakage fields, then we will provide mechanical power on shaft P_2. Then COP of motor

$$\eta = \frac{P_2}{P_1} = \frac{P_1 - \Delta P}{P_1} \quad . \tag{11.31}$$

where $\boldsymbol{\Delta P}$ – total loss in the motor.

Some of the loss (e.g. loss in steel) are virtually independent of the load; others (for example, the electrical windings in the stator and rotor) are associated with the load. In general, with increase of load loss increase, and hence the heat of the motor. Maximum mechanical power on the shaft, which the motor can develop long-term (indefinitely), not overheating over allowable temperature, is called the ***rated power of the motor***. It is indicated in the ticket. COP of motor at rated load largely depends on the rated power of the motor. The larger the power of motor, the less the relative loss in it and more COP. For large asynchronous motors rated COP is very high and reaches values of 0.9 to 0.97, while for engines of small capacity (about 1 kilowatt) is equal to 0.7 to 0.8. In addition, on equal status rated COP higher at the motor with a higher synchronous speed.

The electromagnetic power of the motor P_{EM} can be represented by the product of the moment of electromagnetic forces and angular speed of field

$$P_{EM} = M \cdot \omega_1 = M \frac{2\pi \cdot n_1}{60} \quad . \tag{11.32}$$

In turn, the mechanical power of the rotor $P^{'}_2$ is equal to the product of moment on the angular speed of the rotor

$$P^{'}_2 = M \cdot \omega_2 = M \frac{2\pi \cdot n_2}{60} \quad . \tag{11.33}$$

The electromagnetic power of the rotor:

$$P_{E.R.} = P_{EM} - P^{'}_2 = M \frac{2\pi}{60}(n_1 - n_2) = M \frac{2\pi \cdot n_1}{60} \frac{n_1 - n_2}{n_1} = M \cdot \omega_1 \cdot s = P_{EM} \cdot s \ . \tag{11.34}$$

When the motor is started, when s = 1, the electromagnetic power P_{EM} is equal to the power of loss in the rotor $P_{E.R.}$, with the acceleration – loss in the rotor decrease.

Thus, the slip s is a measure of the loss in the rotor.

In normal mode of work of the motor a small value s of the order of a few percent is an important condition of its economy.

11.7 Torque of asynchronous motor

Let's express the torque through loss in the rotor and slipe

$$M = \frac{P_{Э.Р}}{\omega_1 \cdot s} = \frac{m_2 \cdot I_2^2 \cdot R_2}{\omega_1 \cdot s} \quad . \tag{11.35}$$

In accordance with the vector diagram (fig. 11.6) $I_2 \cdot R_2 = E_2 \cdot \cos\psi_2$, and formulas (11.6) and (11.7) the EMF of the rotor is equal to

$$E_2 = E_{2S} \cdot s = 4{,}44 f_1 \cdot w_2 \cdot k_2 \cdot \Phi \cdot s \, . \tag{11.36}$$

Substituting (11.36) in the formula for the moment, will get

$$M = \frac{4{,}44 \cdot m_2 \cdot f_1 \cdot w_2 \cdot k_2}{\omega_1} \Phi \cdot I_2 \cdot \cos\psi_2 = C \cdot \Phi \cdot I_2 \cdot \cos\psi_2 \ , \qquad (11.37)$$

where $C = \dfrac{4{,}44 m_2 \cdot f_1 \cdot w_2 \cdot k_2}{\omega_1}$ – a constant value for a given motor.

Thus, torque is proportional to the product of flux, the rotor current and the cosine of the phase angle between the current and the EMF of the rotor.

Values Φ, I_2 and $\cos\psi_2$ to a greater or lesser extent dependent on the slip, which leads to dependence on the slip torque. To clarify this relationship is transformed source expression of the moment (11.35), using formula (11.8) and (11.20):

$$M = \frac{m_2 \cdot I_2^2 \cdot R_2}{\omega_1 \cdot s} = \frac{m_2}{\omega_1 \cdot s} \cdot \frac{E_{2st}^2 \cdot s^2 \cdot R_2}{R_2^2 + x_{2st}^2 \cdot s^2} = \frac{m_2}{\omega_1 \cdot k_e^2} \cdot \frac{E_1^2 \cdot s \cdot R_2}{R_2^2 + X_{2st}^2 \cdot s^2} .$$

Have denoted $\dfrac{m_2}{\omega_1 \cdot k_e^2} = C_1$ a constant get

$$M = C_1 \cdot E_1^2 \frac{s \cdot R_2}{R_2^2 + X_{2st}^2 \cdot s^2} . \qquad (11.38)$$

If we neglect the voltage drop in the stator winding $I_1 \cdot Z_1$, it is possible EMF E_1 to replace by the voltage U_1 (11.16). This assumption, however, is only possible within certain limits. At high loads and slipes the voltage drop in the stator winding strongly increases and the difference between the voltage and EMF can be significant. With that said, the formula of the moment takes the form

$$M = C_1 \cdot U_1^2 \frac{s \cdot R_2}{R_2^2 + X_{2st}^2 \cdot s^2} . \qquad (11.39)$$

When U_1 = const, which is usually have place in the normal mode of operation of the motor, torque is a function of the slip, whose graph is depicted in curve 1 (fig. 11.11).

From the formula (11.39) and it follows that at small slips ($s < s_C$) moment approximately proportional to the slip. With increase of slip it becomes larger the influence of s in the denominator of the formula (11.39) and moment, reaching its highest value of M_c at slip $s = s_c$, further decreases to a value of the starting moment M_s when $s = 1$. ***The maximum moment M_c and slip s_c in which it is developed are called critical***.

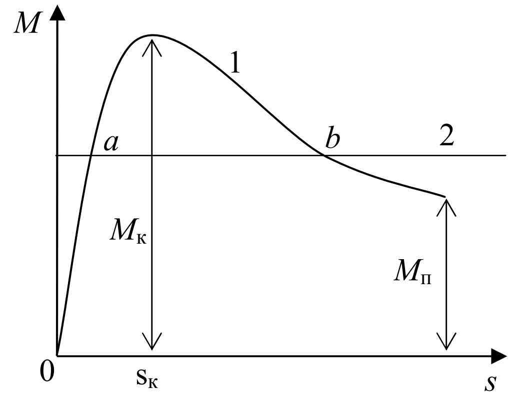


Figure 11.11: Mechanical characteristic of AMT

The motor works with established, i.e. constant, the rotation speed when the balance of torque and drag torque. The curve of the drag torque M_{DT} depending on the slip (or speed) is determined by the properties of the working mechanism. If $M_{DT}=F(s)$ has form of curve 2 (see fig. 11.11), the condition of equilibrium of moments observed at two different slips - point a, to which corresponds to a small slip $s < s_C$, and point b while slip $s > s_c$. However, for the stable operation of the motor it is not enough simple equilibrium of moments. It is necessary that the motor was able to restore the balance when possible its violations. This is possible only if

$$\frac{dM}{ds} > \frac{dM_{DT}}{ds} \ . \tag{11.40}$$

As can be seen from figure 11.11, the indicated inequality takes place only in region **s** < s_C (point *a*), i.e., on the left side of the curve M=F(s).

Therefore, asynchronous motor work stably only at the slips, less critical. The right part of the curve $M = F(s)$related to the field of slips $s > s_C$ is called unstable. When such slips the motor can not operate. So, if you work with a slip corresponding to the point *b*, any accidental unbalance of moments in that or other direction causes the acceleration of motor to the slips $s < s_C$ (move to point *a*) or its stop.

Torque at rated load corresponding to the permissible heat of the motor, must be with a certain margin below the critical value M_C. When the drag torque of load reaches the critical moment, the motor switches on an unstable part of the characteristic $M = f(s)$ and stops. There is a "breakdown" of the engine. The more excess of the critical moment over nominal, the larger short-term overload can be overcome by the motor. Overload capacity of the motor can generally be characterised by ***the coefficient of transshipment capacity*** $\boldsymbol{k_{tc}}$, which is equal to

$$k_{tc} = \frac{M_C}{M_R}, \tag{11.41}$$

where $\boldsymbol{M_R}$ – rated moment of motor.

To AMT it is normally k_{tc} = 1.6 ÷ 2.5. When assessment of the starting property of the motor is important the value of the initial starting moment M_{st} when s = 1.

To determine the critical slip it should equate to zero the derivative $\dfrac{dM}{ds}$ and solve the resulting expression with respect to s. This analysis shows that

$$s_\kappa = \frac{R_2}{X_{2st}}, \tag{11.42}$$

and the critical moment, which can be obtained from the expression (11.39) after substitution of the value $s = s_C$, equal

$$M_C = C_1 \cdot U_1^2 \frac{1}{2X_{2st}} \ . \tag{11.43}$$

It follows that, first, the critical moment, and hence the overload capability of an asynchronous motor depends on the square of the voltage. This leads to a high sensitivity of these motors to the deviation of voltage of circuit. Even relatively small reductions in voltage decreases sharply overload capability that can cause stop ("breakdown") of motors.

Secondly, the value of M_C does not depend on the active resistance of rotor R_2, but from R_2 depends slip s_C. For motors with contact rings due to the additional active resistance introduced into the circuit of the rotor, it is possible to obtain a number of curve $M = f(s)$, but the use of the starting rheostat increases starting moment M_s (see chspter 11.9).

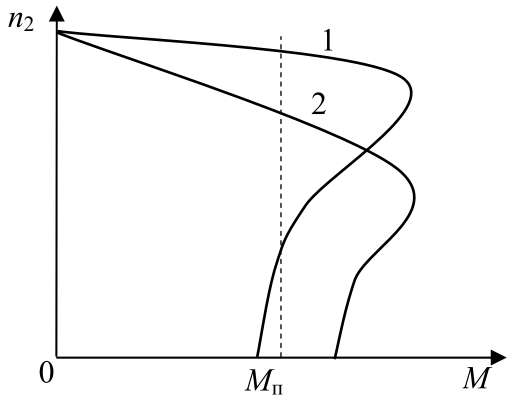


Figure 11.12: Mechanical characteristics of AMT

The dependence of $M = f(s)$ determines the relationship between speed and moment.

Thus, the graph $M=f(s)$ is a variation, i.e. built in other coordinates, ***mechanical characteristic***. In the normal coordinates $n = f(M)$ it has the form shown in figure 11.12. Curve 1 – for short-circuit motor and motor with contact rings without additional resistance in the rotor circuit; curve 2 – for a motor with contact rings when entered in the rotor circuit additional active resistance.

Convert (11.39), have substituted instead of U_1 its value from (11.43):

$$M = \frac{C_1 \cdot 2M_C \cdot X_{2st} \cdot R_2 \cdot s}{C_1(R_2^2 + X_{2st}^2 \cdot s^2)} = \frac{2M_C}{\frac{R_2}{X_{2st} \cdot s} + \frac{X_{2st} \cdot s}{R_2}} . \quad (11.44)$$

Taking into account (11.42), get

$$M = \frac{2M_C}{\frac{s_C}{s} + \frac{s}{s_C}} . \quad (11.45)$$

This simple formula is comfortable to calculate the mechanical characteristic of the motor on two known values – M_C and s_C, that you can get from the catalogue.

11.8 The working characteristics of asynchronous motor

Under working characteristics of AMT it is understood the dependence of some quantities that determine certain properties of the motor, from useful power P_2, developed on the shaft of the motor, at constant applied voltage of circuit. These values are the rotation speed n_2 or slip s, torque M, power coefficient cosφ, COP and the stator current I_1.

Approximate graphs of these dependencies for motor of normal implementation are shown in figure 11.13. Briefly explain them.

The dependence $n_2 = f(P_2)$ or $s = f(P_2)$ is called a ***high-speed characteristic***.

At idle ($P_2 = 0$), the rotation speed n_2 is close to synchronous n_1 (slip is close by zero). With increase of load, the speed of rotation decreases, the slip increases in accordance with the relation $s = \frac{P_{E.R}}{P_{EM}}$ (see 11.34). For reasons of high COP, this relationship is limited to a narrow ranges. Usually when $P_2 = P_{nom}$ slip s = 1.54–5%. Accordingly $n_2 = f(P_2)$ is a weakly inclined to the x-axis curve.

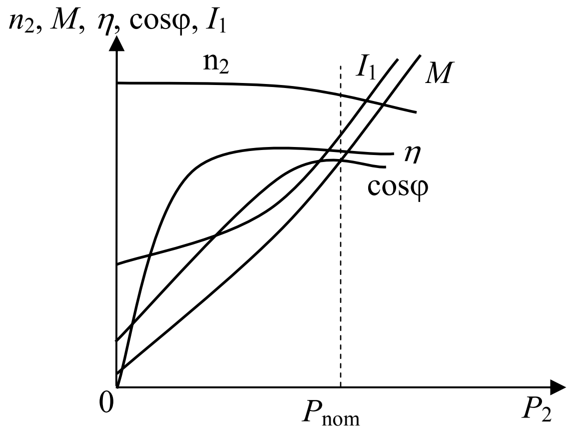


Figure 11.13: Working characteristics of AMT

The moment characteristic $M = f(P_2)$. As the speed of rotation of motor within the working range of load changes very slightly, the relationship $M = f(P_2)$ is very close to linear.

Dependence **cosφ** = $\boldsymbol{f(P_2)}$. Due to the presence in the magnetic circuit of the motor air gap stator current contains a relatively large reactive component. This affects the power coefficient. So cosφ of AMT is always less than one. Its greatest value (0.8–0.9) takes place at rated load. With decrease of load, it decreases sharply, reaching at idle values of 0.15–0.2. Underloaded asynchronous motor has a low power coefficient.

The dependence $\boldsymbol{\eta = f(P_2)}$. The COP of asynchronous motor has a maximum value at nominal or close to it load ($P_2 \approx P_{nom}$). However, it is quite high.

It is important to note that when the load $P_2 = (0.25–1.25)P_{nom}$ changes of COP is negligible. This means that in quite a large load range the motor works efficiently with COP, close to the maximum.

The dependence $\boldsymbol{I_1 = f(P_2)}$. According to the formula (11.24), the stator current

$$\dot{I}_1 = \dot{I}_0 + \left(-\dot{I}'_2\right)$$

consists of significant magnetizing component I_0, which is almost independent of the load, and component - I'_2, balancing the load of the rotor. Mainly due to I_0 asynchronous motor at no load consumes a relatively large idle current I_{idle}. It can be more than 50% of the rated. With increase of load, the stator current increases.

In addition to the above, to work characteristics it is also included overload capacity of the motor and its mechanical characteristics.

11.9 Launching of asynchronous motors

Under the start it is understood the approach of an alternating voltage to the connection terminals of AMT and its subsequent acceleration to the speed determined by the frequency of the supply voltage and the drag torque on the shaft.

At the initial moment of launching the rotor of AMT is stationary and in its winding is induced EMF of maximum value, the frequency of which is equal to the frequency of the supply voltage. The slip of the rotor at this moment of time, $s = 1$ and rotor current reaches a maximum value. Consumed by the motor current at $s = 1$ is called a starting current I_s. The ratio of starting current relative to the rated current I_r is great and usually is $k_s = 5–7$. The ratio is given in handbooks on asynchronous motors.

Depending on the type of motor, its power, the load resistance of executing mechanism, there are various ways to start AMT.

11.9.1 Launching of AMT with short-circuit rotor. Launching of AMT via a direct connection to the AC circuit without current limitation, called the ***direct launching***. Losses in the windings of AMT is proportional to the square of the current, and therefore the duration of the direct launching is limited to avoid overheat of the windings and of the failure of the motor.

Direct launching is used for motors with short-circuit rotor used for mechanisms with a small moment of inertia, the acceleration of which up to steady speed is relatively small. The scheme of direct launching of AMT (fig. 11.14) is very simple, which contributed to its wide dissemination. For direct launching it is necessary that torque of AMT was higher than the moment of resistance of mechanism.

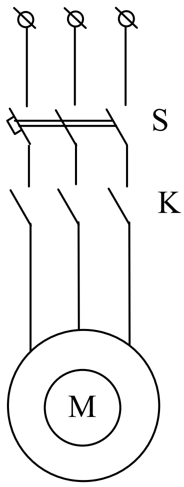
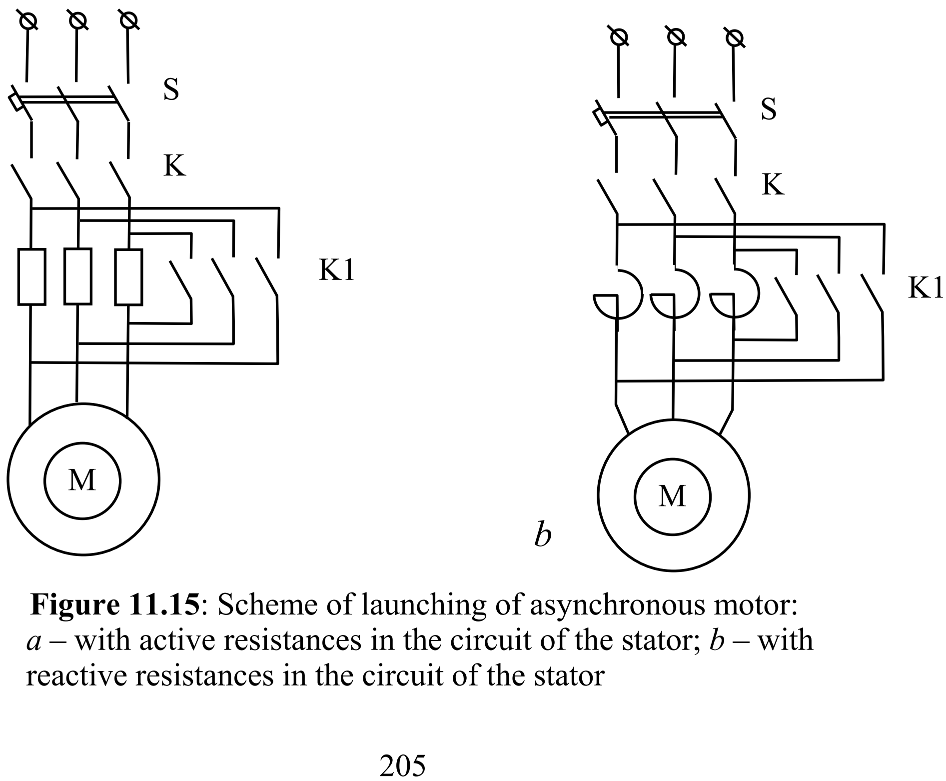


Figure 11.14: Scheme of direct launching of AMT

The motor launching of mechanisms with a large moment of inertia is either supply of a low voltage to AMT, or the implementation into the circuit of the stator current-limiting resistors. At launching on scheme with a current-limiting resistance (fig. 11.15) at first is on the contactor K and the motor acceleration comes to be in the included in circuit of the stator winding active (fig. 11.15, *a*) or reactive (fig. 11.15, *b*) resistances. Then with time delay the contactor K1 switches on and by their contacts bridges the current limiting resistance in the circuit of the stator winding. Launching on the scheme (fig. 11.15, *b*) is called reactor.



a *b*

Figure 11.15: Scheme of launching of asynchronous motor: *a* – with active resistances in the circuit of the stator; *b* – with reactive resistances in the circuit of the stator

11.9.2 Launching of asynchronous motors with phase rotor. The main advantage of these motors is the possibility of implementation of additional resistances in the rotor circuit and, therefore, limit the rotor current during launching.

Figure 11.16, *a* shows a scheme of rheostat launching with the implementation in the rotor circuit of the resistors. In this case, launching of AMT is most often performed in a function of time. For switching on the motor bracket (control circuit are not shown in the figure) is placed at the initial position in which the contacts 1CC – 3CC are open and in the rotor circuit the resistors *R*1, *R*2 and *R*3 are fully brought. After launching the motor (closing of power contacts of actuator K) with time delay t_1 contacts of contactor of acceleration 1CC are closed, which bridge the first step of the resistors *R*1 – *R*3. Then with time delay t_2 contacts of contactor of acceleration 2CC are closed, which bridge second step of the resistors *R*1 – *R*3. And, with time delay t_3 third stage of the resistors *R*1 – *R*3 bridges.

The inclusion of active resistance in the rotor of AMT leads to an increase of the critical slip:

$$s_к = \frac{c_1(R_2' + R_{2A}')}{\sqrt{R_1^2 + (X_{1\sigma} + c_1 \cdot X_{2\sigma}')^2}}, \tag{11.46}$$

where $\boldsymbol{R'_{2A}}$ – additional active resistance in the rotor circuit, reduced to the stator winding of AMT.

Increase $s_к$ in its turn, increases the slope of the mechanical characteristics (fig. 11.16, *b*). Critical moment of AMT does not depend on the resistance of the rotor circuit, and its value, despite the inclusion of R'_{2A}, remains unchanged. The control circuit of rheostat launching is performed so that the moments of switching M_{S1} and M_{S2} for all levels of launching were the same.

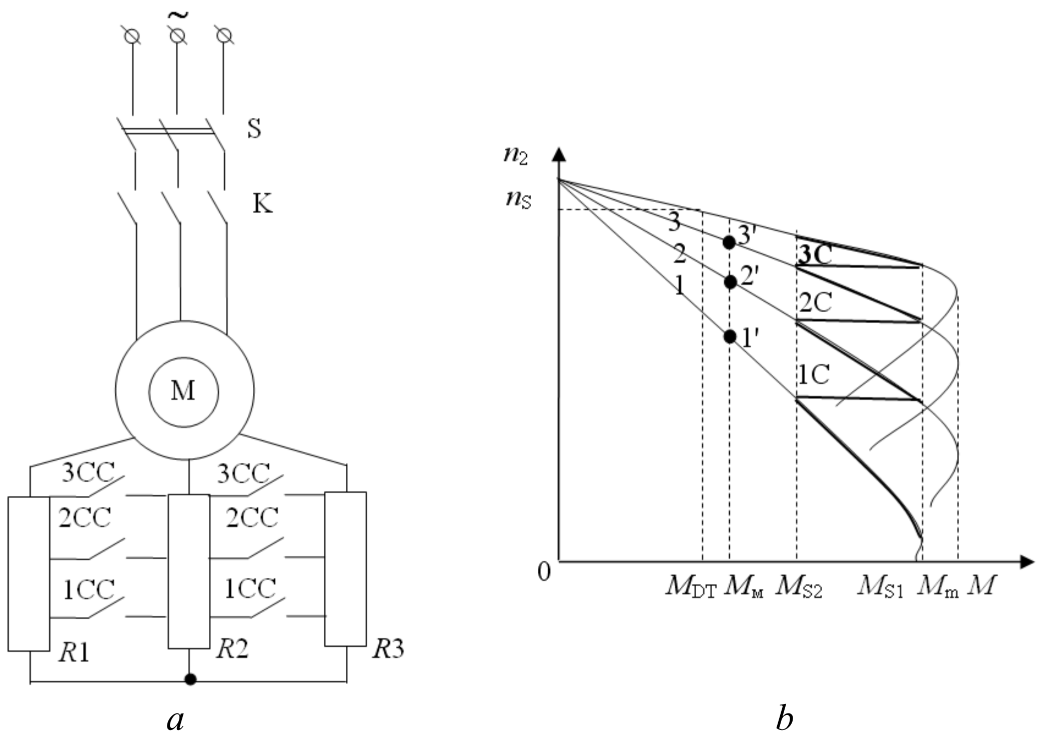


Figure 11.16: Scheme (*a*) and starting diagram (*b*) of rheostat launching of collector AMT

On figure 11.16, *b* by starting mechanical characteristics 1, 2, 3 it corresponds to the additional active resistances $R_{2A1} > R_{2A2} > R_{2A3}$. After switching on the AMT works on feature 1. At the point 1C contactor 1C switch on and AMT goes on characteristic 2, and in point 2C it is the transition to the feature 3. After bridging of additional resistors in point 3C AMT goes on the natural characteristic and reaches steady speed n_s, defined by the moment of resistance M_r. The value of moment of switching M_{s2} is determined by the values of the aging of relay of time of control circuits, which are configured to the required values.

In practice there are other schemes of AMT launching [29].

11.10 The mechanical characteristics of the asynchronous motor in the braking conditions

In chapter 11.7 it was reviewed the mechanical characteristics of AM, working in motor mode. However, the AMT can run in the braking condition: when braking to return energy to the circuit, when braking of opposition circuit and dynamic braking.

Braking with energy output to the circuit (generative mode of work in parallel with the circuit) is possible at speeds above synchronous. The mechanical characteristics of the asynchronous motor in the coordinates M and ω is presented in figure 11.17. In quadrant I they are parts of the motor mode for three different resistances of the rotor circuit. As it approaches the speed of the motor to the speed of the ideal idle speed or synchronous speed, the motor moment approaches to zero.

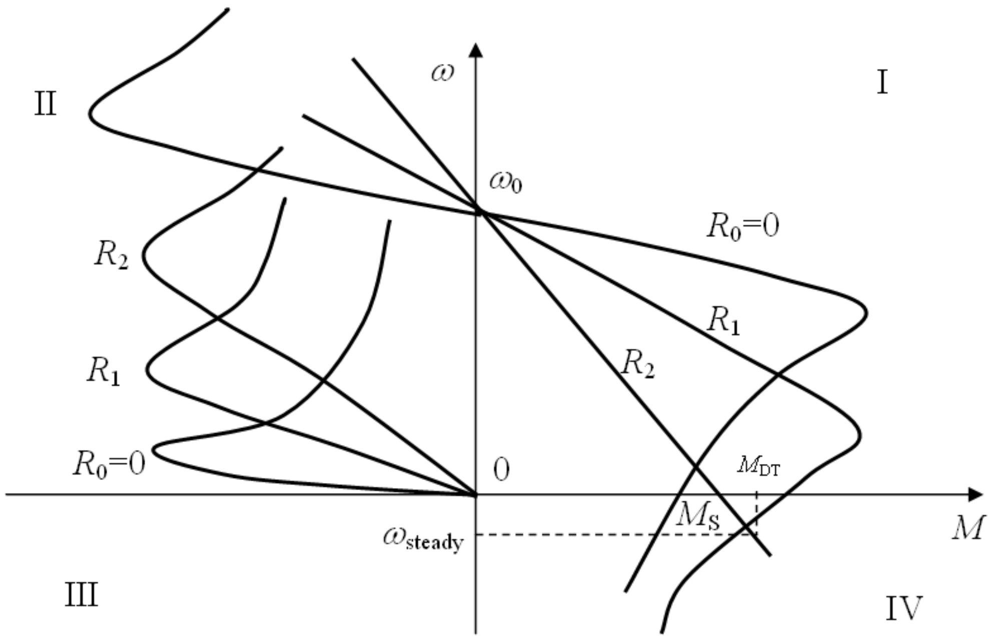


Figure 11.17: Mechanical characteristics of AMT in various modes

With further increase in the angular speed under the influence of external moment when $\omega = \omega_0$, the motor works in the mode of generator in parallel with the circuit, to which he can give electrical energy, consuming meanwhile reactive power for actuation. To braking with energy output in the network sections of characteristics response, located in the upper part of quadrant II. In this mode, the maximum moment is more important than in the motor one. The braking mode with the energy output to the network is used almost for engines with switching of the poles, and for drives of the lifting machine (elevators, excavators, etc.), and in some other cases.

Braking by opposition circuit has significantly greater usage in practice. The braking mode by opposition circuit can be obtained, as well as for a DC motor when driving moment of load $M_{DT} > M_S$ (fig. 11.17). For current limitation and obtain of the corresponding moment it is necessary when using a motor with a phase rotor in its rotor circuit to include additional resistor. The steady mode when braking by opposition circuit corresponds to, for example, $\omega_{уст}$, M_{DT} on the characteristic R_2 (fig. 11.17).

Mechanical characteristics for R_1 in the braking mode by opposition circuit and M_{DT} = const does not provide stable operation. Braking by opposition circuit can also be obtained by switching on the motion of two phases of the stator winding, which leads to the change of direction of rotation of the magnetic field (the move from point A to point B in fig. 11.18). The rotor rotates against the direction of movement of the field and gradually slows down. When the angular speed falls to zero (point C in fig. 11.18), the motor must be disconnected from the circuit, otherwise it will go in the motor mode, though the rotor will rotate in the opposite direction of the previous (point D).

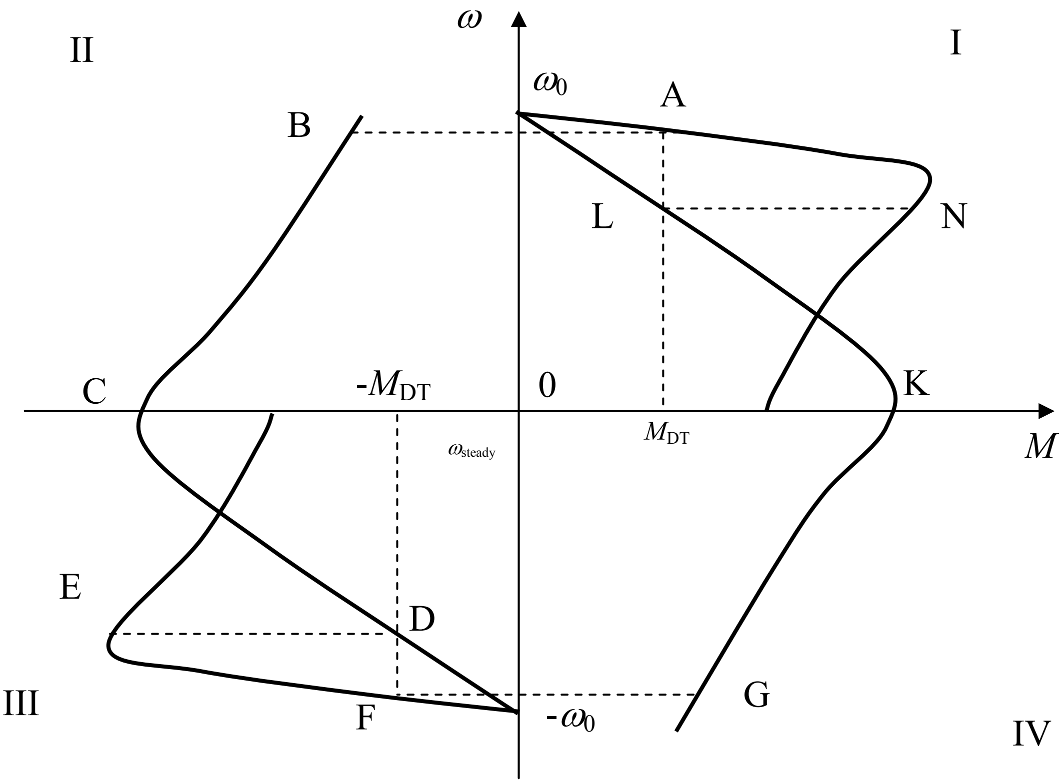


Figure 11.18: Mechanical characteristics of AMT when switching of two phases of the stator

Dynamic braking of an asynchronous motor comes to be usually by the inclusion of the stator winding to the DC circuit; the winding of the rotor meanwhile is locked on the external resistors. To switch from the motor mode to the dynamic braking contactor K1 (fig. 11.19) disables the stator from AC circuit, and contactor K2 connects the stator winding to a DC network. For current limitation and obtain different braking characteristics in the rotor circuit is provided external resistors.

Passing through the stator winding, a direct current forms a stationary field, the fundamental wave of which gives a sinusoidal distribution of induction. In rotating the rotor there is an alternating current that creates its own field, which is also stationary relative to the stator. In the result of interaction of the total magnetic flux with rotor current braking torque occurs, which depends on magnetomotive force (MF) of stator, the rotor resistance and the angular speed of the motor. The mechanical characteristics for this mode are listed in the lower part of the quadrant II (see fig. 11.17). They pass through the origin, as the angular speed equal to zero, the braking torque in this mode is also zero. Maximum moment is proportional to the square applied to the stator of voltage and increases with increase of voltage. Critical slip depends on the resistance of the rotor circuit. It grows in proportion to growth of the resistance. The maximum moment does not change meanwhile.

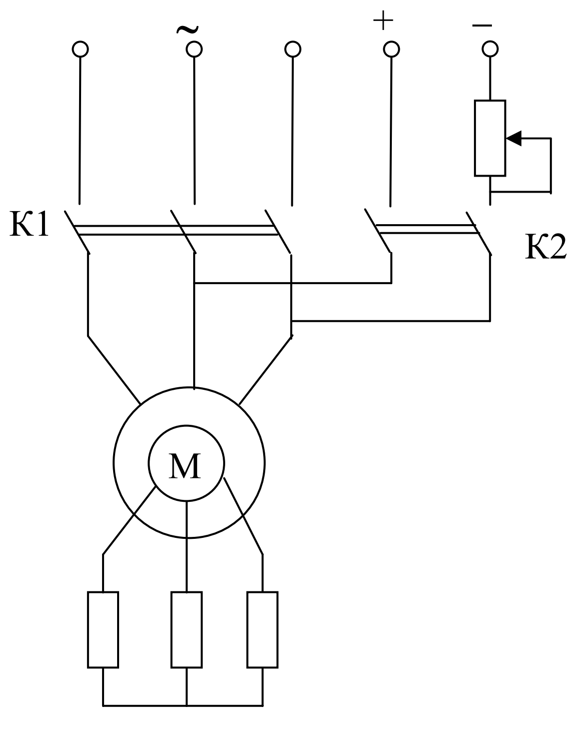


Figure 11.19: Inclusion scheme of AMT for switch to dynamic mode of braking

On figure 11.17 characteristics of dynamic braking are given for three different resistances in the rotor circuit and the same stator current.

Sometimes it is used the braking with self-excitation, connecting to the stator condensing battery. In this case, the machine works by asynchronous generator, receiving the magnetizing current from the condensers. Actuating from the side of the stator, the machine at a certain angular speed generates energy, emitted as heat in the rotor circuit. Such schemes of braking have not yet found wide application because of the high cost of condensers.

In practice, it is often used braking by opposition circuit, especially when in is necessary to make a change of direction of rotation (reverse), or dynamic braking, when the reverse is not required.

11.11 Single-phase asynchronous motor

Single-phase motor has one winding located on the stator. Single-phase winding is supplied by alternating current, creates a pulsating magnetic field. If you place in this field the rotor with short-circuit winding, it will fail to rotate. If you spin the rotor by the third mechanical force in any direction, the motor will operate sustainably. This can be explained as follows.

Pulsating magnetic field can be replaced by two magnetic fields rotating in opposite directions with synchronous frequency n1 and having the amplitude of magnetic flux, equal to half the amplitude of the magnetic flux of the pulsed field. One of the magnetic fields is called direct, the other backward. Each of the magnetic fields induct in the rotor winding eddy currents. In the interaction of eddy currents with the magnetic fields torques generate, directed opposite to each other.

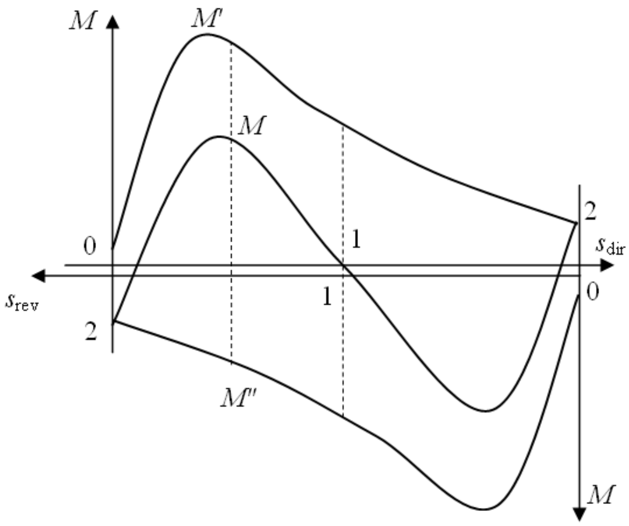


Figure 11.20: Dependences of direct and inverse moments of the single-phase motor from the slip

Figure 11.20 shows the dependence of moment from the direct field *M'*, moment from reverse field *M''* and the resultant moment $M = M' - M''$ as a function of slip. Axis of slips are directed oppositely to each other. In the starting mode to the rotor torques equal on magnitude and opposite in direction act.

If you spin the rotor by strange force in the direction of the direct magnetic field, appears surplus (result) the torque speeding up the rotor to a speed close to synchronous. When the slip of the motor relatively to the direct magnetic field $s_n \approx 0$, $n_1 \approx n_2$.

The slip of the motor relatively to the reverse magnetic field

$$s \approx \frac{n_1 - (-n_1)}{n_1} \approx 2 .$$

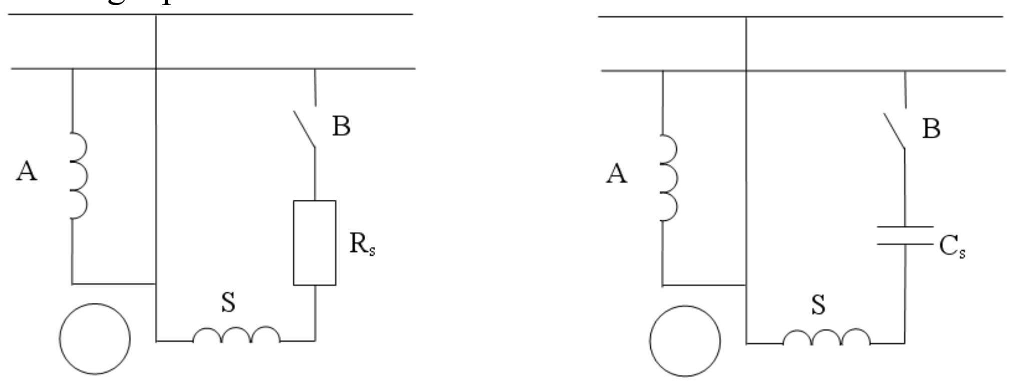
Considering the resulting characteristic, it is possible to make the following conclusions:

1. Single-phase motor has no starting moment. It will rotate in that direction, in which is rotated by an external force.

2. Due to the braking actions of the reverse field characteristics of single-phase motor is worse than three-phase.

Single-phase asynchronous motor (fig. 11.21) is powered by a single-phase circuit and has on stator two windings: working A and starting S; the rotor of the motor is performed short-circuited. Alternating current carrying through the working winding, usually occupying 2/3 grooves of the stator, creates a pulsating magnetizing force, and the last – the pulsating magnetic field.

For launching of single phase asynchronous motor it is used the starting winding S, shifted in space relatively to the working winding on 90°. Thus the currents working and starting windings must be shifted in time, which is achieved by connecting them either to the symmetric two-phase circuit, or to a single-phase circuit. In the latter case to obtain the phase shift between the currents in the windings the working winding A is connected to the circuit directly, and starting S – through active resistance (fig. 11.21, *a*) or the condenser (fig. 11.21, *b*). The starting winding S is included only for the period of start of the motor; when the rotation frequency of the degree 0.7 from the synchronous speed of rotation starting winding with help of button-switch, centrifugal switch or electromagnetic relay is turned off, and the motor further works as a single phase.



a *b*

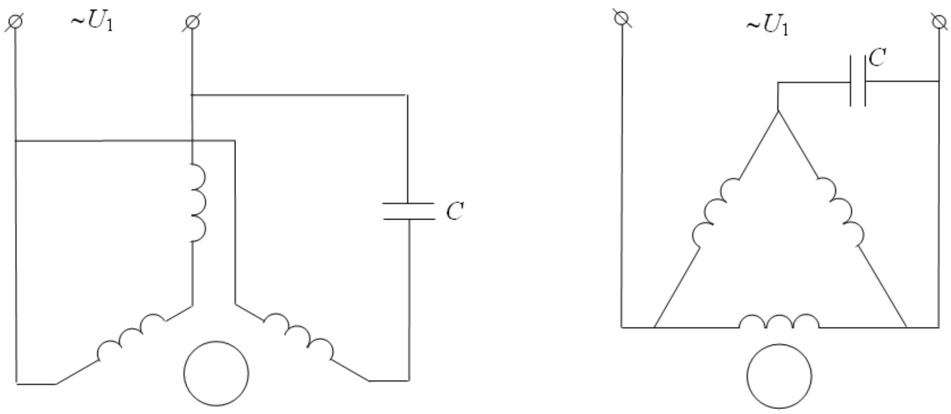
Figure 11.21: Schemes of inclusion of single-phase motor:
a – with active resistance; *b* - with condenser

Single-phase asynchronous motors in comparison with the three-phase have the worst performance, lower COP and power factor.

From a single-phase circuit three-phase asynchronous motors can work, if it is used one of the schemes shown in figure 11.22.

In the scheme of figure 11.22, *a* stator windings are connected in star, and in the scheme of figure 11.22, *b* – by triangle. The condenser C is starting, is included in the network only at the starting time and the magnitude of its capacity $C \approx 60$ microfarad to 1 kilowatt of motor power. There are other schemes of inclusion of three-phase AMT in single-phase circuit, for example, through active resistance.

Three-phase motors, working in single-phase mode, can develop power $P = (0.45–0.65)\ P_{rat}$, where P_{rat} is the nominal power of three-phase motor.



a *b*

Figure 11.22: Schemes of inclusion of three-phase motors in single-phase circuit

Key findings

1. Asynchronous machine can work in motor and generative modes. Depending on the magnitude of the slip it is distinguished motor, generative and the mode of opposition circuit of AM.

2. AMT have a high sensitivity to deviations of the circuit voltage. When a relatively small voltage reduction transshipment capacity decreases sharply, which can lead to stop of the motor.

3. In the moment of switching starting current of AMT in 5–7 times higher than their rated value. To reduce starting currents it is used special schemes of launching of AMT.

4. A distinctive feature of the single-phase AMT is the presence of the starting winding, which is on during launching in the work the motor.

Control questions

1. How is magnetic core of AMT satisfied?
2. The principle of implementation of the stator winding of AMT.
3. The concept of electric angle. The ratio between the electric and geometric angles.
4. How is the rotor winding of short-circuit motor met?
5. The device of a winding of the motor rotor with contact rings.
6. How is connection scheme of the windings of the stator selected and how to determine the same conclusions of phase windings of the stator?
7. From what does the speed depend on of rotation of the magnetic field? By what is the number of motor pole pairs determined?
8. How is torque of AMT created and why is the rotor unable to reach synchronous speed of rotation?
9. How are the slip and the rotation speed of the rotor defined and how does the change of load on the shaft on AMT influence?
10. What is meant by the mechanical characteristics of the motor and what kind of mechanical characteristic of AMT is?
11. How are active values of phase EMF of the stator and rotor determined?
12. How do EMF of the rotor and the current frequency of the rotor depend from slip?
13. How are EMF of leakage of stator and rotor taken into account?
14. What is the role of the EMF of the stator? How is the equation of the stator voltages written?
15. The equation of the voltage of the rotor. How do rotor current and its phase depend from the slip?

16. How is the equation of magnetizing forces of the motor written?

17. How is the equation of currents written and how is the vector diagram of currents built?

18. Based on what equations are full vector diagram and equivalent circuit of an asynchronous motor built? How is a vector diagram built?

19. What kinds of power losses occur in the AMT? What are electromagnetic and mechanical power of the motor meant under?

20. Define the term " rated power of motor ".

21. How does the electrical loss in the rotor depend on from slip?

22. What determines motor torque?

23. Analyze the dependence of the torque from slip.

24. What is meant by the coefficient of overload capacity and what is its value for usual AMT?

25. What is the condition for stable operation of the motor?

26. Form what and how depends on the critical moment and the critical slip?

27. What has possibilities of influence on the mechanical characteristics of AMT?

28. What indicators can starting property of the motor be assessed?

29. The advantages and disadvantages of direct launching of asynchronous short-circuit motors.

30. Advantages and disadvantages of launching in run of the motors at a lower voltage.

31. How do motors with contact rings start? Give a general assessment of their starting properties.

32. What is working performances of the motor meant under? The nature of these dependencies for AMT.

33. The condition for the transition of the asynchronous motor in the generative mode. The practical value of such regime.

34. What are the possible ways of braking of an asynchronous motor? Their assessment and practical value.

35. The device and principle of action of single-phase AMT.

36. How do single-phase motors start?

12 Synchronous Electric Machines

Key concepts: the actuation winding, exciter, implicit-pole rotor, salient-pole rotor, the inductor, the armature, the characteristic of the idle SM, external characteristics SG, rated actuation current, electromagnetic power, the angular characteristic, synchronization.

Synchronous machines (SM) are used as generators at electric stations and as AC motors in the unregulated electric drive of industrial installations (pumps, the compressors, blowers, mills for various purposes, rolling mills, diesel generator plants and other). The main advantage of synchronous motors, which consists in opportunities to work with high power coefficient leads to more their widespread use.

The angular speed of the synchronous motor when work in steady mode with increase of load on the shaft up to a certain value, not exceeding the maximum moment M_{max}, remains strictly constant and equal to the synchronous angular speed:

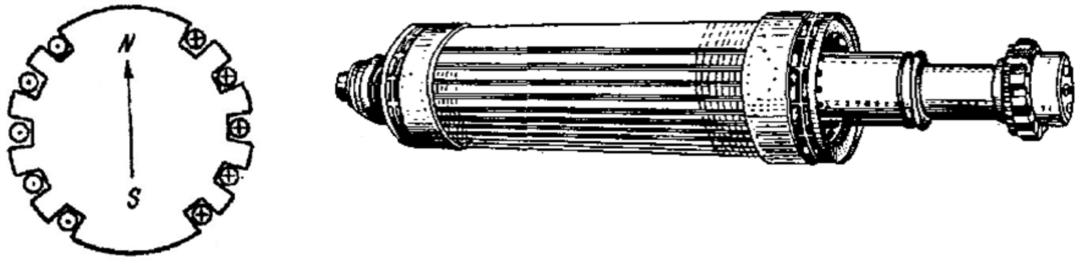
$$\omega_0 = \frac{2\pi \cdot f}{p}. \quad (12.1)$$

12.1 The device of synchronous machines

SM, like other electric cars, reversible, i.e. they can be used both as generators and as motors. The device SM have a lot in common with asynchronous. Consider the differences in their construction.

The stator of SM does not differ from the stator of the asynchronous (see chapter 11.2.1) – such assembled from sheet steel core, the same principle the implementation and the connection of its windings. The SM rotor is an electromagnet of DC. Its winding is powered with direct current from an external source. It is called the ***actuation winding***. The connection of the winding of the rotor come true with a direct current source via two contact rings on the shaft and the stationary brushes. As a direct current source to power the actuation winding of the rotor is typically used DC generator, which is often mounted on the same shaft with the rotor. Such a generator is called ***exciter***. The power required for the power of actuation windings is small and accordingly the exciter power is about 0.3-5% from rated power of SM. For large SM it is installed reserve exciter that is introduced into the work with faults mainly. It is also possible the power winding of actuation from the AC network connected to the stator, through the rectifiers.

There are two types of rotor of synchronous machines – ***implicit-pole rotor***, or implicitly expressed poles, and a ***rotor*** with explicitly expressed poles, or ***salient-pole***. In the first case, the core of the rotor is a solid cylindrical body of steel (barrel of rotor); with longitudinal grooves in which is laid the actuation winding (fig. 12.1, *a*). Grooves and winding are arranged to receive a sinusoidal distribution of induction in the gap between the cores of the rotor and stator. General view of implicit-pole rotor is shown in figure 12.1, *b*. This rotor is used when the number of pairs of poles $p \le 2$.

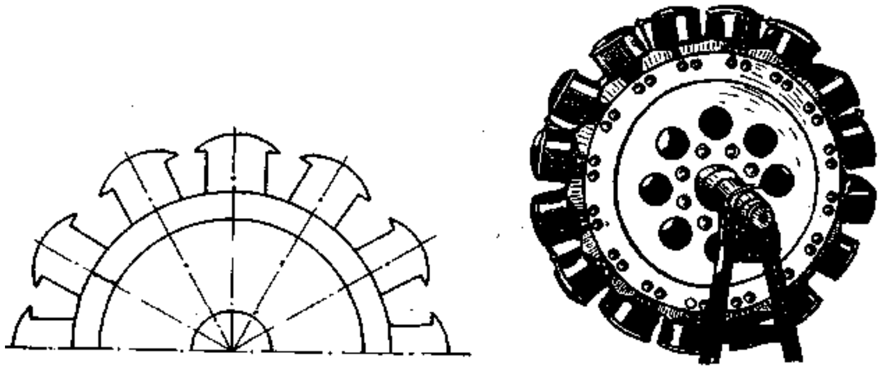


a *b*

Figure 12.1: Rotor implicit-pole SM: *a* – cross section; *b* – general view

Salient-pole rotor consists of a massive steel wheels putted on the shaft. On the outer surface of the wheel are attached to the steel cores of the poles (fig. 12.2, *a*). The last ones, and sometimes the rim, are made of sheet steel. For small machines and at not too many poles instead of the wheel on the shaft are settled a steel sleeve, to which poles are mounted. The excitation winding in the form of coils placed on the cores of the poles. To obtain a sinusoidal distribution of the induction gap between the surface of the pole bit and the inner surface of the stator is made uneven due to the special form of pole bits. Sinusoidal distribution of induction in the gap it is necessary to obtain a sinusoidal EMF in the windings of the stator.

General view of the salient-pole rotor is shown in figure 12.2, *b*.

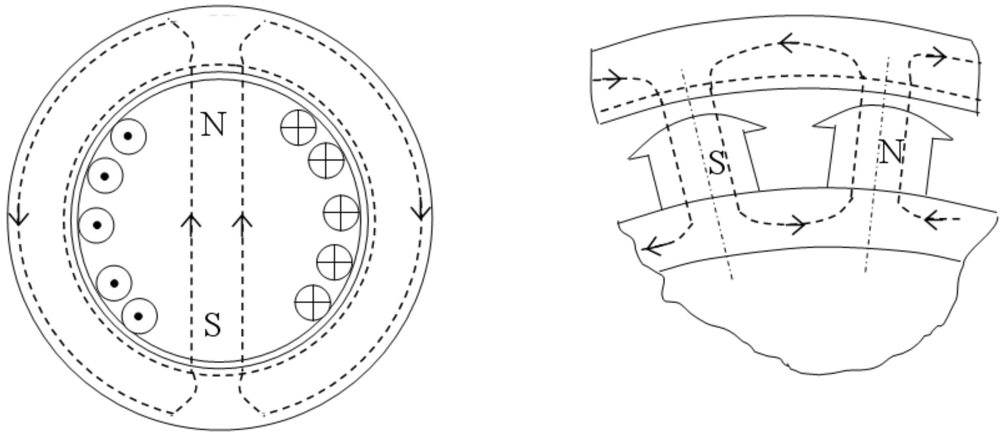


a *b*

Figure 12.2: Rotor salient-pole SM: *a* – cross section; *b* – general view

Thus, a ***synchronous machine*** as asynchronous, ***consists of a magnetic core, windings and mechanical parts***. The magnetic core includes a core of poles, rim of the rotor, or just the body of implicit-pole rotor, the stator core and the gap between the rotor and stator (fig. 12.3).

Winding AC of SM is placed in the grooves of the stator core. For three-phase machine – it is a three phase winding, spatially offset by 120 electrical degrees. The rotor carries the actuation winding was powered by a direct current. Mechanical parts – jar cast or welded, shaft, bearing boards or column and other parts needed for installation and assembly of the machine. Machines with implicit-pole rotor shaft is usually forged along with the core of the rotor. The rotor of the synchronous machine as a source of magnetic field is called an ***inductor***. Part of the machine, in the winding of which when work is induced EMF, is called the ***armature***. At the SM of usual design armature is the stator.



a *b*

Figure 12.3: Magnetic flux CM: *a* – implicit-pole; *b* – salient-pole

12.2 Synchronous generator

In a ***synchronous generator*** (SG) it is the conversion of mechanical energy into electrical energy alternating, usually three-phase current. The generator rotor with the permanent magnetic field is leaded into rotation by the primary engine (steam or hydraulic turbine, diesel engine and so on). Due to electromagnetic induction in the windings of the stator (armature) are induced variables EMF. As in the stator of three-phase generator are three identical, symmetrical spatially offset by 120 electrical degrees of winding, their EMF are equal to its active and amplitude values and are symmetrically shifted in phase by 120°. Frequency of EMF of the armature is determined by the rotation speed and the number of pole pairs of the rotor (12.1). The current frequency is standardized and must be constant. From (12.1) it is shown that for any given frequency of alternating current it is required to ensure well-defined, constant, independent of load, and equal to ω_0 speed of rotation. With this purpose the primary motors on electric power plants equipped with automatic speed control.

Required for a given frequency, the speed of rotation of the rotor the less, the greater the number of pairs of poles p. Therefore the generators, working from low-speed primary engines are made by multipolar.

By the nature of the primary motor there are ***two main types of synchronous generators*** – ***turbogenerators and hydrogenerators***. First ones are installed in thermal power plants and work from a steam turbine, the second ones are used for hydropowers. The power of modern turbo – and hydrogenerators often reach several hundred (500 and more) thousand kilowatts.

For steam turbines in thermal power plants high speed of rotation (typically 3000 turns/minute) is typical. The standard turbine generator of frequency $f = 50$ Hz, working with such speed, must have a number of pairs of poles

$$p = \frac{2\pi \cdot f}{\omega} = \frac{2\pi \cdot 50 \cdot 60}{3000 \cdot 2\pi} = 1,$$

i.e., the rotor must be bipolar. In addition, at high speed of rotation it is very important issue of ensuring the mechanical strength of the rotor. Therefore, the

turbogenerators are made with implicit-pole rotor. They are characterized by a relatively small radial and significant axial dimensions.

The generators typically have low rotation speed (50–300 turns/minute). To receive an alternating current of standard frequency $f = 50$ Hz, the generators are implemented on a large number of pairs of poles. So, for example, at a speed of $n = 50$ turns/minute) number of pole pairs must be equal

$$p = \frac{2\pi \cdot f}{\omega} = \frac{2\pi \cdot 50 \cdot 60}{50 \cdot 2\pi} = 60\,.$$

For their placement it is necessary to increase the lateral dimensions of the generator. The generators are salient-pole and have a relatively large radial dimensions at moderate axial. As a rule, they are made for vertical installation. SG are made on the voltage of 0.4; 6,3; 10,5; 15; 20 kiloV.

At idle of generator armature current is zero and the magnetic field is generated only by the actuation winding of the rotor. By distributing the windings of the stator and rotor of implicit-pole machine, and by giving special form to pole bits of salient-pole rotor it is achieved that change of flux linkage of armature windings when the rotor is rotated it is practically sinusoidal. This is necessary to obtain a sinusoidal EMF in the armature. This shape of the curve of EMF is a requirement of the standard to AC generators. In this case, the active value of the EMF in the windings of the armature is determined by a formula similar to the windings of AM (11.7), i.e.:

$$E_0 = 4{,}44 \cdot f \cdot w \cdot k \cdot \Phi_0\,; \tag{12.2}$$

where: $\boldsymbol{E_0}$ – EMF of phase armature winding;
$\boldsymbol{f}$ – frequency of EMF anchor;
$\boldsymbol{w}$ – the number of turns of the phase winding;
$\boldsymbol{k}$ – winding coefficient of the armature winding;
$\boldsymbol{\Phi_0}$ – the rotor flux penetrating the stator core.

At a constant frequency f on the magnitude of the EMF of the armature it can be influenced by the flow Φ_0, which is created by the actuation current of the rotor I_{act}. The dependence of the EMF of the armature from the actuation current at rated speed and no load armature($I = 0$) is called the ***characteristic of idle***. Its usual view is presented in figure 12.4. As $E_0 \equiv \Phi_0$, then $E_0 = f\,(I_{act})$ is determined by the magnetic circuit of the machine and at a different scale repeats the curve $\Phi_0 = f(I_{act})$.

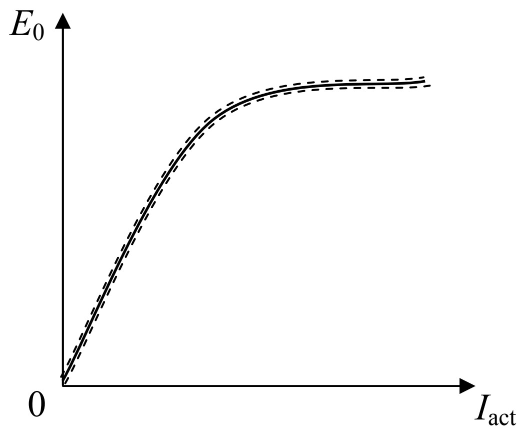


Figure 12.4: Characteristic of idle of SG

Characteristic of idle can be built according to the calculation of the magnetic circuit or on the basis of the experience of idle. The last one is at rated speed and load off. The rotor voltage is measured with a gradual increase in the actuation current from zero, and then at its decrease. Due to hysteresis, these voltages may not be the same. To build the characteristic of idle it is accepted average values.

12.3 External characteristic of synchronous generator

The external characteristic of the generator is called the voltage dependence of the load voltage $U = f(I)$ for n = const, I_A = const, $\cos\varphi$ = const. The greatest interest is the external characteristic $U = f(I)$ at rated speed, $\cos\varphi$ and the actuation current in the range of load current from $I = 0$ to $I = I_{rat}$.

The actuation current, which should be at rated load, i.e. at U_R, I_R, $\cos\varphi_R$, is called the ***rated actuation current***. Generators typically are calculated for work with rated power coefficient $\cos\varphi_R = 0.8$ to 0.9 for mixed active-inductive load. In this case, the longitudinal demagnetizing reaction of armature significantly influences. Therefore, when the decrease of load from rated one the rated voltage quickly increases (curve 1 fig. 12.5).

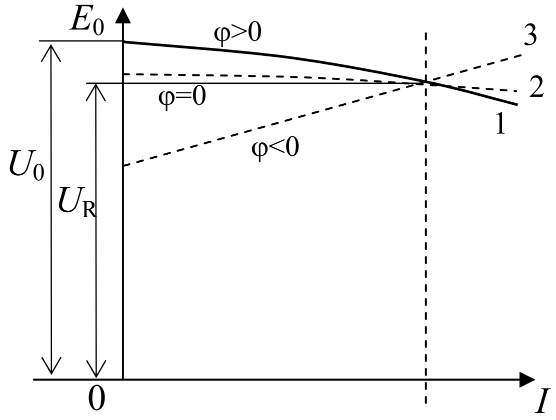


Figure 12.5: External characteristics of the SG

The relative change (increase) of the voltage may reach several tens of percent. ***Under the relative change in the voltage it is understood expressed in percentage to the rated voltage rise of voltage when transition from the rated load to idle.*** It is equal

$$\Delta U = \frac{U_0 - U_R}{U_R} 100 . \qquad (12.3)$$

As $\cos\varphi$ of load may differ from the rated, this in turn affects on the external characteristic of the generator. Figure 12.5 curve *2* depicts the external characteristic with the active load ($\varphi = 0$), curve 3 with active-capacitive load ($\varphi < 0$). External characteristics can be constructed using vector diagrams or according to experience.

To maintain the generator voltage constant when the load is changed according to the size and nature it is regulated the actuation current of the rotor.

12.4 Electromagnetic torque of synchronous generator

The mechanical power acted to the generator shaft, after deduction loss, converted to electric. Useful electric power of the generator is equal to

$$P = m \cdot U \cdot I \cdot \cos\varphi \ , \qquad (12.4)$$

where: ***m*** – the number of phases of the armature;
U – external voltage;
I – the armature current;
cos*φ* – power coefficient.

Some part of the electric power consumed in the windings of the armature in the form of electrical loss in the armature: $P_{EA} = m \cdot I^2 \cdot R$. *Full electric power of armature is equal to the sum of useful electric power of generator and power losses in the armature, is called an **electromagnetic power** generator:*

$$P_{EM} = P + P_{EA} = m \cdot U \cdot I \cdot \cos\varphi + m \cdot I^2 \cdot R = m \cdot I(U \cdot \cos\varphi + I \cdot R) \ . \quad (12.5)$$

Also the ***electromagnetic power of the motor*** is expressed. Only for the motor *it is that part brought with the electrical power that is converted into mechanical*.

Formula of electromagnetic torque of SM has the form

$$M = \frac{m}{\omega} \cdot \frac{E_0 \cdot U}{X} \sin\Theta , \quad (12.6)$$

where: $\boldsymbol{\omega}$ – the angular speed of rotation of the rotor;
$\boldsymbol{E_0}$ – the EMF created by the main thread of the rotor;
$\boldsymbol{X}$ – the inductive resistance of the armature;
sinΘ – the phase angle between the EMF E_0 and voltage U.

Figure 12.6 shows a simplified vector diagram of the generator (without active voltage drop in armature $I{\cdot}R$, since the resistance of the armature is usually very small).

From (12.6) it is shown that the electromagnetic torque depends on EMF E_0, voltage U and sinΘ. At generator electromagnetic torque is opposed, it balances the moment of the primary motor (steam or hydroturbines). For synchronous motor electromagnetic torque is torque. The necessary condition for transformation of energy in SM (mechanical to electrical generator and electrical to mechanical in the motor) is the presence of a phase shift between EMF E_0 and the voltage U on the angle Θ. When Θ = 0 angle ψ = 90°, since the I_x and I are mutually perpendicular (see fig. 12.6). Electromagnetic torque M and the capacity of P_{EM} at it are zero.

The phase angle Θ on the vector diagram of the voltages of the generator (see fig. 12.6) corresponds to a spatial offset between the axes of the fields of the rotor and the result of the field angle Θ/p when the leading field of the rotor. The synchronous motor, on the contrary, the rotor field lags behind the result on the corner Θ/*p*.

If you take EMF E_0 and the voltage U constant, independent of load (this mode is possible, for example, with parallel work of synchronous machines with the circuit, the electromagnetic torque will depend only on sinΘ. The moment dependence from the angle Θ is called the ***angle characteristics of synchronous machines*** (fig. 12.7). It is used for evaluation of the static stability of the machine when the parallel operation with the circuit.

As follows from formula (12.6) and graph (see fig. 12.7), maximum torque machine develops when Θ = 90°. It is equal

$$M = \frac{m}{\omega} \cdot \frac{E_0 \cdot U}{x} \ . \quad (12.7)$$

The magnitude of the maximum moment and AM characterizes overload capacity. It can be viewed as the limit of the static stability of the machine in terms of its parallel work with the circuit. The less time corresponding to the given load compared to the maximum, the more stability margin. At the same time for the stable operation is necessary for at increase of the angle Θ electromagnetic torque increases. This condition is observed only when Θ < 90°.

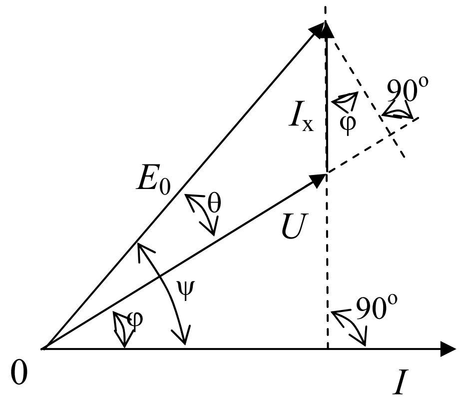


Figure 12.6: Vector diagram of SG

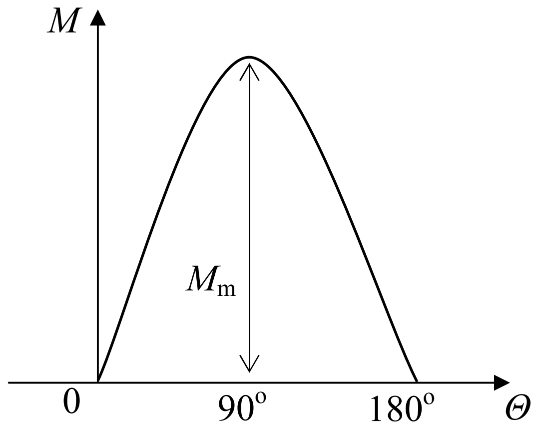


Figure 12.7: Angular characteristic of SM

12.5 Parallel work of synchronous machine with electric circuit

In the modern practice autonomous work of the SG on its load out of touch with other sources of three-phase current is rare. Normal is the power supply of consumers of energy from energy systems. The power system on the high voltage lines unite for parallel operation electric power plants. The presence of power systems provides substantial economic benefits, chief of which is the rise of reliability of power supply and reduction of the cost of electricity production.

The power of modern power systems amount in the millions and tens of millions of kilowatts. Each of the generators included in the power system, is in the mode of parallel work with the circuit of high power. You can assume that the work mode of the individual generator does not affect the circuit work mode, as ***the frequency and the voltage on the connection terminals rigidly it is defined by external circuit and do not depend on the actuation current, load and power of the generator***. Synchronous motor is always connected in parallel to supply circuit.

At inclusion of SG in parallel with the circuit, it is necessary to ensure possible lesser inrush current at the time of connection of the generator to the network. Current at the time of connection to the circuit will be equal to zero, if the condition of equality of the instantaneous values of the voltages of the generator and the circuit comes true.

$$U_m \sin(\omega_g - \alpha_g) = U_{cm} \sin(\omega_c - \alpha_c), \tag{12.8}$$

where: $\boldsymbol{U_m}$, $\boldsymbol{U_{cm}}$ – the amplitude values of the voltages of the generator and the circuit;

$\boldsymbol{\omega_g}$, $\boldsymbol{\omega_c}$ – frequency of generator and circuit ($\omega = 2\pi f$);

$\boldsymbol{\alpha_g}$, $\boldsymbol{\alpha_c}$ – the initial phases of the generator and the circuit.

From (12.8) it is resulted ***conditions of inclusion of SG in parallel with the circuit***: the equality of the voltages $U_m = U_{cm}$; the equality of frequencies $\omega_г = \omega_c$; the equality of the initial phases $\alpha_g = \alpha_c$. In addition, it is necessary to align the phase alternation order.

The creation of these conditions with the subsequent inclusion of the generator in the circuit is called ***synchronization****.*

After inclusion of SG in the circuit voltage U becomes equal to the voltage of the circuit U_c. Relatively to the external load voltage U and U_c match in phase, and on the contour “generator – network” are in antiphase, i.e., $\acute{U} = - \acute{U}_c$.

On the basis of equation of voltages the armature current is determined by the expression

$$\dot{I} = \frac{\dot{E}_0 - \dot{U}}{jX} . \tag{12.9}$$

As the voltage of the generator and the circuit are equal and opposite in phase, can be written

$$\dot{I} = \frac{\dot{E}_0 + \dot{U}_c}{jX} , \tag{12.10}$$

where $\boldsymbol{U}_c$ – voltage of circuit.

The generator after inclusion in the circuit works in the idle mode. Vector diagram corresponding to this case is shown in figure 12.9.a. At increase of actuation current the absolute value of EMF of the armature E_0 increases. Since the voltage at the connection terminals is set by the network and remains constant, then appeared voltage difference $\Delta U = E_0 - U = E_0 + U_C$ will cause current in the armature

$$\dot{I} = \frac{\dot{E}_0 - \dot{U}}{jX} = \frac{\Delta \dot{U}}{jX} . \tag{12.11}$$

The armature current during this lag on phase from ΔU and E_0 on the angle $\psi = 90°$ (fig. 12.9, *b*). The mode turns out to be the same as if when offline of generator it was loaded on a purely inductive load. In relation to circuit generator in this mode is a source of reactive power. On the contrary, when the reduction of the actuation current (underexcitation) EMF E_0 reduced, which leads to a phase change ΔU and current I on the opposite one (see fig. 12.9, *c*). The armature current when it is ahead of the voltage U and EMF E_0 on 90°, and in relation to the circuit voltage U_c will lag at the same angle.

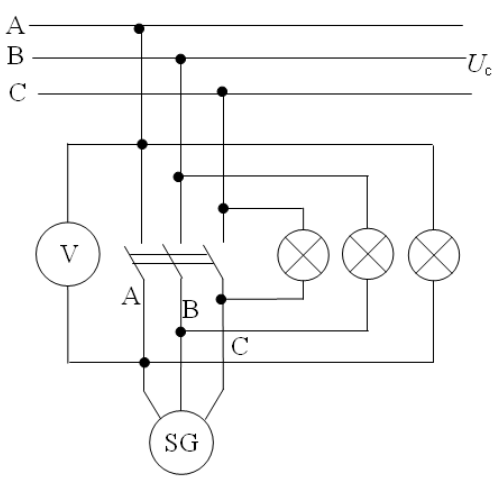
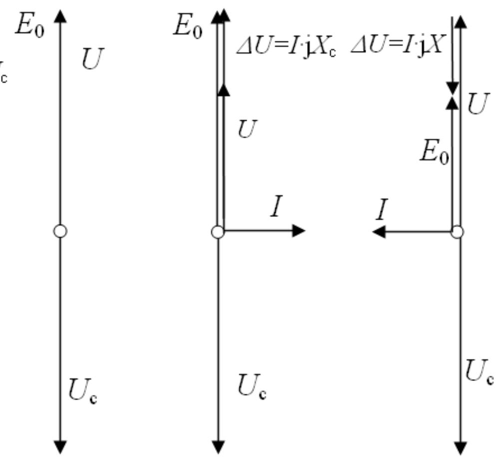


Figure 12.8: Scheme of inclusion of SG in circuit



a *b* *c*

Figure 12.9: Vector diagrams

In this mode, the generator consumes from the circuit reactive power required to maintain the resulting field at the same level as the voltage at the connection terminals are push-type. The reaction of the armature will be longitudinally magnetizing. The load on the primary motor in both cases remains at idle, because of the electromagnetic power at ψ = 90° is equal to zero.

Thus, the regulation of the actuation current causes only a change of reactive load of generator.

To load the generator by active power, it is necessary to influence on the primary motor to give the rotor a certain acceleration. With this aim the intake of steam or water into the turbine increases. During acceleration of the rotor the phase shift appears between EMF E_0 and the voltage U at an angle Θ, which is a consequence of the spatial shift between the axes of the fields of the rotor and the result field on the corner $\frac{\theta}{p}$.

The phase shift between the EMF and the voltage will cause the voltage difference ΔU, which in turn will cause the armature current

$$\dot{I} = \frac{\dot{E}_0 - \dot{U}}{jX} = \frac{\Delta \dot{U}}{jX} . \quad (12.12)$$

This current, lagging behind from ΔU in phase on 90°, will have a phase shift with respect to EMF E_0 on the angle $\psi < 90°$ (fig. 12.10, *a*).

As $\Theta > 0$ and $\psi < 90°$, then the generator will develop electromagnetic power $P_{эм} = m \cdot E_0 \cdot U \cdot \cos\psi > 0$, incoming in circuit, and electromagnetic torque

$$M = \frac{m}{\omega} \cdot \frac{E_0 \cdot U}{X} \sin\Theta , \quad (12.13)$$

which will balance the moment of the primary motor at the same synchronous speed of rotation.

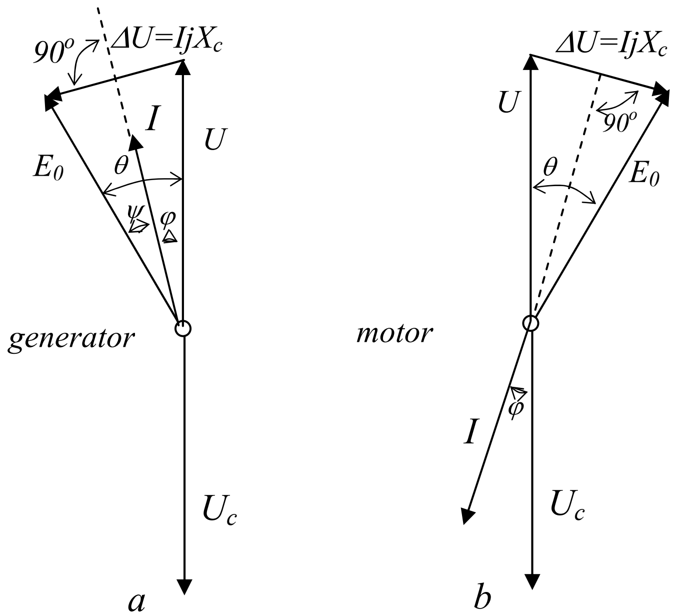


Figure 12.10: Vector diagrams of SM: *a* – the generator mode, *b* – mode of motor

With the increase of moment of primary motor the angle Θ, the electromagnetic power and the bucking moment will increase. The speed will be kept synchronous up until the time of its primary motor will be balanced by the electromagnetic torque of generator. When the angle Θ = 90°, the electromagnetic torque is maximum (see ratio 12.7).

Its value is the limit of the generator load on moment (accordingly and given power) at which the generator is able to operate synchronously with the circuit. If the moment of primary motor exceeds this value, the generator will not be able to balance and falls out of synchronism. Parallel operation with the circuit becomes impossible.

If after switching on the parallel work of synchronous machines not to increase the torque of the primary motor, and, conversely, to reduce or even attach to the shaft the braking torque, the rotor will slow down and fall behind from the result of the field on angle Θ/p. Meanwhile EMF E_0 will lag behind from the voltage U on the angle Θ. This will cause the phase change ΔU and armature current almost on the opposite (fig. 12.10, *b*). As a consequence, the direction of flow of electromagnetic power and electromagnetic torque will change, which of the bucking moment will be torque. *The machine will switch to the synchronous motor.* Rotating electromagnetic torque will balance the braking torque and the rotation speed will remain synchronous.

Thus, synchronous machine, is connected in parallel with a circuit of high-power, counteracts within its static stability as the acceleration and deceleration of the rotor and keeps the speed constant. The only change is the angle between the axes of the fields of the rotor and the resultant field within ±90 electrical degrees. For stable operation of the synchronous machines are designed and manufactured so that at rated load angle Θ does not usually exceed 20±30°. Synchronous machine counteracts the change of the actuation current by corresponding change in reactive load, as the voltage at the connection terminals are push-type.

12.6 Synchronous motor

In a synchronous motor electric power of AC received from the circuit on the stator is converted into mechanical on shaft. On a device synchronous motor does not differ from the generator. The basis of the torque of a synchronous motor is the interaction between AC stator with the permanent magnetic field of the rotor.

To clarify the nature of this interaction we will use figure 12.11. It shows the elements of the arc of the stator and rotor. Let at some point in time the direction of current in the conductors of the stator and the position of the poles corresponds to figure 12.11, *a*. According to the rule of the left hand [36] each of the conductors of the stator will experience a mechanical force from the interaction of its current to the rotor field directed counterclockwise. The force acting on the poles of the rotor is reversed. Through half of period of AC T/2 in the conductors of the stator will be the same in magnitude, but oppositely directed current.

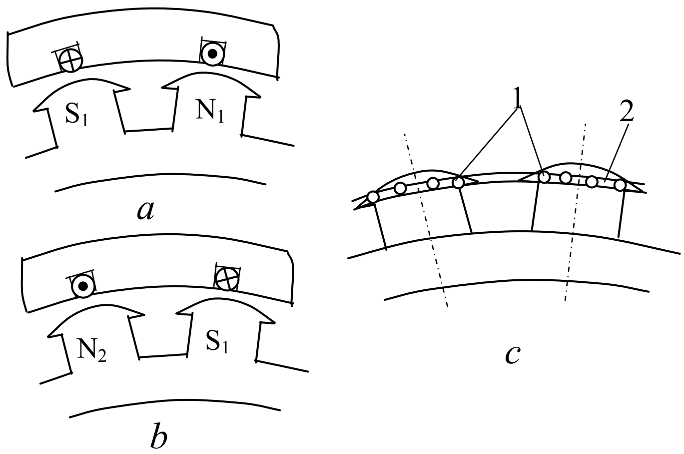


Figure 12.11: Principle of action of SMT

To the direction of the force acting on the rotor at the same time preserve, it is necessary that at the same time the rotor is turned at one pole and take the position shown in figure 12.11, *b*. However, due to the inertia the rotor at time T/2 at frequency f = 50 Hz will remain in place and therefore in the following half-period of time it will force the opposite direction. Consequently, synchronous motor can't turn around. But if due to external forces to give the rotor previously such speed, which provides a specified condition, the motor will continue to rotate at synchronous speed due to its own electromagnetic torque. If the rotor has *p* pairs of poles, the time of one full turn will be *pT* seconds, and the speed of rotation

$$\omega = \frac{2\pi}{p \cdot T} = \frac{2\pi \cdot f}{p}. \tag{12.14}$$

The direction of rotation of three-phase synchronous motor is determined by the phase alternation of the supply circuit.

Thus, the motor work is carried out at synchronous, independent of the load speed of rotation. Load change impact only on the angle of the spatial offset between the axes of the result fields of the armature and field of the rotor. In contrast to the generator rotor of synchronous motor lag behind from the result field of the anchor on Θ electrical degrees. In the motor the field of armature is leading, followed by the rotor.

Rotating electromagnetic torque synchronous motor is expressed by the same formula as for the generator, i.e.,

$$M = \frac{m}{\omega} \cdot \frac{E_0 \cdot U}{X} \sin\Theta \ . \tag{12.15}$$

If the drag torque of the load exceeds the maximum value (when Θ = 90°), then the equilibrium of moments is not possible; the motor falls out of synchronism and stops. When the rotation speed less than synchronous position of the rotor are inevitable, when 360 > Θ > 180. When the motor moment changes direction and will further slow down the rotor, accelerating it to stop.

To include synchronous motor in the circuit it must be beforehand synchronized with it. The requirements are the same as turning on parallel work of generators.

To give the rotor of synchronizing motor synchronous speed prior to inclusion in the circuit, you need a special boost motor. This inconvenience is greatly hindered the application of synchronous motor. Modern synchronous motor usually let go in motion by way of the asynchronous start, when boost motor is not required.

12.7 Setting of synchronous motor

For independent setting (without boost motor) in the rotor of synchronous motor is placed in a special short-circuited starting winding on type of squirrel cage. Its elements are shown in figure 12.11, *c*. It consists of rods 1, which go in specified for this purpose grooves, pole bits and two locking rings 2, by which on the butts of the rotor rods are connected between each other. Rods are made from brass, aluminum bronze and other alloys with high specific resistance. Sometimes without starting winding, but in this case, the pole bits are made continuous. On the butts they are electrically connected by dutchmans forming a short-circuit contour, which performs the role of starting cells. In high-speed motors with implicit-pole rotor functions of squirrel cage perform the superficial layers of the solid cylindrical core of the rotor. Meanwhile synchronous motor starts the course by way of asynchronous start. The scheme of such launching is presented in figure 12.12, *a*.

The stator of the motor 1 is switched on three-phase current at full voltage (direct launching) or on low voltage, if it is necessary to reduce the impact of starting current. The rotating magnetic field of stator induct in the starting short-circuit winding 2 EMF and current, which, interacting with the field creates a torque. The rotor comes into rotation, as in asynchronous machine. Due to the asynchronous moment it reaches the speed of rotation at which the slip is usually not greater than 0.05. Execution of starting winding with increased active resistance allows to obtain sufficient starting moment. On the time of acceleration of the rotor to the specified slip (s = 0.05) actuation winding 3 is closed to the external resistance R_s, which should be 10–15 times more than its own.

This is because the rotating field induct also in the actuation winding of EMF, which in the initial moment of launching can be significant, dangerous for winding insulation and staff. After reaching the rotor speed close to synchronous (s ≈ 0.05), the actuation winding is switched to the DC power from the exciter. Meanwhile in addition to the asynchronous moment, proportional to slip, the synchronous moment appears from the interaction of the armature current with a constant magnetic field of the rotor, which depends on the actuation current (EMF E_0) and the angle Θ.

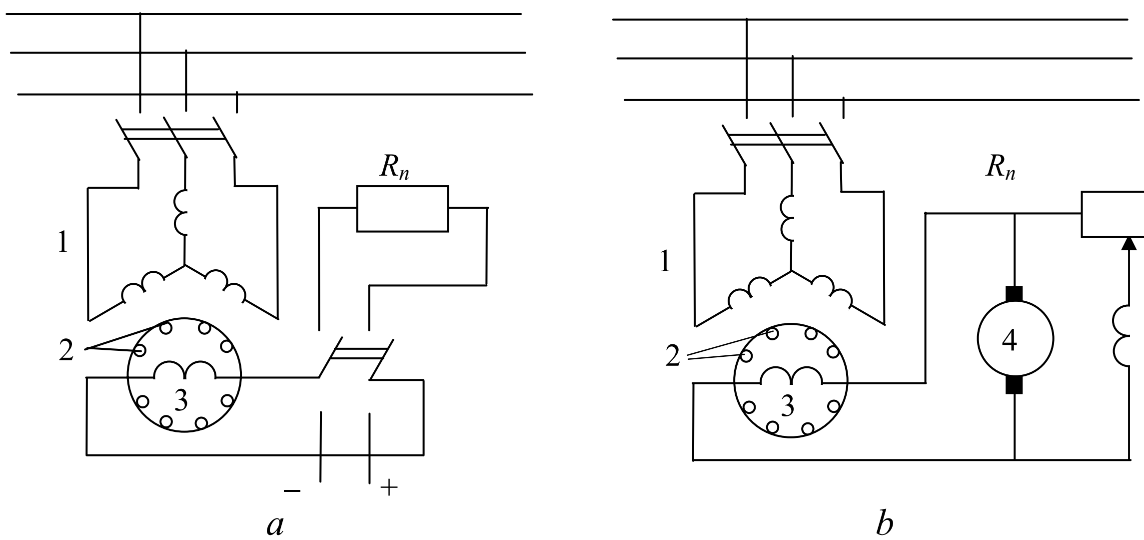


Figure 12.12: Schemes of asynchronous launching of synchronous motor: *a* – with a starting rheostat; *b* – with connected exciter

Due to the slip angle Θ is continuously changes and in intervals of time, when $0 < \Theta < 180°$, synchronous moment, adding to the asynchronous, speeds up the rotor and slip decreases. At achievement s = 0 the motor after some oscillations near synchronous speed enters in synchronism and continues to work as synchronous. The angle Θ at it it is greater than zero and less than 90°. In working mode, the current and EMF of starting winding are equal to zero, equal to zero and its asynchronous moment, as the rotor rotates at synchronous speed. However, when the load changes, when there is a change of the angle Θ and the angle between the axes of the fields Θ/p , the starting cell has a positive effect, reducing the swing of the rotor about the new position corresponding to the changed load.

Also it is used asynchronous launching of synchronous motor with tightly connected with the exciter (see fig. 12.12, *b*). In this case, the actuation winding 3 is connected to the armature of the exciter 4, mounted on the one shaft, with the beginning of the launching. As acceleration the motor becomes excited and on achievement a speed close to synchronous, entered in synchronism. This method requires less starting equipment for control and automation of launching.

Asynchronous launching of synchronous motor is characterized by the same drawbacks as the launching of asynchronous short-circuit motors, the main of which is a large starting current. To limit the starting current of large motors they have resorted to reduction on the launching time the voltage feeded to the stator, using reactor (inductive impedance), included serially to circuit of the stator (reactor launching) or by using autotransformers (autotransformer launching). Management of launching of synchronous motor in modern systems are usually automated.

12.8 Regulation of the power coefficient of synchronous motor

In chapter 12.5, it was noted that at regulation of the actuation current of SM connected in parallel with the circuit, reactive power changes. As the synchronous motor always runs in parallel connection to the circuit, that to it it fully concern said. The change of reactive power at a given active one also means change of the power coefficient cosφ. Hence the ability to control by the actuation current of the rotor power coefficient of synchronous motor results.

Let us explain this using vector diagrams of SMT (see fig.12.13). Let for a given load, the actuation current is set such that the angle φ = 0, cosφ = 1, the stator current coincides with the voltage on phase. Let's call the angle φ = 0, cosφ = 1, the stator current coincides with the voltage on phase. Let's call this actuation current is normal. Corresponding to this mode vector diagram shown in figure 12.13 by vectors:

$$\overline{U_C} = \overline{OB}; \quad -\overline{E_0} = \overline{OA}; \quad \overline{I} \cdot X = \overline{AB}; \quad \overline{I} = \overline{OC}\,.$$

If, without changing the load on the shaft, to change the actuation current, it will change the EMF of the armature E_0 and the angle θ, but

$$M = \frac{m}{\omega} \cdot \frac{E_0 \cdot U}{X} \sin \Theta = \frac{m}{\omega} \cdot \frac{E_0^{'} \cdot U}{X} \sin \theta^{'} \tag{12.16}$$

remains unchanged, i.e.

$$E_0 \sin \theta = E_0^{'} \sin \theta = E_0^{''} \sin \theta^{''} = const\,. \tag{12.17}$$

From here it result that the end of the vector $-E_0$ at change of the actuation current moves in a straight line A'A" parallel to the voltage vector U_c. This is followed by change of the magnitude and phase of inductive voltage drop $I{\cdot}X$ and the stator current I. As

$$I{\cdot}X{\cdot}\cos\varphi = E_0{\cdot}\sin\Theta = \text{const, then and } I{\cdot}\cos\varphi = \text{const.}$$

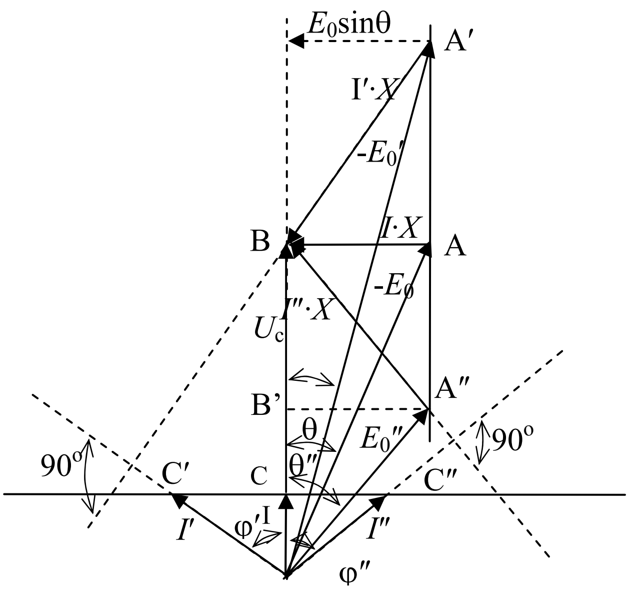


Figure 12.13: Vector diagrams of synchronous motor

Therefore, the active component of the stator current is kept unchanged. The end of the vector of current I at the regulation of the actuation current moves in a straight line C'C", perpendicular to the voltage vector U_c (see fig.12.13).

Thus, due to the actuation current, it is possible to set any synchronous motor mode on power coefficient. By increase of the actuation current in comparison with the normal stator current is ahead of the voltage. In relation to the circuit motor behaves as an active-capacitive load. The motor in this mode is called ***overexcited***.

Engine the vector diagram corresponds to ***overexcited motor*** (see fig. 12.13), represented by the vectors $-E_0'$, $I'{\cdot}X$, U_c and I'. On the contrary, when the reduction of the actuation current (***under excitation***) engine consumes lagging current, and is an active-inductive load network (vectors I'', E_0'', $I''{\cdot}X$). The greatest interest is mode of normal work with cosφ = 1 and mode of overdrive when the motor consumes leading current. In the latter case, due to the reactive (capacitive) component of the motor current it is reached the increase of cosφ in the circuit as a whole, as the main load of the circuit often has active-inductive character.

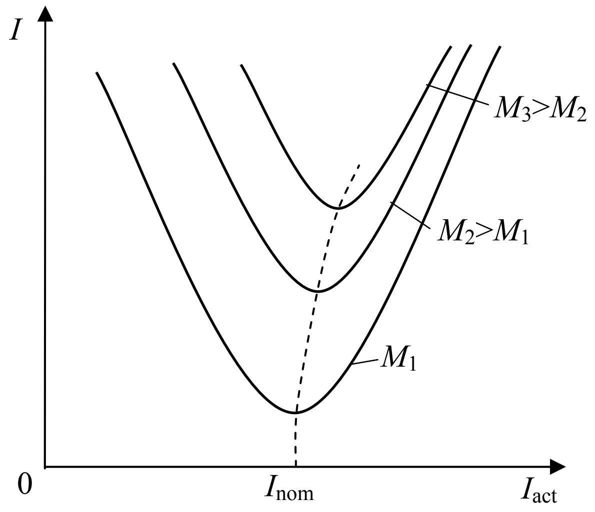


Figure 12.14: Dependence of the stator current od SMT from the actuation current

Synchronous motor is usually calculated for work at nominal load in overexcited mode with cosφ = 0,8. With the change of the actuation current not only the phase changes but also the magnitude of the stator current. The dependence of the stator current from the actuation current when the load is depicted as a U-shaped curve (fig. 12.14). The minimum stator current is at normal actuation current, when cosφ = 1. With increase of load, the minimum of U-shaped curves shifts to larger currents of actuation.

When reduction of the mechanical load on the shaft the active component of the stator current decreases, which expands the possible range of the reactive power component. Synchronous motor on idle without load can be used as a controlled source of reactive power of circuit. For such purposes, however, it is used special SM, which are called ***synchronous compensators***.

Unlike the motor and generator synchronous compensator does not try mechanical loads and therefore is calculated only on a small mechanical power associated with the idle loss. Synchronous compensators are used in electric circuits of power systems for voltage regulation.

12.9 Working characteristics of the synchronous motor

Under the operating characteristics of Synchronous motor it is understood the dependence of speed, torque on a shaft, power factor, cosφ, COP, consumed power and current from the useful power P_2, taken down from the motor shaft, at constant voltage and the actuation current. The graphs of dependencies have the form of the curves presented in figure 12.15.

As the speed of rotation of motor is constant, the speed characteristic $n = f(P_2)$, as well as mechanical $n = f(M)$, is depicted as a straight line, parallel to the abscissa axis, and the line of moment on shaft M – line passing through the origin.

The nature of the dependence $\cos\varphi = f(P_2)$ is caused by established actuation current of the motor. If impart at the expense of actuation current $\cos\varphi = 1$ at rated load, when the underloading the motor will consume leading current (overexcitation), when the overload – lagging current (underexcitation). In both cases, the power factor decreases.

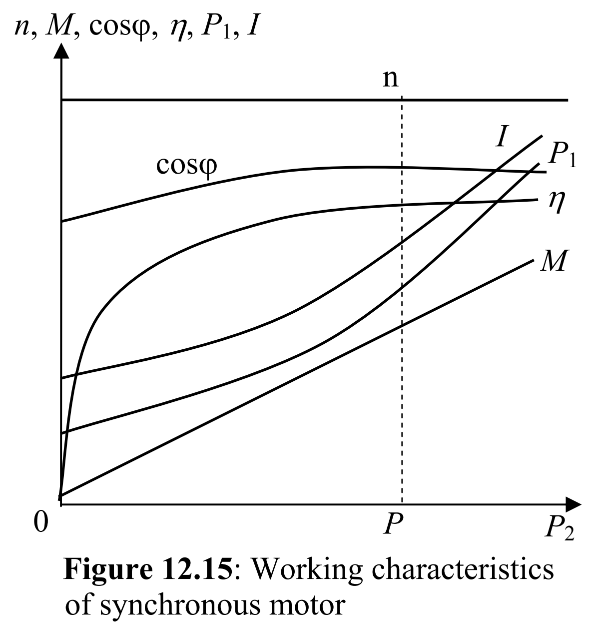


Figure 12.15: Working characteristics of synchronous motor

COP curve has the same form as for AM. In the field of loads from 0.3 to 1.3 P_L changes in COP are low. Characteristic $\eta = f(P_2)$ determines the dependence of the consumption power $P_1 = f(P_2)$, as $P_1 = P_2/\eta$. In turn, from $P_1 = f(P_2)$ and $\cos\varphi = f(P_2)$ it follows the dependency for the current consumed by the stator from the circuit, $I = f(P_2)$, so as

$$I = \frac{P_1}{m \cdot U \cdot \cos\varphi} .$$

12.10 Comparison of synchronous and asynchronous motors

On device synchronous motor is harder then asynchronous, it will cost you more. The relative difference in values is higher at low power of machines. For synchronous motor it is required two kinds of current. It should be noted that at the present time, along with rotating actuation motors with self-excitation from the supply circuit of alternating current through the semiconductor rectifiers get spread.

On starting properties synchronous and asynchronous motors are about the same. The first one has a lower sensitivity to fluctuations in the circuit voltage, as its torque, including maximum, proportional to the first degree of voltage. At second one, this dependence is quadratic. In addition, using in necessary cases automatic forcing (increase) of current of actuation of synchronous motor, it is possible to maintain the stability of its work, even when significant short-term reductions in circuit voltage. In relation of regulation of the speed of rotation synchronous motor yields asynchronous, although adjusting properties of the latter also cannot be considered good.

The major advantage of synchronous motor is its ability to work with $\cos\varphi = 1$ *or even with leading (capacitive) current of the stator.* This advantage is especially important for motors designed for low speed rotation as asynchronous motor with low speeds are characterized by low power coefficient.

Based on all properties synchronous motors are competitive or even excel asynchronous ones at a power of about 100 kiloWatt and above. In some cases it is turned out the appropriate use of synchronous motors also at lesser power.

Key findings

1. Synchronous machine is reversible. It can be used both as a generator and as a motor.
2. In a synchronous generator it happens conversion of mechanical energy into electrical energy of three-phase alternating current. In industry there are two main types of synchronous generators - turbogenerators and hydrogenerators.
3. Frequency of EMF of the generator is determined by the rotation speed and the number of pole pairs of the rotor. Required for a given frequency the speed of rotation of the rotor the less, the greater the number of pairs of poles p. Generators working from low-speed primary motors are multipolar.
4. External characteristics of the generator characterizes the dependence of voltage from the load current $U = f(I)$ at n = const, I_E = const, $\cos\varphi$ = const. To maintain the generator voltage constant when the load is changed according to the size and nature it is regulated the actuation current of the rotor.
5. At the generator electromagnetic torque is opposing, it balances the moment of the primary motor (steam or hydroturbine).
6. At the synchronous motor electromagnetic torque is torque.
7. Mandatory condition of energy conversion in the machine (mechanical to electrical in generator and electrical to mechanical in the motor) is the presence of a phase shift between EMF E_0 and the voltage U at an angle Θ.
8. To turn on the generator for parallel work with the circuit, you must comply with equality of frequency and equality of voltages.

9. Reactive load of the generator is regulated by the actuation current.

10. In synchronous motor, the electrical energy of alternating current coming from the circuit to the stator is converted into mechanical on shaft.

11. Synchronous motor start by the method of asynchronous launching, for what in its rotor a special short-circuit starting winding goes.

12. At the expense of change of actuation current it can be set to synchronous motor any mode on power coefficient. At increase of the actuation current (mode of overexcitation) in relation to the circuit motor behaves as an active-capacitive load. At decrease of the actuation current (mode of underexcitation) motor is an active-inductive load of circuit.

Control questions

1. What is the main feature of SM and area of their use?
2. Explain the structure and functions of the main parts of a synchronous electric machine.
3. The device and the purpose of the main parts of SM.
4. What are the differences in structure of the magnetic core and windings of SM compared to asynchronous one?
5. What is the principle of work of synchronous generator?
6. Structural differences of turbo - and hydrogenerators.
7. What is meant under the characteristic of the idle of generator?
8. On the basis of what data is characteristic of the idle of generator built?
9. What is meant by external characteristics of the synchronous generator and how it depends on the nature of the load?
10. What is meant by electromagnetic power of generator and motor? The role of the electromagnetic torque in the generator and motor.
11. What is the angular characteristic? What range of angle Θ is limited to its stable part?
12. The inclusion conditions of the SG in parallel in the circuit.
13. How on the work mode of SM does the regulation of the actuation current influence?
14. How does the active load of the generator change included in parallel with the high-power circuit?
15. Under what conditions SM goes in mode of synchronous motor? What load range does stable work of SM included in parallel with the circuit remain?
16. How does the actuation current of the rotor influence on the power coefficient of the SMT? Analyze this influence by using vector diagrams.
17. For what purpose are synchronous compensators used and how do they differ from the usual SM?
18. What dependencies are called working characteristics of synchronous motor and what their character?
19. Give a comparative assessment of the electromechanical properties of the synchronous motor with respect to asynchronous one.
20. What is the appropriate application domain of synchronous motor?

SECTION V.
INDUSTRIAL ELECTRONICS AND ELECTRIC DRIVE

Industrial electronics is a branch of science and engineering that studies the structure and work of various electronic devices and their applications in industry. Today electronics has infiltrated all branches of modern science, technology and industry. Electronic devices are used in automation, telemechanics, communications, medicine, physics, mechanical engineering, construction industry and so on.

Electric drive has become one of the major areas of effective use of elements of industrial electronics. It solves control tasks of motors of various types and purposes. Wide industrial application of thyristors has caused significant progress in the field of variable electric drives of AC and DC. Highly effective devices have been created to convert the current of the industrial frequency into alternating current of regulated frequency for speed control of electric motors.

Electronic amplifiers, rectifiers, measuring instruments and other devices have become a powerful tool for automation and control of production processes. The use of managed high-speed semiconductor devices in conventional schemes significantly expands their capabilities in providing new modes of work and, consequently, new functional properties of equipment developed on their basis.

13 FUNDAMENTALS OF INDUSTRIAL ELECTRONICS

Key concepts: diode (valve), thyristor, transistor, emitter, base, collector, rectifier, smoothing filter, amplifier, bandwidth, gain.

Modern scientific and technological progress in many industries, particularly in construction, associated with the development of electronics. Success of electronics is the result of creation of variety of electrovacuum and semiconductor devices. Currently, the number of different types of electronic devices is so great that a full review is beyond the scope of the course "Electrical engineering in construction". Therefore, consideration is limited to the main types of semiconductor devices, which are widely used in various devices, in the electric drive control system of construction machines and mechanisms in particular.

There are power and data electronics in electronic engineering. One of the main tasks of power electronics is to convert various kinds of electrical energy, AC to DC in particular. Information electronics is mainly used for solving problems of information flux management, in particular for the amplification of the signals.

In the frames of the topic the issues of organization and working principles of semiconductor diodes, thyristors and transistors and their use in rectifiers and amplifiers were considered.

13.1 Elements of semiconductor equipment

13.1.1 The principle of operation of semiconductor devices is based on the phenomenon of unidirectional conduction of partition boundaries of two semiconductors with different types of electroconductivity: e (electroconductivity n-type) and hole (electroconductivity p-type). The region of electroconductivity of n-type is characterized by the fact that the current passage here is due to the migration of negatively charged electrons, an excessive amount of which can be created by entering in the single crystal semiconductor of donor admixtures, such as antimony, arsenic, phosphorus. In the field of electroconductivity of p-type current passage is conditioned by the transfer of positively charged “holes” (the hole is an atom with one electron is missing, and which, therefore, has a positive charge, the absolute value is equal to the electron charge). Holes are obtained by the introduction of the single crystal semiconductor of acceptor impurities, such as indium, boron, aluminum.

In addition in doped semiconductors along with the main carriers, the concentration of which is high, there are also minor carriers: holes in semiconductors of n-type and electrons in semiconductors of p-type. In semiconductors without impurities the number of electrons is always equal to the number of holes.

With the direct contact of the two semiconductors, one of which has an electronic, and the other has a hole electroconductivity, we obtain by the so-called electron-hole junction (p-n junction), which main attribute is the dependence of its resistance from the polarity of the applied voltage. Ohmic contacts with leads are created at the p-n regions of the semiconductor to connect to an external circuit.

Using the example of two-layer crystal of silicon let’s consider the processes happened in p-n junction under the influence of the external voltage on it. If a positive potential applies to the p-region, and a negative one applies to the n - region, then the main current carriers will move in the boundary layer towards each other (fig. 13.1, *a*). As a result the resistance of the p-n junction reduces and the direct current I_{dir}, limited practically only by the load resistance R_{L} passes through the boundary. The external voltage U_{dir} of such polarity is called direct or conductive.

When the polarity of the applied voltage changes (fig. 13.1, *b*) the holes in the p-region and the electrons in the n-region of a semiconductor will be deleted from the boundary of a section, which leads to an increase of the resistance of the p-n junction, and the main stream media reduces to zero. A small amount of current generated by minor carriers, for which the applied potential difference is accelerating, passes through the p-n junction. External voltage of such polarity is called reverse U_{rev} or locking, and the resulting small current is called reverse current I_{rev}.

Thus, the value and direction of current flowing through the p-n junction of two-layer semiconductor structure depends on the value and sign of the external voltage, i.e. the p-n junction has rectifying (valve) properties.

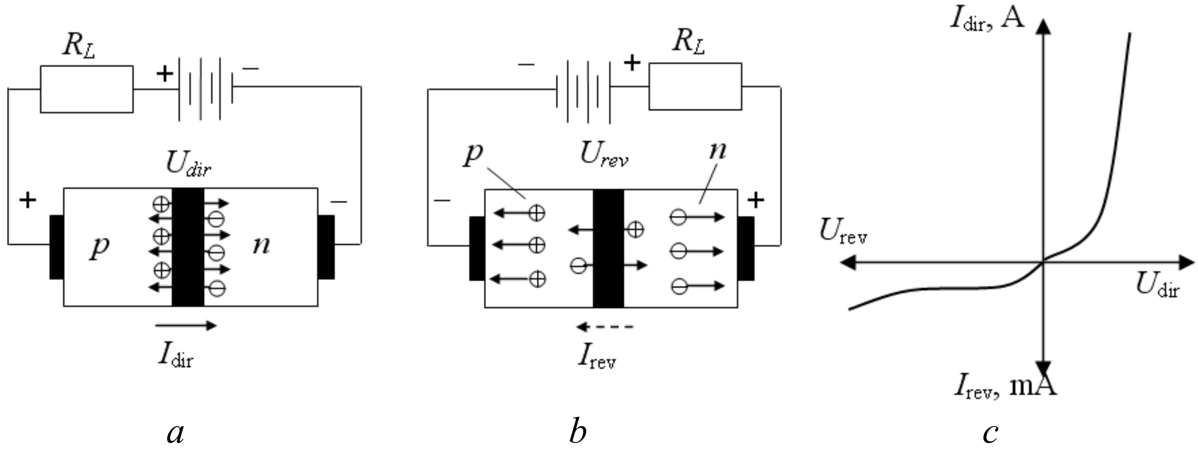


Figure 13.1: The current passing through the p-n junction of semiconductor diode: *a* – open (conductive) state; *b* – closed (no conducting) state; volt-ampere characteristic

The dependence of the current *I* passing through the p-n junction, from the applied voltage *U* is called a ***volt-ampere characteristic*** **(VAC)** ***of the junction***. This characteristic has two branches (fig. 13.1, *c*): one is located in the first quadrant and corresponds to the conductive direction in the p-n junction (direct current in it) the second is located in the third quadrant and characterizes the locking properties of the junction.

13.1.2 Unmanaged diodes. Silicon, germanium and selenium unmanaged diodes are used in devices of industrial electronics. Sometimes they are called valves.

Silicon diodes. Let's consider the device and VAC of a silicon diode (see fig. 13.2). Thin plates cut from a single crystal of silicon with electronic type of electroconductivity are the source material of these diodes. In these plates a layer with the electroconductivity of *p* - type is created by the alloying with aluminum or diffusion of atoms of aluminium or boron into the silicon.

Silicon disc with a p-n junction is soldered between the molybdenum plates (fig. 13.2, *a*) having approximately the same linear expansion coefficient as silicon, and good thermal conductivity. Electrode connected to the semiconductor layer with the electroconductivity of n-type is the ***cathode*** K, and the electrode attached to the layer with the electroconductivity of p-type is the ***anode*** A (fig. 13.2, *a* and *b*).

Thus obtained two-layer single-crystal *p-n* structure is placed in a nonseparable hermetic glass-to-metal or ceramic jar, protecting it from external influences (moisture, dirt, mechanical damages).

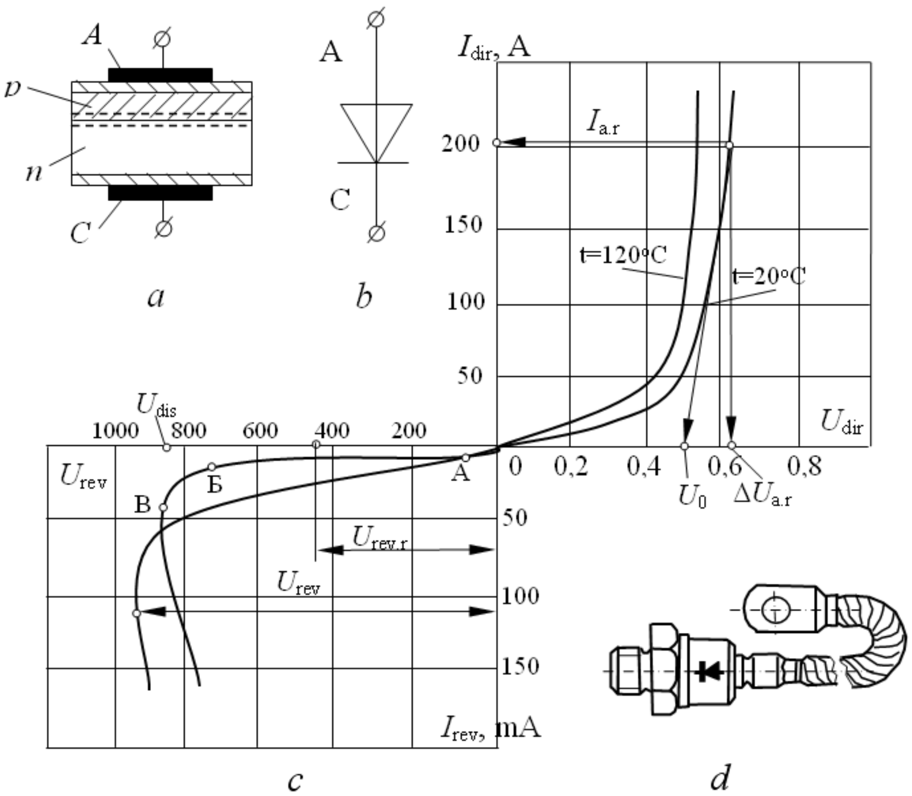


Figure 13.2: Silicon diode: *a* – device, *b* – symbolic notation, *c* – static VAC at different temperatures of the p-n junction; *d* – construction

The lower part of the jar is in the form of hexahedral nut and it ends by the stud with thread for screw driving the valve in the cooler (fig. 13.2, *d*). This diode construction of the jar provides good heat tap from the *p-n* junction in the environment and serves as an external lead of the cathode. External lead of anode is a flexible copper wire with a bit, insulated from the jar by insulator.

In figure 13.2, *c* VAC of a silicon diode at rated current 200 A is shown. Direct branch contains two characteristic sections: in the first section, which coincides with the x-axis, the valve has a relatively large resistance and with the growth of direct voltage the current increases slightly; in the second section with increase $U_{dir} > U_o$ the resistance of the valve decreases sharply, and the direct current I_{dir} increases to values determined by the load resistance.

On the return branch, there are three characteristic sections: the first section of *OA* (fig. 13.2, *c*) is relatively small, the valve has low conductivity, and a small current I_{rev}, measured in milliamperes passes through the junction; in the second section *АБ* with a significant increase in the reverse voltage the current I_{rev} reaches saturation and slightly increases; the third section *БВ* is characterized by the fact that for certain values of the reverse voltage the current I_{rev} sharply increases and the breakdown of the *p-n* junction occurs. The voltage U_{dis}, at which the return branch bends sharply is called the disruptive voltage.

For normal work of the valve maximum permissible (rated) reverse voltage $U_{rev.r}$ is taken twice less compared to the disruptive voltage U_d.

Silicon power valves are available for currents from 10 to 1000 A and the reverse voltage from 100 to 1500 V.

In comparison with silicon **germanium diodes** have lower direct voltage drop, and lower values of permissible reverse voltage (500–600 V compared with 700–1500 V in the silicon diodes). The reverse current of these diodes is significantly larger than that of the silicon diodes.

Selenium diodes allow much lower density of the direct current 50–60 miliA/cm^2 and smaller values of the reverse voltage 40–50 V than germanium and silicon diodes, allowing the density of the direct current 40 to 80 A/cm^2 and the value of the reverse voltage 400–1200 V.

Serial connection of selenium elements in one rectifier column is widely used to increase the value of $U_{rev.acc}$, meanwhile it does not require the use of voltage dividers (equalizing resistance) which germanium and silicon diodes need.

A characteristic feature of selenium rectifiers is large overload capacity (as they have a large thermal inertia) compared to germanium and silicon, as well as less sensitivity to momentary overvoltages. This is because at accidental overvoltages the place of a breakdown is covered by amorphous selenium and selenium element recovers its properties.

Denoted properties of selenium rectifiers are an incentive for their use in rectifiers in low voltage and high currents: chargers, galvanic and electrolytic installations and other.

VAC of the semiconductor diodes depends on the temperature of the *p-n* junction. With the increase of temperature in all types of diodes we have the following: a lower straight voltage drop (see fig. 13.2, *c*) under the same direct currents; a significant increase of the reverse current, accompanied by some increase of the breakdown voltage of the silicon and selenium diodes.

13.1.3 Managed diodes – thyristors. The main element of the managed diodes (the other name is thyristors) is a silicon disc with electronic type of electroconductivity in which a four-layer semiconductor structure is created by special technological methods. Layers with different types of electroconductivity (*p-n-p-n*) alternate in this structure. The result is a monocrystalline structure with three *p-n* junctions Π_1 - Π_2 - Π_3, connected in series (fig. 13.3, *a*).

Thyristor semiconductor structure is mounted either in glass-to-metal or ceramic jar, the base of which has a stud with thread and serves as an external lead of the anode and the cathode is considered to be a flexible copper lead with a bit; a control electrode is taken out to the side of the cathode (fig. 13.4, *c*); or it is placed in a metal-ceramic tablet jar of a round shape, which is sealed by cold welding.

With the help of holding down devices a tablet jar of a diode is connected with coolers from aluminum alloys, providing electrical and thermal contact connections of structure of jar and coolers, which have a developed surface. Lead from the anode and cathode of the thyristor is carried out directly from coolers using copper bars, the lead from the control electrode is located on one side.

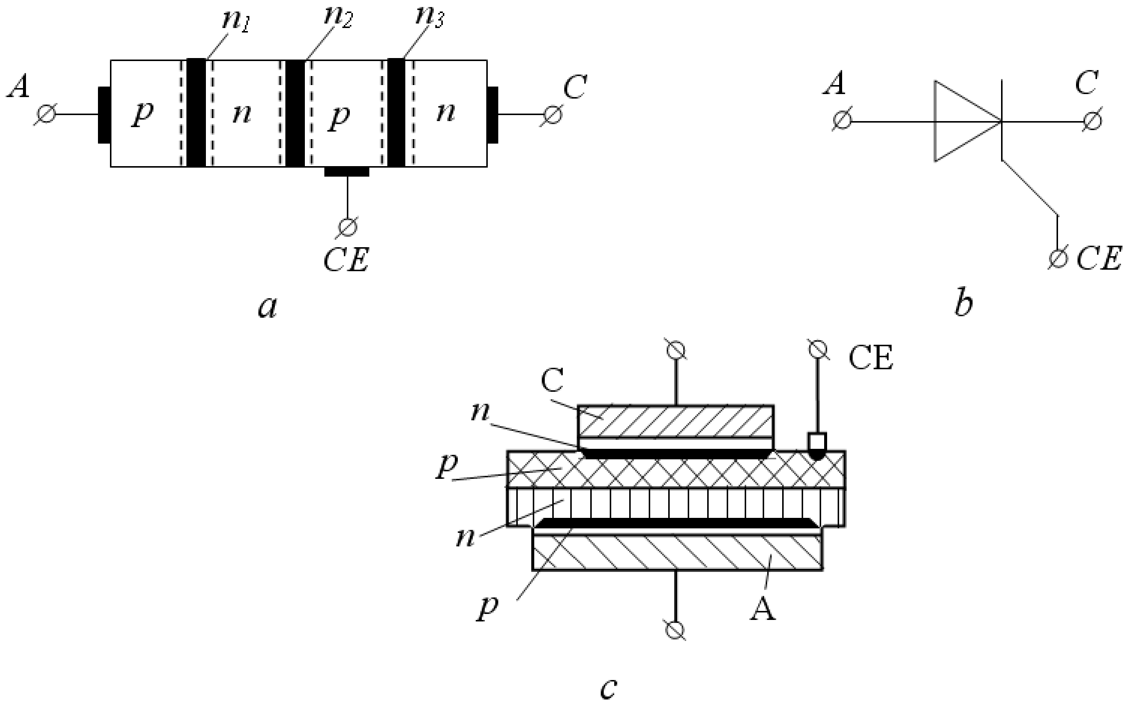


Figure 13.3: Controlled silicon diode - thyristor. *a* – scheme of four-layer structure; *b* - symbolic notation; *c* – structure of thyristor (schematic)

When a thyristor is included in the electric circuit with an adjustable source of DC (fig. 13.4, *a*), the polarity of which can be changed, the relationship between the current, passing through the thyristor in the straight and reverse directions, and the voltage between the anode and cathode, reflects the static VAC (fig. 13.4, *b*). If the circuit of the control electrode of the thyristor is not connected to the source U_c, and the voltage between the anode and cathode does not exceed the permissible value of switching U_{swit} then independently from the polarity of the applied voltage U_{source} between the anode and cathode the current practically does not pass.

Indeed, if a negative potential is fed on the anode of the thyristor and a positive one is fed on the cathode then a straight voltage will be applied to the middle junction J_2 (fig. 13.3, *a*), and a reverse voltage U_{dir} will be applied to the junctions J_1 and J_3 which are connected serially. The thyristor is locked. Through it a small reverse current I_{rev} passes in the external circuit that corresponds to an inverse branch of VAC of the thyristor, which is similar to a branch $I_{rev} = f(U_{rev})$ of power silicon diode (see fig. 13.2, *c*). If potentials of opposite polarity are applied to the anode and cathode of the thyristor then a reverse voltage will be applied to the junction J_2 that again determines the closed state of the thyristor.

The transfer of a thyristor from a closed state to an open state can be accomplished in two ways:

1. by feeding a straight voltage exceeding the voltage of the switching $U_{\text{пер}}$ (fig. 13.4, *b*)to the anode of the thyristor, meanwhile its resistance (thyristor opens) sharply decreases and the current increases, value of which is limited by the resistance of the external circuit. However, such opening on the anode (voltage) for thyristors is usually not allowed;

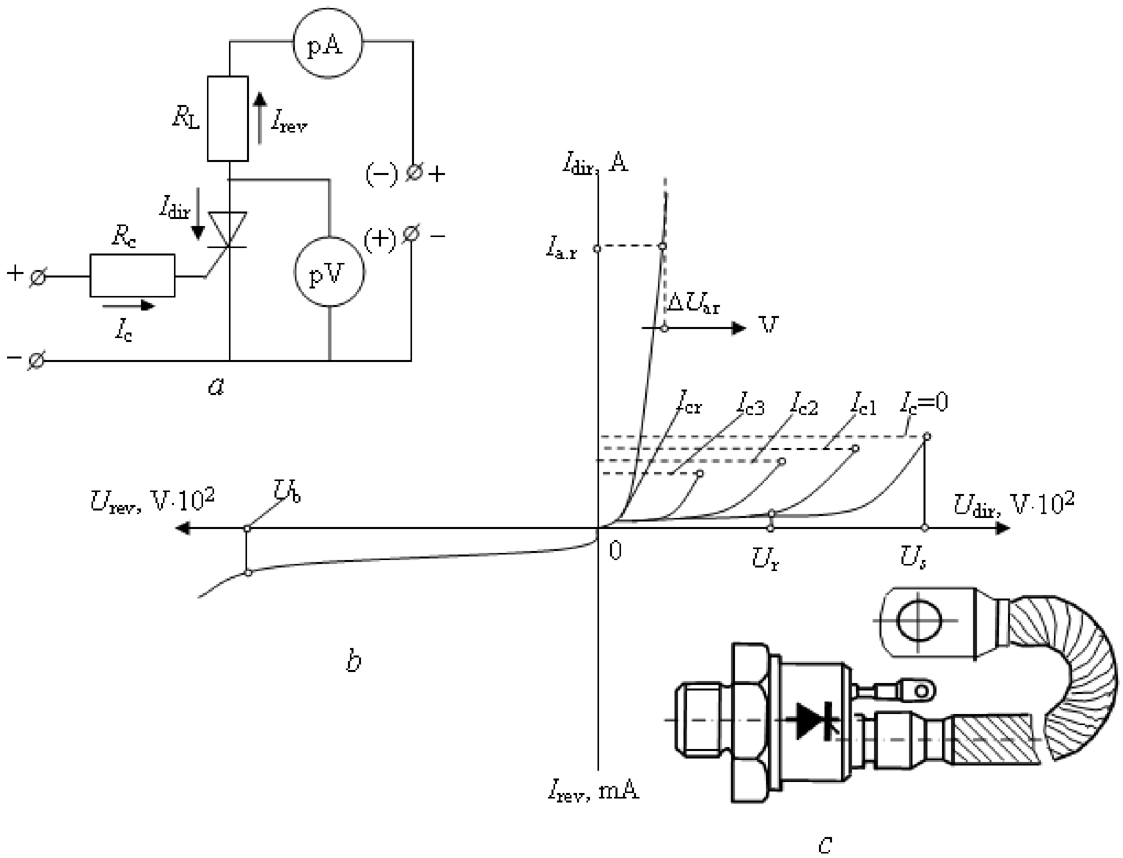


Figure 13.4: Thyristor: *a* – scheme for taking off characteristics; *b* – static VAC; *c* – general view of the thyristor without heatsink

2. by feeding a positive pulse of voltage U_c at the straight voltage at the anode of the thyristor to the control electrode. A small current I_c will pass through the junction J_3 under the action of U_c neutralizing the effect of the closed junction J_2, and the thyristor is opened at a lower value U_{switch}. A direct current I_{dir} which value is practically limited only by the resistance of the external circuit R_L, since the voltage drop in the open thyristor is very small and does not exceed 0.5–1.2 V (fig. 13.4 – direct branch) will pass in the direction from the anode to the cathode of the thyristor.

This process of firing of the thyristor occurs very quickly (no more than 15–20 microseconds). With the growth of I_c the voltage of switching U_{switch} of the thyristor reduces and an open state of the device corresponds to VAC of a normal unmanaged diode. If you change the polarity of the voltage applied between the anode and cathode of the thyristor then the previously open junction J_2 for 25–250 microseconds (depending on the power thyristor) recovers its blocking properties and the thyristor is again ready for work.

The thyristor powering from the direct current source can restore its blocking properties only by breaking the anode circuit or creating briefly the negative voltage on the anode using special devices. Powered from a source of alternating voltage the thyristor is closed during the negative half-wave of voltage.

13.1.4 Semiconductor triodes – transistors are electronic devices based on two properties located on two, located very close to each other electron-hole *p-n* junctions. The presence of three layers with different conductivity causes on the borders of their division two *p-n* junctions, characterized by a dynamic equilibrium.

The transistors are divided into two groups such as bipolar and unipolar. Transistors the current in which is due to the carriers of two types (electrons and holes) rate to bipolar ones. In unipolar (also called field) transistors the current is caused by carriers of the same sign (or electrons or holes). Let's consider the device and working principle of the transistor using an example of bipolar transistors.

The bipolar transistor is a three-layer structure type *n-p-n* (fig. 13.5) and type *p-n-p*. In figure 13.6, *a* and *b* the conventional images of these transistors are shown. The transistor is called bipolar because the physical processes in it are connected with the movement of the charge carriers of both signs (free holes and electrons).

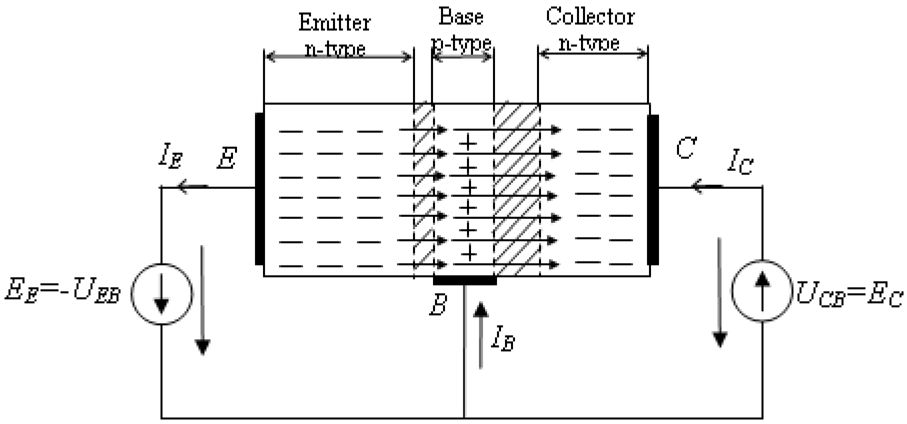


Figure 13.5: Bipolar transistor of type *n-p-n*

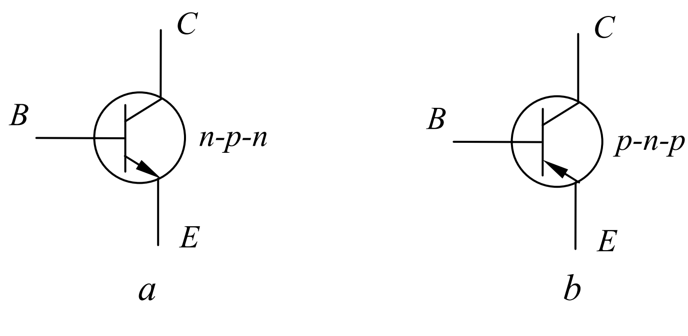


Figure 13.6: Designations of transistors:
a – type *n-p-n*; b – type *p-n-p*

The middle layer of a bipolar transistor is called the ***base*** *B*, one extreme layer – ***collector*** *C*, and the other extreme layer – the ***emitter*** *E*. Each layer has a lead with which the transistor is included in the circuit.

There are three variants of the inclusion scheme of the transistor (see tabl. 13.1): with a common emitter (CE), with a common base (CB) and a common collector (CC).

Table 13.1: Schemes of inclusion of the transistors

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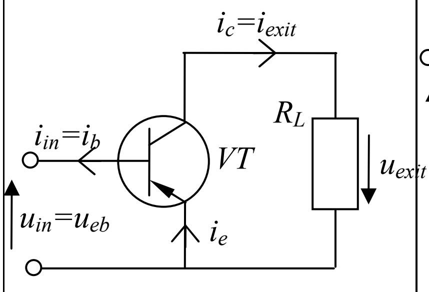
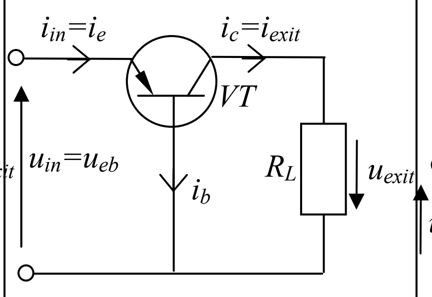
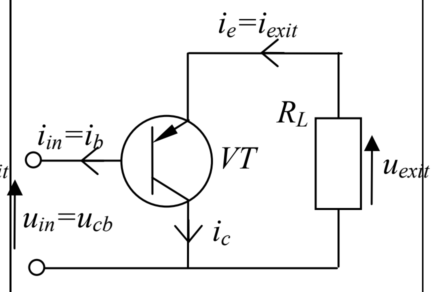
<tr><td>with a common emitter</td><td>with a common base</td><td>with a common collector</td></tr>
<tr><td>
</td><td>
</td><td>
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Regardless of the inclusion scheme transistors can work in one of four modes with different polarity of the voltage at the junctions of the emitter-base and collector-base.

1. Normal active mode in which the emitter-base junction is included in the straight direction and the collector-base junction is turned on in reverse one;

2. Inverse active mode in which the emitter-base junction is included in the reverse direction and the collector-base junction is turned on in straight one;

3. The cutoff mode in which both junctions are turned on in the reverse direction;

4. The saturation mode in which both transitions are turned on in the straight direction.

The saturation mode and the mode of cutoff are used in digital and pulse devices.

In schemes in which the transistor is used to amplify the signals the basic is its active mode. When the positive pole of the source of constant EMF $E_E = -U_{EB}$ is connected to the base, the potential barrier of the *p-n* junction (*n-p-n* transistor in fig. 13.5) between the base and emitter decreases. Free electrons are diffused (are injected) from the emitter into the base, forming a current I_E in the circuit of the emitter. If between the collector and base the source of constant EMF $E_C = U_{CB}$ is included by negative pole to the base, then the potential barrier of the *p-n* junction between the base and collector increases. A large part of the electrons injected from the emitter into the base, is pulled by a strong electric field with intensity E_{CB} of this *p-n* junction, forming a current I_C in the circuit of the collector. Note that the electric field at the junction of the collector-base also exists at open branches from the source of EMF E_c. Therefore, the collector current depends a little from the voltage values $U_{CB} > 0$. A small part of free electrons injected from the emitter into the base forms a current I_B in the base circuit.

The relationship between the currents of collector and emitter circuits of the transistor is characterized by the current transfer ratio

$$\alpha = i_C/i_E \,. \tag{13.1}$$

The number of recombining in the base main charge carriers of the emitter determines the base current: $i_B = i_E - i_C$. When studying the amplifying properties of transistors for variable signals the schemes of their inclusion are considered without power sources, as on comparison with other resistances the internal resistances of the power sources are very small. The scheme with the CE is the most commonly used (see tabl.13.1). With the help of this scheme amplification on current, voltage, power is realized. For this circuit the gains of current, voltage and power are determined from the expressions:

$$k_i = \frac{i_{\text{exit}}}{i_{\text{in}}} = \frac{i_{\text{C}}}{i_{\text{B}}} = \frac{\alpha}{1-\alpha} = \beta > 1; \tag{13.2}$$

$$k_u = \frac{u_{\text{exit}}}{u_{\text{in}}} = \frac{i_{\text{C}} \cdot R_{\text{L}}}{i_{\text{B}} \cdot R_{in}} = \alpha \frac{R_{\text{L}}}{R_{\text{EB}}} > 1; \tag{13.3}$$

$$k_p = k_i \cdot k_u = \frac{\alpha^2}{1-\alpha} \cdot \frac{\text{R}_{\text{L}}}{\text{R}_{\text{EB}}} > 1 \ ; \tag{13.4}$$

where: $\boldsymbol{R}_{\mathbf{EB}}$ – the resistance of the junction emitter-base; $\beta = \dfrac{\alpha}{1-\alpha}$.

The output voltage u_{exit} is in antiphase with the input voltage u_{in}.

For the scheme of the transistor with CB the gain of current, voltage and power are of expressions:

$$k_i = \frac{i_{\text{exit}}}{i_{\text{in}}} = \frac{i_{\text{C}}}{i_{\text{B}}} = \alpha < 1 \ ; \tag{13.5}$$

$$k_u = \frac{u_{\text{exit}}}{u_{\text{in}}} = \frac{i_{\text{C}} \cdot R_{\text{L}}}{i_{\text{B}} \cdot R_{\text{in}}} = \alpha \frac{R_{\text{L}}}{R_{\text{EB}}} > 1; \tag{13.6}$$

$$k_p = k_i \cdot k_u = \alpha^2 \cdot \frac{R_{\text{L}}}{R_{\text{EB}}} > 1 \ . \tag{13.7}$$

Inclusion of the transistor on the scheme with a common base is usually used at higher frequencies however this scheme is characterized by the gain current smaller one $k_{\text{i}} < 1$. The output voltage u_{exit} is in the phase with the input voltage u_{in}.

For the scheme of the transistor with CC the gain of current, voltage and power are of expressions:

$$k_i = \frac{i_{\text{exit}}}{i_{\text{in}}} = \frac{i_{\text{E}}}{i_{\text{B}}} = \frac{i_{\text{C}} + i_{\text{B}}}{i_{\text{B}}} = \beta + 1 = \frac{1}{1-\alpha} > 1 \ ; \tag{13.8}$$

$$k_u = \frac{u_{\text{exit}}}{u_{\text{in}}} = \frac{u_{\text{exit}}}{u_{\text{exit}} + u_{\text{EB}}} = \frac{i_{\text{E}} \cdot R_{\text{L}}}{i_{\text{E}} \ (R_{\text{L}} + R_{\text{EB}} \ )} = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{eb}}} \le 1; \tag{13.9}$$

$$k_p = k_i \cdot k_u \cong k_i = \beta + 1 = \frac{1}{1-\alpha} > 1 \ . \tag{13.10}$$

The output voltage for the scheme with CC is in the phase with the input.

Considering the main amplifying schemes of inclusion we proceed from the fact that the work of the transistor takes place in the linear parts of its characteristics, which corresponds to small input signals, and at the calculation of the gain of the transistor-resistor amplifiers, taking into account the working conditions at medium frequencies, the influence of input transition and output capacitances are neglected.

The main characteristics of the transistors are ***static input and output characteristics***, the type of which depends on the scheme of the inclusion of the transistor.

As an example, let's consider these characteristics for the scheme with the CE (fig. 13.7).

The input characteristic of a transistor is considered to be the dependence of the input current from the input voltage at the constant output voltage. For schemes with CE it is $I_B(U_B)$ at U_C = const.

The output characteristic is the dependence of the output current from the output voltage at a constant input current. For schemes with the CE it is $I_C(U_C)$ at I_B = const.

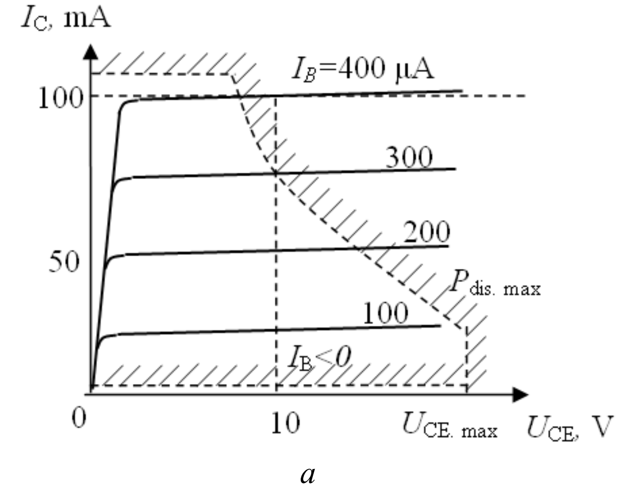


Figure 13.7: Static characteristics of the transistor in the scheme with the CE: *a* – output characteristic; *b* – input characteristic

Static characteristics of a bipolar transistor are shown in figure 13.7. The area of working modes of the transistor on its characteristics is limited to the maximum allowable values of current $I_{C.max}$ voltage U_{CEmax} and power of dissipation $P_{dis.max} \approx U_{CE} \cdot I_C$, and also nonlinear distortion at low values of collector current.

The main advantage of the bipolar transistors is high performance at high collector currents. The presence of external heat sinks allows the bipolar transistors in the power dissipation of 50 W and currents up to 10 A to work. Their main disadvantage is a relatively small resistance of the input circuit at inclusion on the scheme with the CE (1 to 10 kΩ).

13.2 Semiconductor rectifiers

Rectifier is a device designed to convert AC to DC. In practice, many schemes of rectifiers such as single-phase and three-phase current are used. The choice of a particular scheme is determined by the properties of the used diodes and working conditions of the rectifier. For example, in rectifier aggregates for charging of rechargeable batteries that require a small value of the rectified voltage, the most appropriate was the scheme of single-phase rectification with selenium diodes. When straightening the high voltages up to 1000–1500 V we often resort to the serial connection of diodes or use the diodes for large values U_{rev}.

Let's consider the work of the main schemes of rectification of single-phase and three-phase current, assuming for simplicity of calculations and facilitation of understanding of the physical nature of processes in the elements of the schemes that the rectifier works in the active load and consists of ideal diodes and transformer, which allows to neglect the voltage drop and the reverse current of the diode, the inductance and the magnetizing current of the transformer.

Usually the main elements of the rectifier (fig. 13.8) are: a power transformer 1 that is used to match the input and U_c output U_d voltage of the rectifier, as well as for the electrical separation of the supply circuit and the load circuit; a unit of rectifier elements 2 carrying out the rectification of the alternating current; a smoothing filter 3 that is designed to reduce the pulsation of the rectified current in the load circuit 4. If the rectifier is controlled, then another node 6 is included in the block scheme. This node contains the system of control by rectifier unit (thyristors). To protect the rectifier from damage during emergency conditions a block protection and alarm 5 is included in its scheme.

In some cases in the rectifier scheme some elements may be absent. For example, the filter 3, when the rectifier works to the load of an inductive nature, or the power transformer 1, when the rectifier is included without transformer.

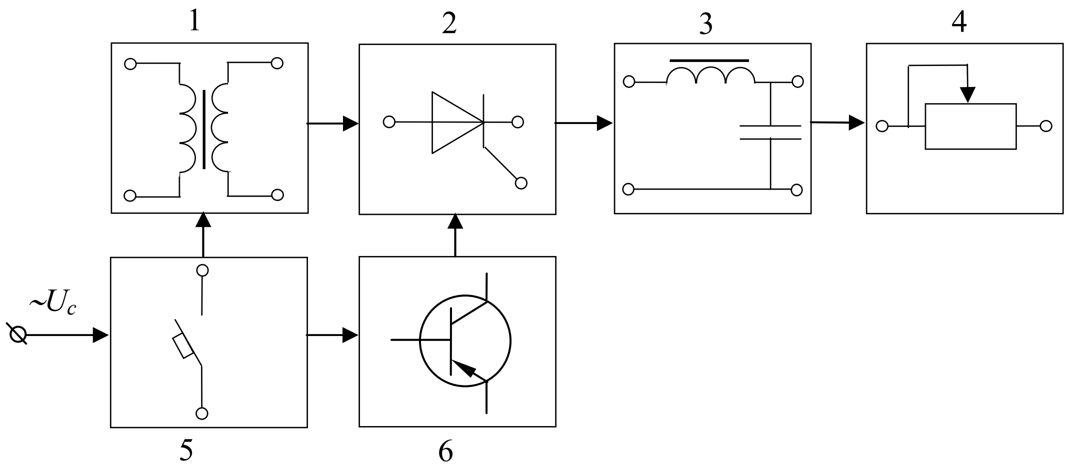


Figure 13.8: Structural scheme of the rectifier

13.2.1 Rectifiers of single-phase current. Let's consider the scheme of ***single-phase half-wave rectifier*** (fig.13.9). In this scheme the transformer has a single secondary winding the voltage u_2 of which varies according to the sine law $u_2 = U_{Max2}\ \sin\omega t$. The current in the load circuit R_d passes only the positive half-periods (fig. 13.9, *b*), when the point *a* of the secondary winding, to which the anode of the diode *VD* is attached, has a positive potential relatively to the point *b*. In the negative half-periods (the time interval $t_1 - t_2$) reverse voltage is applied to the diode *VD*, and it will be closed.

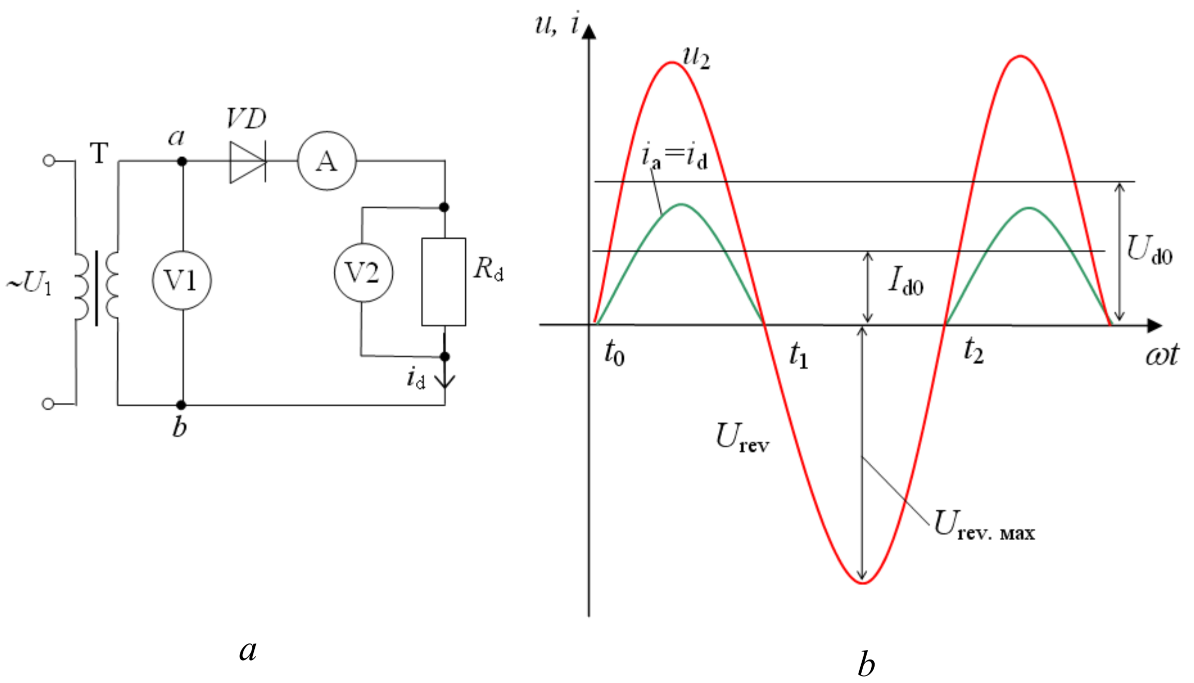


Figure 13.9: Single-phase half-wave rectifier: *a* – scheme;
b – diagram of voltages and currents

The rectified voltage u_d is described by the positive half-wave of voltage u_2 of the secondary winding of the transformer. *The average for the period value of the voltage at the load is called **rectified voltage*** U_d. Load current R_d is in one direction, but has pulsating character and represents the rectified current i_d.

The rectified voltage u_d and current i_d contain a constant component U_d and I_d and a variable component (pulsation) $U_{d\sim}$ and $I_{d\sim}$. The qualitative aspect of the work of the rectifier is estimated by the ratio between the constant component and a pulsation of voltage and current.

For half-wave schemes the following relations between voltages, currents and powers in the individual elements of the rectifier are fair.

The average value of the rectified voltage

$$U_{d0} = 0{,}45 \cdot U_2. \tag{13.11}$$

The maximum value of the reverse voltage applied to the diode in a nonconducting part of the period:

$$U_{\text{rev.max}} = \sqrt{2} \cdot U_2 = 3{,}14 \cdot U_{d0} \ , \tag{13.12}$$

where $\boldsymbol{U_2}$ – the active voltage of the secondary winding of the transformer *T*.

The average value of the current passing through the diode and the load

$$I_a = I_d = U_{d0}/R_d \ . \tag{13.13}$$

The average power delivered to a load is determined by the product of the voltage U_d and current I_d, i.e. $P_d = U_d \cdot I_d$.

Estimated (typical) power of the transformer defining its dimensions, in 3.09 times greater than the power in the load R_d :

$$S_{tr} = 3{,}09\ U_d \cdot I_d. \tag{13.14}$$

Thus, *the rated power of the transformer, working on the half-wave rectifier, is greater than the power in the load, as non-sinusoidal current with constant and variable components passes in its secondary winding, and the current of fundamental frequency f_1 and the currents of highest harmonics pass in the primary winding.* In relation to power circuit these currents are reactive, and without producing useful power, only heat up the windings of the transformer of rectifier.

The active value of current of the secondary winding of the transformer is determined by the ratio

$$I_2 = 1{,}57 \cdot I_d . \tag{13.15}$$

From the formula (13.15) it is followed that the readings of the ammeter of electromagnetic system A_2 included in the circuit of the secondary winding of the transformer T (fig. 13.9, *a*), in 1.57 times exceed the readings of magnetoelectric ammeter A_d, because the first one measures the active current and the second one – the average current in the load circuit.

The active value of the voltage of the secondary winding

$$U_2 = 2{,}22 \cdot U_d . \tag{13.16}$$

The active current of the primary winding subject to coefficient of transformation $k_{tr} = w_1/w_2$

$$I_1 = 1{,}51 \frac{1}{k_{tr}} I_d . \tag{13.17}$$

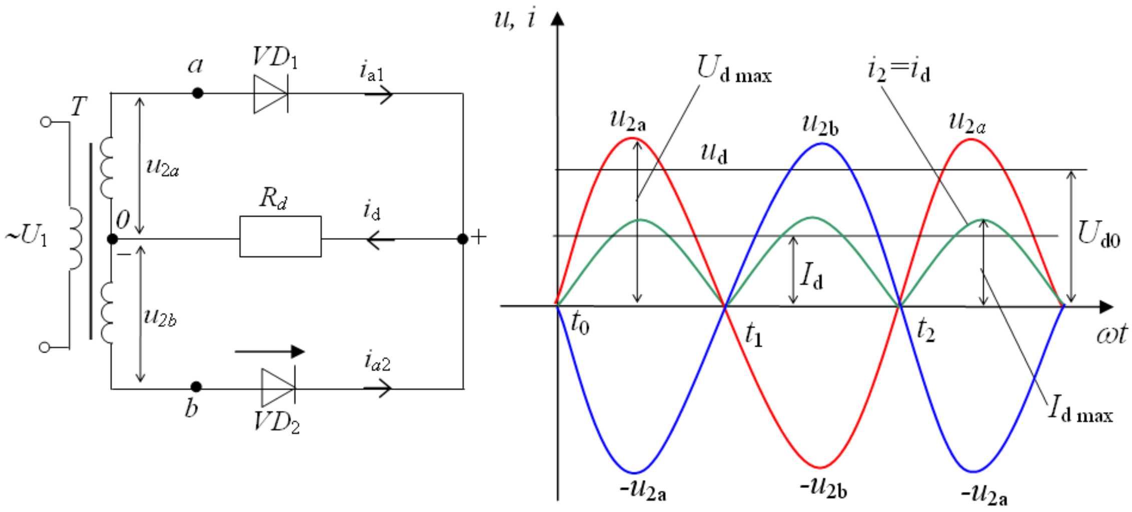
The considered scheme of a half wave rectifier has the following disadvantages: poor use of the transformer, a high reverse voltage on diodes, a large coefficient of pulsation of the rectified voltage.

The advantages of this scheme include its simplicity (it is used only one diode) and the simplicity of the feeding transformer (no zero point on the secondary winding as at the full wave circuit).

A full-wave single-phase scheme consists of a transformer with one primary and two secondary windings connected serially with the lead of common (zero) point of these windings (fig. 13.10, *a*). The transformation ratio k_{tr} is determined by the ratio U_1/U_2 , where U_2 is the voltage of one secondary winding (phase voltage).

The free ends of the secondary windings a and b are joined to the anodes of the diodes VD_1 and VD_2, the cathodes of which are connected together. The load R_d is turned on between the cathodes of the diodes (the positive pole of the rectifier), and a zero lead 0 of the transformer (the negative pole of the rectifier).

The diodes in this scheme, and the secondary winding of the transformer work alternately letting the current at positive values of anode voltage u_{2a} and u_{2b} (fig. 13.10, *b*), which usually take direction coinciding with the conductivity of valves, pass in the load.



a *b*

Figure 13.10: Single-phase full-wave rectifier: *a* – scheme; *b* – diagram of voltages and currents

Indeed, changing the voltage at points *a* and *b* according to the law $u_2 = U_{max2} \cdot \sin\omega t$ in the half-cycle when the voltage in the winding 0 is positive, the current conducts the diode VD_1, the anode of which is positive relative to the cathode connected through the resistance R_d with the point 0 of the secondary winding. The anode of the diode VD_2, as well as the lead *b* of winding *0-b*, in this half-period ($t_0 - t_1$) is negative with respect to the zero lead *0* and, hence, it does not let the current pass.

In the next half-cycle (time interval $t_1 - t_2$ in the figure 13.10, b), when the voltages on the primary and secondary windings of the transformer change their polarity into opposite one, the current will pass diode VD_2, and the diode VD_1 is locked by negative voltage. The current in the load R_d all the time flows in one direction – from the cathodes of the diodes to the zero point *0* of the secondary winding of the transformer.

For this scheme the following relations between voltages, currents and powers in the individual elements of the rectifier are fair.

The average value of the rectified voltage with ideal diodes and the transformer

$$U_{d0} = 0{,}9 \cdot U_2 . \qquad (13.18)$$

Diode, not working in the negative part of the period, is under the influence of a reverse voltage equal to double phase one, as the positive potential of the lead a (b) of the secondary winding of the transformer through the conducting diode VD_1 (VD_2) moves to the cathode of the diode VD_2 (VD_1), and the anode of the closed diode has a negative potential.

The maximum value of the reverse voltage is equal to:

$$U_{\text{rev.max}} = 2\sqrt{2} \cdot U_2 = 3{,}12 \cdot U_{d0} . \qquad (13.19)$$

The average value of the rectified current in the load

$$I_d = \frac{U_d}{R_d} = \frac{U_2}{1{,}11 \cdot R_d} . \qquad (13.20)$$

The average value of the current through each diode is 2 times less than the current I_d flowing through the load, i.e. $I_a = 0,5 \cdot I_d$.

The effective value of the current of the diode $I_{a.eff}$ is equal to the effective value of the current of the secondary winding of the transformer I_2 and is determined by the ratio

$$I_2 = 0,785 \cdot I_d = 1,57 \cdot I_a . \quad (13.21)$$

The active voltage of the secondary winding

$$U_2 = 1,11 \cdot U_{d0} . \quad (13.22)$$

The active current of the primary winding with regard to the transformation ratio k_{tr} will be equal:

$$I_1 = \sqrt{2}\frac{1}{k_{tr}} I_2 = 1,11\frac{1}{k_{tr}} I_d . \quad (13.23)$$

The estimated capacity of the windings of the transformer is determined by the products of the active values of current and voltage:

$$S_2 = 2 \cdot I_2 \cdot U_2 = 2 \cdot 0,785 \cdot I_d \cdot 1,11 \cdot U_{d0} = 1,74 \cdot P_d ; \quad (13.24)$$

$$S_1 = I_1 \cdot U_1 = 1,11\frac{1}{k_{tr}} I_d \cdot 1,11 k_{tr} U_{d0} = 1,23 P_d . \quad (13.25)$$

Estimated power of transformer:

$$S_{tr} = \frac{S_1 + S_2}{2} = \frac{1,23 + 1,74}{2} P_d = 1,48 \cdot P_d . \quad (13.26)$$

The frequency of the main harmonic of the variable component of the rectified voltage in this scheme is equal to double frequency of circuit $2 \cdot f_1$. The coefficient of pulsation of voltage at the lead of the rectifier is equal to:

$$q = \frac{2}{m^2 - 1} = \frac{2}{2^2 - 1} = 0,67 , \quad (13.27)$$

where $\boldsymbol{m}$ – the number of phases of the rectifier, i.e., the number of half-wave rectified voltage per one period of the AC supplied rectifier.

Single-phase bridge scheme consists of a transformer *Tr* with two windings and four diodes VD_1, VD_2, VD_3 and VD_4, united under the scheme of the bridge (fig. 13.11, *a*). Secondary winding joins to one diagonal of the bridge (points 1, 3), load R_d is included to another (points 2, 4). The common point of the cathodes of the diodes VD_1 and VD_2 is the positive pole of the rectifier, and the negative one is the point of connection of the anodes of the diodes VD_3 and VD_4.

The diodes in this scheme work in pairs alternately. In the positive voltage half-period u_2 the diodes VD_1 and VD_3 conduct the current, and the reverse voltage is put to the diodes VD_2 and VD_4 and they are closed. In the negative half-cycle voltage u_2 the diodes $VD_2$and VD_4 will conduct the current, and the diodes VD_1 and VD_3 withstand the reverse voltage.

The current i_d in the load passes all the time in one direction – from the united cathodes of the diodes VD_1 and VD_2 to the anodes of the diodes VD_3 and VD_4. The current i_2 in the secondary winding of the transformer (fig. 13.11, *b*) changes its direction every half-period and will be sinusoidal. The constant component of current in the secondary winding is absent. Therefore, there is no magnetization of the transformer core by constant magnetic flux. The current in the primary winding of the transformer is also sinusoidal.

The average value of the rectified voltage U_{d0} and current I_a through the valve in this circuit turn out the same as in the scheme with a zero point.

A reverse voltage applied to the closed diodes is determined by the voltage u_2 of the secondary winding of the transformer (fig. 13.11, *b*), so as not working in this half-cycle the diodes are connected to the secondary winding of the transformer through two other working diodes, the voltage drop in which can be neglected, i.e.,

$$U_{\text{rev.max}} = \sqrt{2}U_2 = 1{,}57 \cdot U_{d0} \ . \quad (13.28)$$

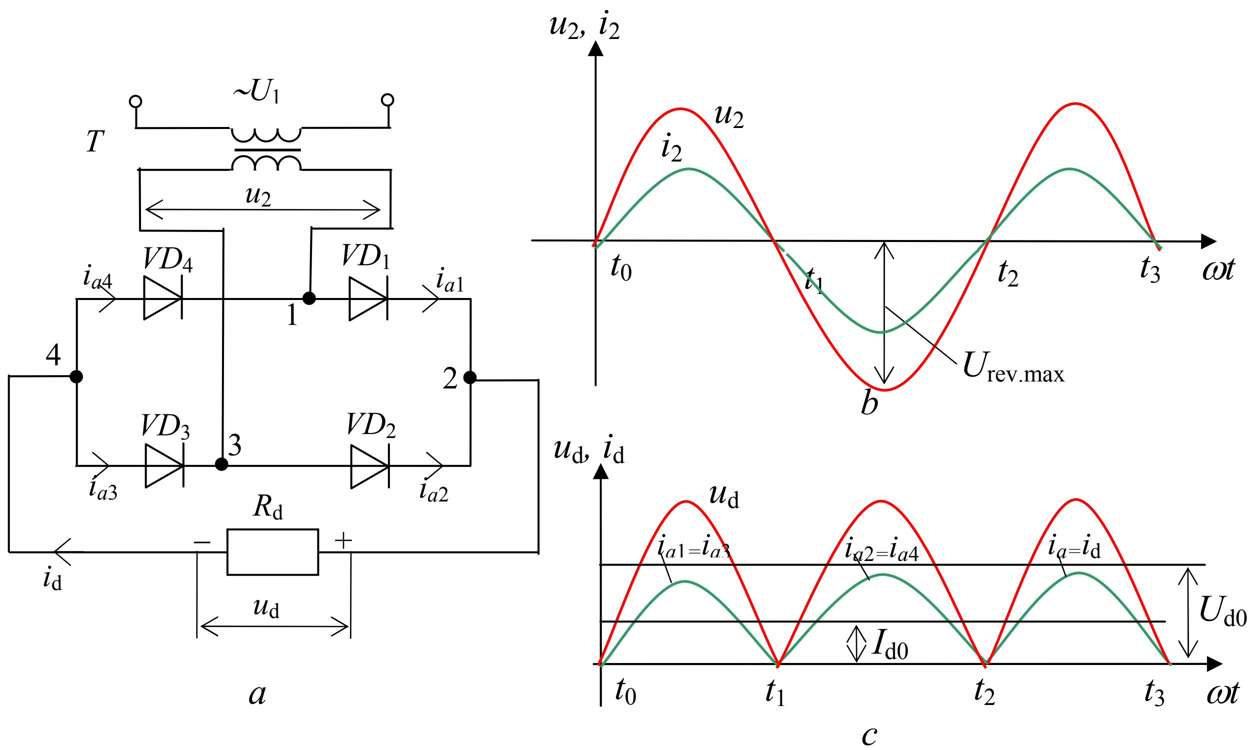


Figure 13.11: single-phase bridge rectifier: *a* – scheme; *b* and *c* – diagrams of voltages and currents on the schema elements

Correlations between other variables for single-phase bridge circuit are shown in the table 13.2.

Let's compare single-phase circuit of rectification.

Single-phase zero scheme:

1. The number of diodes is in 2 times less than in the single-phase bridge.
2. The power loss in the rectifier will be less, as in the zero scheme the current passes through one diode and in the bridge scheme it passes sequentially through two.

Table 13.2: The ratio between the currents and voltages in rectifiers

The rectifier scheme	The dependence U_d from angle of regulation in continuous mode	$U_{d0}/U_{2ф}$	$U_{rev.max}/U_{d0}$	I_a/I_d	$I_{a.active}/I_d$	I_2/I_d	S_{tr}/P_d	Coefficient of pulsation, %
Single-phase full-wave (zero)	$U_d = U_{d0} \cdot \cos\alpha$	0.9	3.14	0.5	0.785 / 0.71	0.785 / 0.71	1.48 / 1.34	67
Single-phase bridge	$U_d = U_{d0} \cdot \cos\alpha$	0.9	1.57	0.5	0.785 / 0.71	1.11 / 1.0	1.23 / 1.11	67
Three-phase zero	$U_d = U_{d0} \cdot \cos\alpha$	1.17	2.09	0.33	0.585 / 0.577	0.585 / 0.577	1.37 / 1.35	25
Three-phase bridge	$U_d = U_{d0} \cdot \cos\alpha$	2.34	1.05	0.33	0.577	0.817	1.05	6

***Notes**: 1. For uncontrolled rectifiers $\alpha = 0$, $\cos\alpha = 1$ and $U_d = U_{d0}$.*
2. For all schemes it is accepted U_2 – phase voltage and $X_2 = 0$.
3. Ratios for $I_{a.active}$, I_2, S_{tr} specified at $L_d = 0$ (the numerator) and $L_d = \infty$ (the denominator).

Single-phase bridge circuit:

1. The reverse voltage on the diodes is in 2 times less than in the zero scheme.
2. The voltage (the number of turns) of the secondary winding is twice less with the same value U_{d0} .
3. The transformer has a standard version, as there is no lead of the middle point on the secondary winding.
4. The rated power of the transformer is 25% less than in the zero scheme, therefore, lower copper and iron is used, the smaller the size and weight will be.

This scheme of the rectifier can operate without a transformer if the voltage u_1 is suitable in value to obtain the required values $U_{в0}$ and does not require isolation of the circuit of rectified current from the circuit.

13.2.2 Rectifiers of three-phase current. Three-phase scheme with a zero point and a bridge scheme are used by analogy with the schemes of single-phase current for rectification of three-phase current.

The three-phase rectification circuit with a zero point (or three-phase zero scheme) is shown in figure 13.12. The transformer *T* is connected to the circuit of three-phase voltage. Its three primary windings can be connected in a star or triangle, the secondary winding - only in a star. The free ends *a, b, c* of each phase of the secondary winding are attached to the anodes of the valves VD_1, VD_2, VD_3. The cathodes of the valves are connected together and used as a positive pole for the load circuit R_d, and the zero point *0* of the secondary winding of the transformer is used as a negative pole.

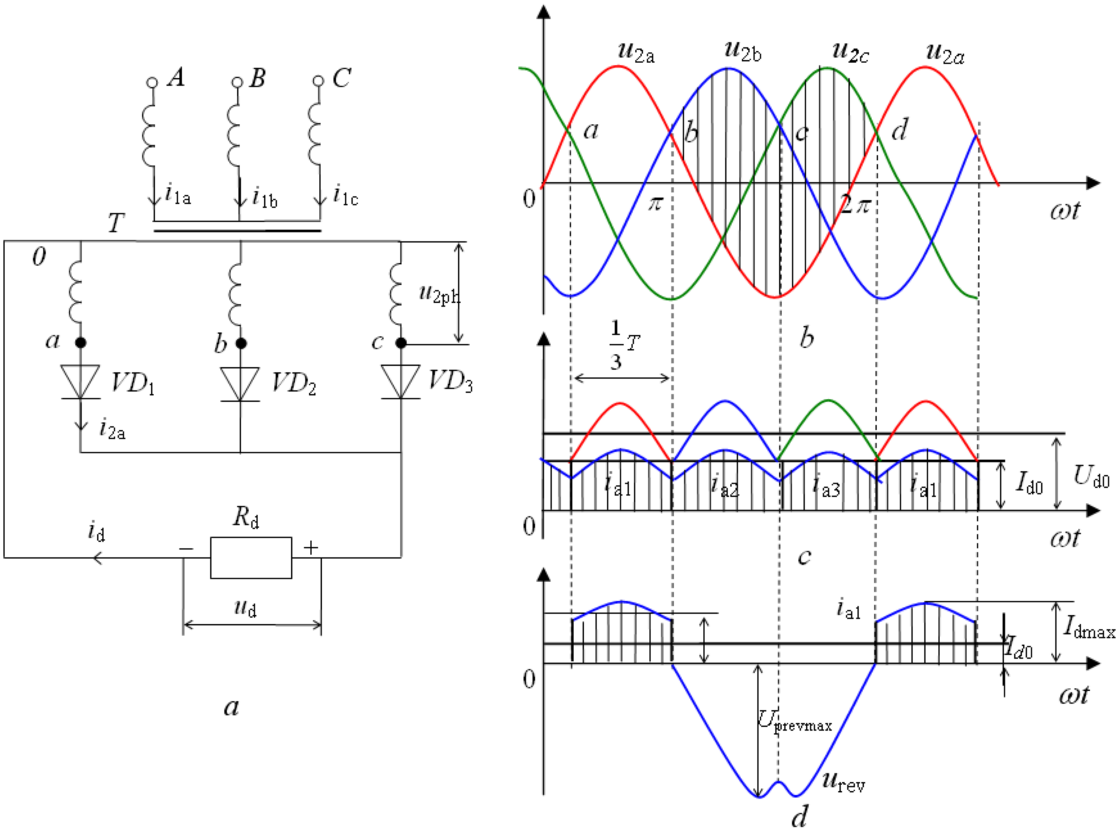


Figure 13.12: Three-phase rectifier with zero point: *a* – scheme; *b*-*d* – diagrams of the voltages and currents on the elements

From the temporary diagram (see fig. 13.12, *b*) it is shown that the voltages u_{2a}, u_{2b} and u_{2c} are shifted in phase by 2π/3 (degree) and during 1/3 the period (1/3T) voltage of one phase is higher than the voltage of the other two phases relatively to the zero point of the transformer. The current through the valve i_a, associated with it secondary winding and the load will pass during that third period, when the voltage in this phase is higher than the other two. Working valve ceases to conduct the current when the potential of its anode becomes lower than the common potential of the cathodes.

The junction of the current from one valve to another (switching of current) occurs at the crossing moment of curves of phase voltages (points a, b, c and d in fig. 13.12, *b*). Rectified current i_d passes through the load continuously R_d (fig. 13.12, *c*).

The voltage on output of the rectifier u_d at any time is equal to the instantaneous value of the secondary winding voltage, in which the valve is open, and the rectified voltage represents the envelope of the tops of the sine waves of phase voltages u_{2ph} .

When changing the secondary voltage by the law $u_{2ph} = U_{max2} \cdot \sin\omega t$ the current of each phase will be a sinusoidal function

$$i_2 = \frac{u_2}{R_d} = \frac{U_{max2}}{R} sin\,\omega t\,. \tag{13.29}$$

Consequently, the anode current i_a will be in the form of a rectangle with base 2π/3, which is limited from above by the segment of the sinusoid.

Figure 13.12, *d* shows the current of phase a, the currents of phase b and c are shown with such curves, shifted by 2π/3 relative to each other.

Three-phase zero scheme of rectification is characterized by the following ratios between voltages, currents and powers in the individual elements of the rectifier.

The average value of the rectified voltage at idle is

$$U_{d0} = 1{,}17\ U_{2ph}\ . \tag{13.30}$$

The rectified voltage has a constant component U_d and imposed on it a variable component $U_{d\sim}$, which has a triple frequency relative to the frequency of the circuit. The pulsation coefficient of the rectifier output voltage is

$$q = 2/(m^2 - 1) = 2/(3^2 - 1) = 0{,}25\ . \tag{13.31}$$

Reverse voltage u_{rev} applied to a broken valve, is equal to interphase voltage of the transformer's secondary winding, as the anode of the closed valve is attached to one of the phases, and the cathode through the working valve is connected to the other phase of the secondary winding. Figure 13.12, *d* shows the reverse voltage u_{rev} between the anode and cathode of the valve VD_1.

The maximum value u_{rev} is equal to the amplitude of the phase-to-phase (line) voltage of the transformer's *T* secondary winding:

$$U_{rev.max} = \sqrt{3} \cdot \sqrt{2} \cdot U_{2ph} = 2{,}09 \cdot U_{d0}\ . \tag{13.32}$$

The load current is equal to the ratio of the rectified voltage to the load resistance, i.e., $i_d = u_d/R_d$. The average value of the rectified current in the load is

$$I_d = \frac{U_{d0}}{R_d} = 1{,}17\frac{U_{2ph}}{R_d}\ . \tag{13.33}$$

Every valve in this scheme works once per period during 1/3T. Therefore, the average current through the valve is 3 times less than the load current, i.e., $I_a = 1/3 \cdot I_d$.

The active value of the current in the secondary winding I_2 and the valve $I_{2.действ}$ is determined by the formula

$$I_2 = I_{2.active} = \sqrt{3} \cdot I_a = 0{,}585 \cdot I_d \tag{13.34}$$

With the same number of phases of the primary and secondary windings of the transformer *T* ($m_1 = m_2$) and the same schemes of winding connection (star-star) the active value of the primary phase current I_1 is less than the reduced value of the current I_2', as there is no permanent component, which is not transformed in the current curve of the primary winding i.e.,

$$I_1 = \frac{1}{k_{tr}}\sqrt{I_2^2 - I_a^2} \approx \frac{1}{k_{tr}} 0{,}47 I_d\ . \tag{13.35}$$

Alternate passing of unidirectional currents in the transformer's secondary windings, which is not fully compensated by the currents in the primary winding, creates in all three cores the flux Φ_0 of one direction, the value of which varies with triple frequency in accordance with the pulsation of anode current and which is closed through the air and transformer shroud. The presence of single-phase flux or forced magnetization Φ_0 in the core leads to an increase of the magnetizing current of the transformer as well as the need to increase the core cut to avoid saturation.

The typical power of the transformer at connection of the secondary windings in a star nonmetering the increase of the mass of the magnetic system, caused by the presence of flux Φ_0 is equal to:

$$S_{\text{tr}} = \frac{S_1 + S_2}{2} = \frac{3U_{\text{1ph}} I_1}{2} = 1.35 P_{\text{d}} . \tag{13.36}$$

Three-phase bridge circuit is shown in figure. 13.13. The rectifier in this scheme consists of a transformer, the primary and secondary windings of which are connected in a star or triangle, and six valves, which can be divided into two groups:

1) cathode or odd (the valves VD_1, VD_3 and VD_5), in which the cathodes of the valves are electrically connected with each other and the common lead is a positive pole to the external circuit, and the anodes are connected to the lead of the secondary winding of the transformer;

2) anode or even (valves VD_2, VD_4 and VD_5), in which the anodes of the valves are electrically connected with each other, and the cathodes are connected with the anodes of the first group. The common point of connection of the anode is the negative pole to the external circuit.

The cathode group of valves repeats the operation mode of the three-phase zero scheme. In this group of valves during each third of period the valve works with the highest potential of the anode (see fig. 13.13, *b*). In the anode group in this part of the period that valve works, in which the cathode has the most negative potential relative to the common point of the anodes.

The valves of the cathodic group open in the moment of crossing of the positive areas of sinusoids (see points a, b, c and d in fig. 13.13, *b*), and the valves of the anode group – in the moment of crossing the negative areas of sinusoids (points k, l, m and n). Each of the valves works during the one-third of period (see fig. 13.13, *f*).

At the instant commutation of current in three-phase bridge scheme at any time two valves conduct current which one from cathodic and the other from anodic group, meanwhile any valve of one group works alternately with the two valves of the other group connected with different phases of the secondary winding (fig. 13.13, *d* and *e*). Through each phase of the transformer, the current i_2 will be within 2/3 of the period: 1/3 period – positive and 1/3 – negative. The current i_{d} in the load all the time goes in one direction.

During the working interval currents pass simultaneously in the secondary windings located on different cores of the magnetic system (see the currents i_{a2} and i_{b2} in fig. 13.13, *a*), through the two primary windings located on the same rods, currents also pass. Magnetizing forces from the currents i_1 and i_2 on each of the rods in this case are balanced, and unidirectional flux Φ_0 does not occur.

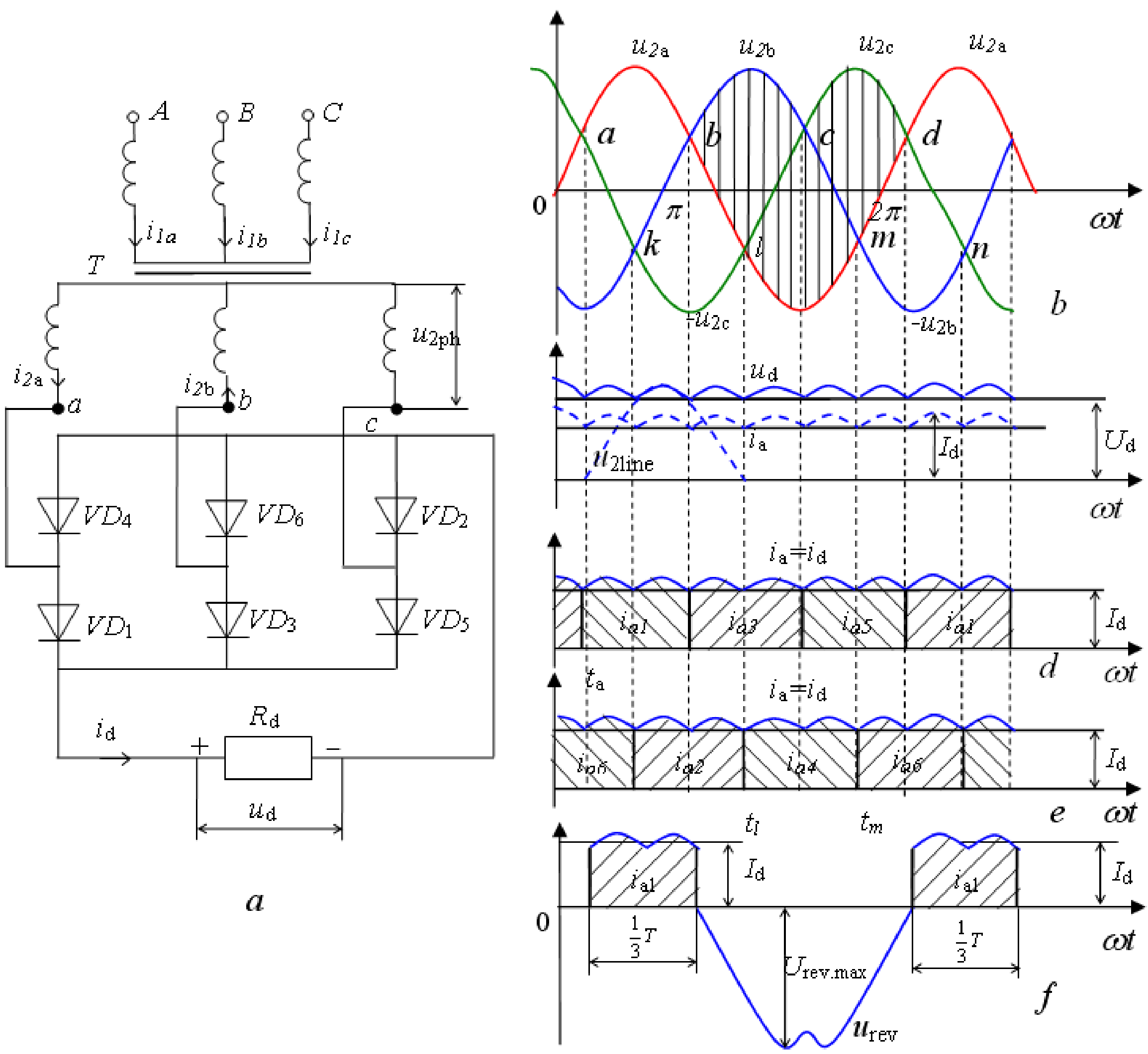


Figure 13.13: Three-phase bridge scheme of a rectifier: *a* – connection of elements; *b–f* – diagram of voltages and current

The rectified voltage u_d in this scheme is described by the upper part of the curves of the interphase (line) voltage u_{2L} (dashed curve in fig. 13.13, *c*). Pulsation frequency of the curve u_d is equal to $6 \cdot f_1$, the coefficient of the pulsation of voltage at the output of the rectifier is equal to:

$$q = \frac{U_{\max}}{U_{d0}} = \frac{2}{m^2 - 1} = \frac{2}{6^2 - 1} = 0{,}057\,. \qquad (13.37)$$

Reverse voltage at a closed valve is determined by the potential difference of its cathode and anode. The ordinates of the curve u_{rev} for the valve VD_1 are shown in figure 13.13, *b* by hatching, and in figure 13.13, *f* the curve u_{rev} is shown fully.

The maximum value of the reverse voltage on the valve in a three-phase bridge scheme is equal to the amplitude of the line voltage of the secondary winding of the transformer.

Rectified current i_d at work on a purely active load repeats the curve u_d (see dotted curve in fig. 13.13, *c*).

The ratios between voltages and currents in three-phase bridge circuit are shown in table 13.2.

Let's compare the advantages of three-phase schemes of rectification at equal values of the power P_d, the voltage U_d, the absence of parallel and serial connections of the valves in the arms of the rectifier.

Three-phase scheme with a zero point:

1. The scheme is simple. The number of valves is in 2 times less than in the bridge or double three-phase schemes.
2. There is less loss in the valves, as in this scheme, the current passes through one diode and in bridge scheme it passes serially across the two diodes.

Three-phase bridge scheme:

1. The reverse voltage applied to the valves is in 2 times less than in the schemes with zero and equalizing reactor.
2. The voltage (the number of turns) of the secondary winding is twice less, but a cut of the wire is in $\sqrt{2}$ times more.
3. There is no forced magnetization of the core. Execution of the transformer windings is normal.
4. Overall power of the transformer is on 30% less than in the scheme with zero and on 26% less than in the circuit with equalizing reactor, the current of the primary winding has the form of a sinusoid.

Using semiconductor valves the bridge scheme has the advantages. It can work directly from the circuit, if the voltage U_1 is suitable in value to obtain the desired value U_d and does not require isolation from the feeding circuit of the rectified current circuit.

13.2.3 Controlled rectifiers. In many practical cases, the rectifiers should provide possibility of smooth control of the average value of the rectified voltage U_d, for example for regulation of frequency of rotation of DC motors at the charging of accumulator batteries and so on.

Using uncontrolled diodes in rectifiers the average value of the rectified voltage U_d, as it is seen from the expressions (13.18), (13.30) and table 13.2, is proportional to the voltage U_{2ph}. Therefore, the regulation U_d in this case is possible only at the expense of change of the secondary winding voltage of the transformer that is not always convenient.

The usage of managed diodes – thyristors in the rectification schemes gives more opportunities to regulate the rectified voltage.

The principle of action of the controlled rectifier. Figure 13.14, *a* shows single-phase zero scheme of the controlled rectifier, which is different from the scheme figure 13.10, *a* in the fact that uncontrolled diodes VD_1 and VD_2 are replaced by thyristors VS_1 and VS_2. The anodes of the thyristors are connected to the lead of the secondary winding and the controlling electrodes are connected with the control system CS, which forms the controlling pulses of voltage U_{c1} and U_{c2} with the supply-line voltage synchronously and allows to change their phase relatively to the phase voltage u_{2a} and u_{2b} of the power source.

Using unmanaged diodes in the scheme the diode VD_1 would open at time t0 (fig. 13.14, *b*), which is the moment of natural unblanking of diode. As noted in the chapter 13.3.1the thyristor comes unlocked if we have positive voltage on the anode and enabling pulse on the controlling electrode. Let's assume that on the control electrode of the thyristor VS_1 unlocking pulse U_{y1} will be delivered at time t_1, therefore, it will open with some delay relatively to the beginning of the positive voltage u_{2a}, as a result of which in the interval t_0 - t_1 the voltage at the load R_d will be zero, as both thyristors VS_1 and VS_2 are closed.

The angle of delay, counted off from the moment of natural unblanking of diode and expressed in electrical degrees, is called the ***angle of control*** and is denoted by the Greek letter *α*. At the time of unblanking of the thyristor VS_1 the voltage u_d on the load R_d increases by lump and is further modified by the curve of the phase voltage u_{2a}. At the moment t_2 the voltage u_{2a} changes sign, the thyristor VS_1 locks itself up, in the interval $t_2 - t_3$ both thyristors will be closed and the current i_d in the load will not pass, and at the moment t_3 the thyristor VS_2 comes in work and remains open until t_4. Then in the interval equal to the angle α, the thyristor VS_1 reenters the work, etc.

When rectifier works on the active load the rectified current curve i_d repeats the shape of the voltage curve u_d (fig. 13.14, *b* and *c*). On figure 13.14, *d* the curve of the reverse voltage u_{rev} was built on the thyristor VS_1 for the scheme with angle of regulation $\alpha = 60°$. In the interval $t_0 - t_2$ the forward voltage $u_{dir} = u_{2a}$ is applied to the thyristor VS_1, in the interval $t_1 - t_2$ the thyristor VS_1 is open and the voltage drop is almost zero. At the time t_2, when the current i_d is zero, the thyristor VS_1 is closed and a reverse voltage is applied to it. The reverse voltage is equal to the phase $-u_{2a}$, as the thyristor VS_2 is also closed. At the moment $t_3 = \pi + \alpha$ the thyristor VS_2 opens and the interphase voltage of the secondary winding of the transformer is applied to the thyristor VS_1. This voltage will act on it until $t_4 = 2\pi$, when the thyristor VS_2 closes. In further the processes in the scheme will be repeated through every period.

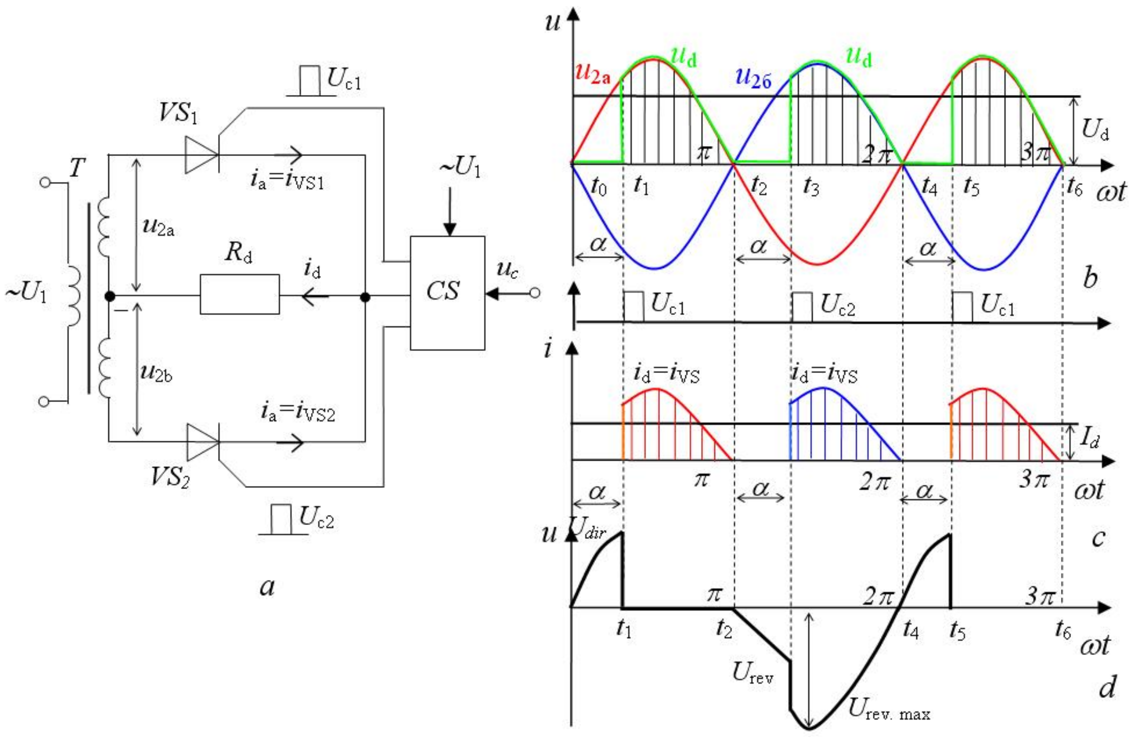


Figure 13.14: Single-phase full-wave controlled rectifier: *a* – scheme; *b* – d – diagram of voltages and currents in the elements

Obviously, if the angle α changes (move on phase controlling pulses U_c relatively to the voltage at the anodes of thyristors), the time of work of the thyristors and accordingly the magnitude of the rectified voltage will change too. In this case the average value of the rectified voltage is determined by the expression

$$U_d = U_{d0} \cdot \frac{1+\cos\alpha}{2}, \tag{13.38}$$

where $\boldsymbol{U_{d0}}$ – the largest value of the rectified voltage at fully open ($\alpha = 0$) thyristors VS_1 and VS_2 can be calculated by the formula (13.18).

When rectifier works on the active load and the rectified voltage is regulated from 0 to U_{d0}, as it can be seen from the formula (13.38), the angle of regulation α must be changed from $\alpha_{max} = 180°$ to $\alpha_{min} = 0$. Really, if $\alpha = 180°$, then $\cos 180° = -1$ and $U_d = 0$; at $\alpha = 0$ $\cos 0 = 1$ and $U_d = U_{d0} = 0{,}9 \cdot U_{2ph}$.

Therefore, the working mode of uncontrolled rectifier is a maximum to which controlled rectifier approximates at the angle of control $\alpha = 0$.

In contrast to the uncontrolled rectifier, the diodes of which can only withstand a reverse voltage, the diodes of the managed converter must be able to withstand both direct and reverse voltage. At the active load the maximum value of the reverse voltage at closed in the half-period thyristor when the angles $\alpha < 90°$ is equal to the amplitude of the voltage across the secondary winding of the transformer and (as in the unmanaged scheme) is determined by the expression (13.19).

The value of the direct voltage U_{dir} on closed thyristor at $\alpha < 90°$ depends on the angle of regulation as follows:

$$U_{dir} = \sqrt{2}\sin\alpha. \tag{13.39}$$

At $\alpha = 90°$ the value U_{dir} reaches its maximum. The average value of the rectified current is defined as $I_d = U_d/R_d$ where U_d can be calculated by the formula (13.38). At the angle of regulation $\alpha = 0$ in the load the current will be the greatest $I_d = U_{d0}/R_d$.

The average value of current through the thyristor $I_{a.av} = 0{,}5\ I_d$, the active value of current of the thyristor $I_{a.active}$ and the secondary winding of transformer I_2, and also the current of the primary winding I_1 with $\alpha = 0$ are determined respectively by the formulas (13.21) and (13.23). The quantitative ratios between other variables for single-phase zero scheme on managed valves are shown in the table 13.2.

Work of single-phase bridge scheme on thyristors differ from a single-phase zero scheme of rectification on diodes in that controlling pulses must be submitted simultaneously on two thyristors arranged in the opposite shoulders of the rectifier bridge.

The curves of the rectified voltage u_d and rectified current I_d of a single-phase bridge circuit on thyristors are similar to the corresponding curves for a single-phase zero scheme on the diodes. Quantitative ratios for the currents and voltages of the scheme are shown in the table 13.2.

In a three-phase zero scheme with thyristors (fig. 13.15, *a*) controlling pulses are fed on them with some offset in time relatively to the moment of natural unblanking of diodes in the unmanaged scheme, which corresponds to the intersection points of the sine of the phase voltages (points a, b, c and d in fig. 13.15, *b*).

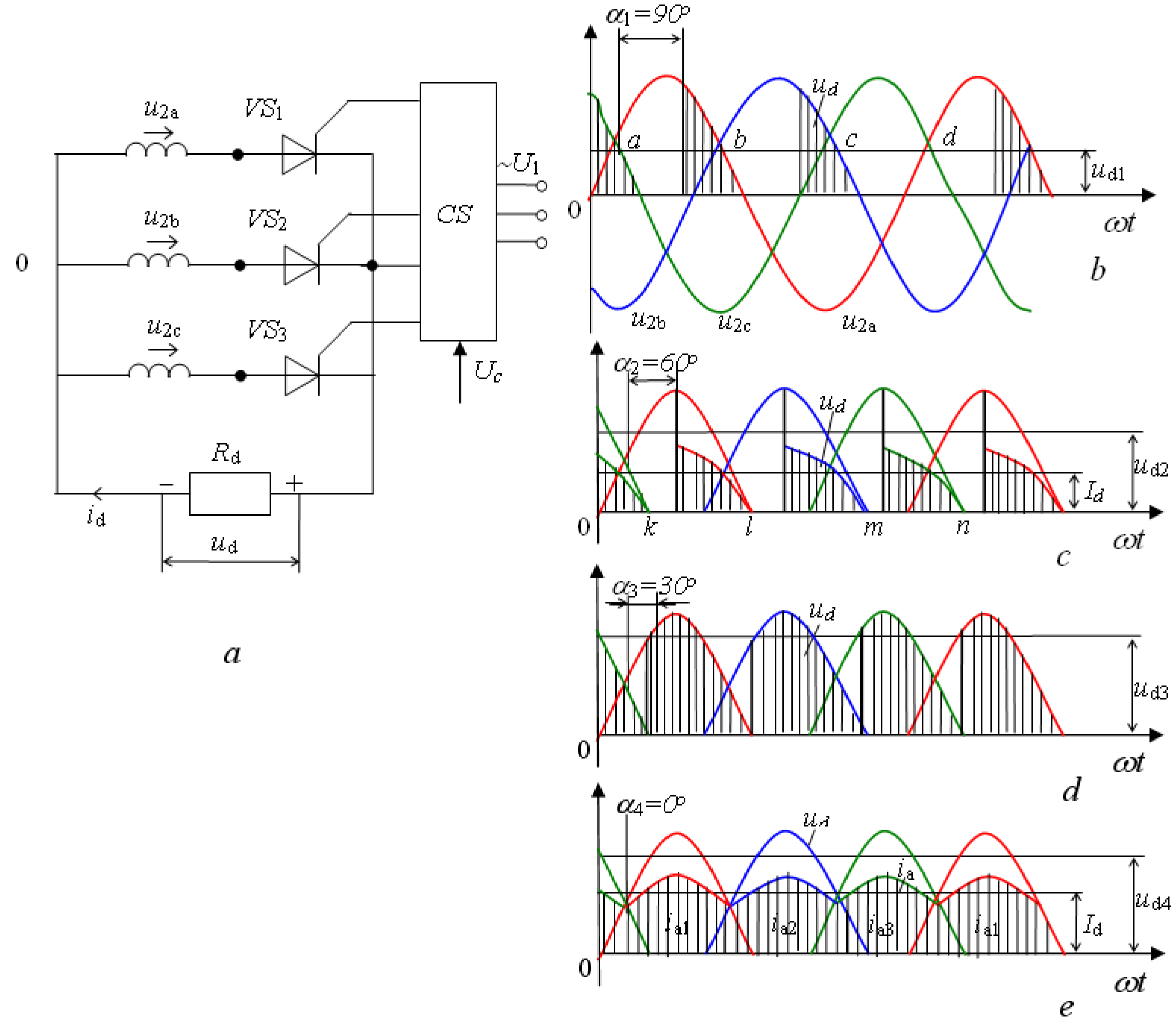


Figure 13.15: Three phase controlled thyristor rectifier with zero point:
a – scheme; *b* – e – diagrams of voltages and currents in the elements

Let, for example, the control pulses on the thyristors VS_1, VS_2, VS_3 move at the moments corresponding to the middle of the positive half-wave phase voltages (this corresponds to the angle α = 60°). In this case, on the load pulses of rectified voltage u_d appear in the form of a quarter of sine wave (fig. 13.15, *c*).

Phase change (shift) of the control pulses in the direction of increasing or decreasing the angle of control α causes corresponding decrease (fig. 3.15, *b*) or increase (fig. 13.15, *d*) of the voltage pulses u_d. At the angle α = 0 the curve of the rectified voltage (fig. 13.15, *e*) will have the same form as the uncontrolled rectifier (fig. 13.12, *b*). It is obvious that the curve of the current i_d in its form will repeat the curve of the rectified voltage u_d at the work of the rectifier on the active load.

From these curves it is seen that there are two typical areas of work of the controlled rectifier. The first one corresponds to the change of angle of regulation in the range $0 < \alpha < 30°$, meanwhile the rectified current will continuous and the average value of the rectified voltage is determined by the expression

$$U_d = U_{d0} \cos\alpha \qquad (13.40)$$

In this case each thyristor of the scheme works one third of the period. The second area corresponds to the angles $\alpha > 30°$ and is characterized by the fact that with the passage of phase voltage through zero (points k, l, m, n in fig. 13.15, *c*) the working thyristor is closed, but as for the next coming into work thyristor the enabling pulse has not yet fed, then pauses arise (zero suncircuits) in the curve of the rectified voltage, during which the current is $i_d = 0$.

The duration of passing of the current through the valve in this case is less than 1/3 T and the average value of the rectified voltage is calculated by the formula

$$Ud = \frac{\sqrt{3}}{3} U_{d0} [1 + \cos(30^0 + \alpha)] \tag{13.41}$$

For three-phase zero circuit when working on the active load the limit angle of regulation, at which $U_d = 0$, is the angle $\alpha_{max} = 150°$. The voltage on the valve is determined by the potential difference of the anode and the common point of the cathode, the potential of which varies on the voltage curve u_d. The maximum value of the reverse voltage on the thyristor, as well as in the scheme with unmanaged valves is equal to the voltage amplitude $u_{rev.max}$ (see equation (13.32)).

In three-phase bridge scheme with thyristors (see fig. 13.16, *a*), as well as with unmanaged diodes, two thyristors work simultaneously: one of the cathode (odd) group, the other of the anode (even) group, and the load at any point in time is attached to the two phases of the secondary winding of the transformer. Enabling pulses on the thyristors of the odd groups are fed with lead on 180° with respect to the thyristors of the even groups attached to the same terminal of the secondary winding, as the first ones work in case of positive values of phase voltages on the anodes, the second ones – of negative values on the cathodes.

The work of the considered circuit of rectification is illustrated in the diagrams of instantaneous values of phase voltages on thyristors (see fig. 13.16, *b*); curves of rectified voltage u_d (see fig. 13.16, *c*), which is obtained by summation of the instantaneous values of the voltages of currently working thyristors; curves of anod currents (fig. 13.16, *d*) of the cathode group – above the time axis, the anode group - under the axis. Each of the charts is constructed for three values of the angles of regulation: $\alpha_1 = 30°$, $\alpha_2 = 60°$ and $\alpha_3 = 90°$.

At an angle of regulation $\alpha = 0$ enabling pulses must be fed on the thyristors at the moments corresponding to the intersection points of the curves of the phase voltages (see points a, b, c, and k, l, m in fig. 13.16, b). In this case, each thyristor conducts current for a 1/3 of the period, as in the unmanaged scheme, and alternation of pairs of working thyristors occurs through 60° (fig. 13.16, *c*).

While the angle of regulation $\alpha << 60°$, the curves of the rectified voltage, and hence the curves of the rectified current (fig. 13.16, c and d) at the active load are continuous. For this mode ($0 << \alpha << 60°$) the average value of the rectified voltage is determined by the expression (13.35). As can be seen from figure 13.16, *d*, through the incoming into work thyristor the current may pass if at the same time the corresponding (adjacent on serial number) thyristor of another arm of the bridge is opening or has already opened. Otherwise, the circuit of current will not be closed and another incoming into work thyristor will not open.

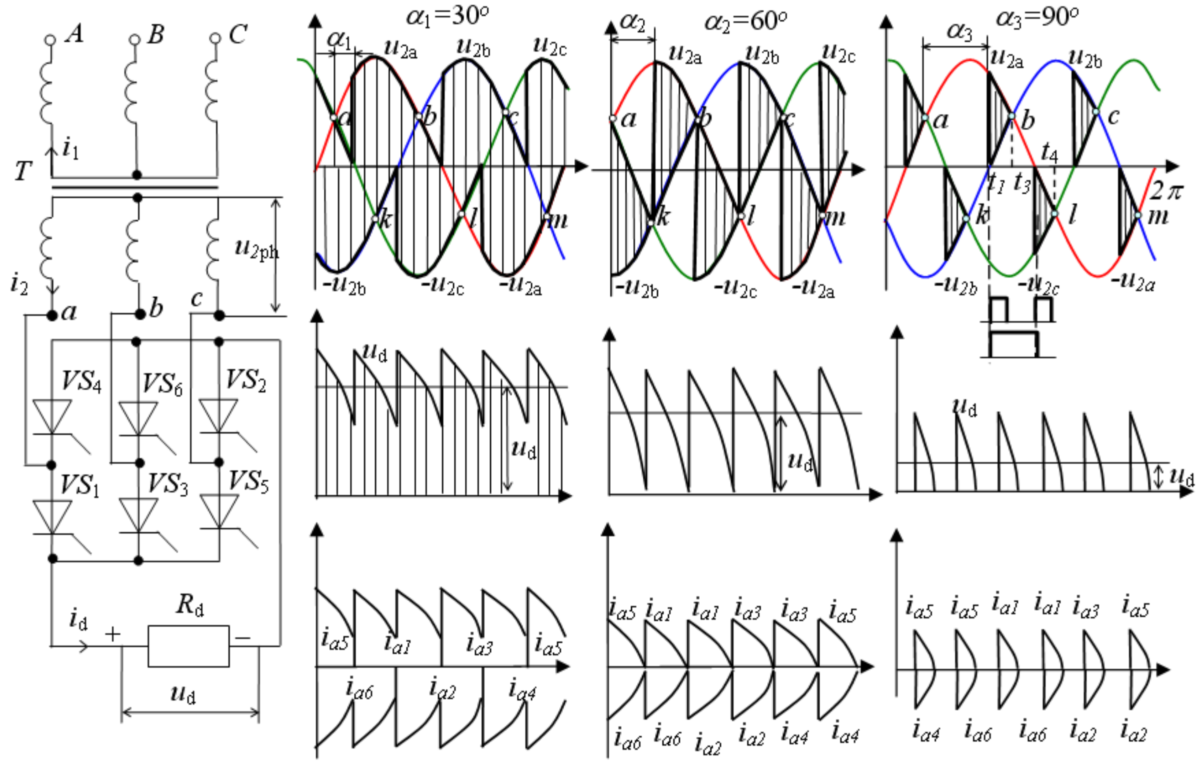


Figure 13.16: Three-phase bridge scheme: *a* – scheme; *b*–*d* – diagram of voltages and currents in the elements

At starting rectifier with zero (U_d = 0) or at passage it in the mode of the intermittent currents (α > 60°) there may be a breach of the above conditions. Therefore it is necessary to apply a pulse width greater than 60° or two narrow pulses with an interval between them in 60°on the control electrodes of the thyristors in a three-phase bridge rectification scheme (see fig. 13.16, *b*, α_3 = 90°).

The control scheme of the rectifier must be constructed so that the feed of the enabling pulse on the coming into work thyristor of one arm of the bridge and the feed of the pulse at the control electrode of the thyristor of lagging phase of the opposite shoulder of the bridge would come true at the same time. For example, when rectifier works with α = 90° (see fig. 13.16, *b*) in order to open the thyristor VS_1 at moment t_1 it is necessary to simultaneously feed an enabling pulse and on the thyristor VS_2, then both valves will conduct current until the moment t_2, when the difference of the instantaneous values of voltages u_{2d} and $u_{2й}$ will be zero and the thyristors VS_1 and VS_2 will close. Then at time t_3 the thyristor VS_3 must join the work, which will open only on condition of the availability of repeated enabling pulse on the thyristor VS_3 or on condition that at time t_1 the pulse duration of greater than 60° will be fed to the control electrode of the thyristor. The thyristors VS_2 and VS_1 will conduct current until moment t4 then the next pair of thyristors VS_3 and VS_2 will come into work, etc.

The average value of the rectified voltage, when the current i_d is intermittent (>60°), is determined by the expression

$$U_d = U_{d0}\,[1 + \cos(60^0 + \alpha)] \qquad (13.42)$$

From the formula (13.42), it follows that when this scheme works on the active load the maximum angle of regulation at which the $u_d = 0$, is considered to be the angle $\alpha_{макс} = 120°$.

13.3 Smoothing filters

The presence of the rectified voltage pulsations impairs the work of the consumers supplied from the rectifier. For example, if DC motors are powered with pulsating voltage the conditions of the current commutation become worse and the losses in the motor increase. When radio equipment is powered the pulsations of voltage u_d sharply degrade the work of devices, creating the background at the output of amplifiers, i.e., additional oscillations of the output voltage of the low frequency. Consequently, the pulsations of voltage on the load must be reduced to values that would not affect their negative impact on the work of the installation.

To reduce the pulsation of voltage at the consumer at the withdrawal of the rectifier a special device is installed, called a **smoothing filter**. The scheme of the filter f is shown in figure 13.17, *a*.

The value of the pulsation of voltage at the output of the rectifier is estimated by the ratio of the pulsations q, which is equal to the ratio of the amplitude of the primary (first) harmonic of pulsations $U_{\sim 1}$ to constant component of the rectified voltage U_d, i.e. $q = U_{\sim 1}/ U_d$.

The pulsation of voltage on the load is characterized by the coefficient q_1, which is equal to the ratio of the amplitude of the main harmonic of pulsation $U_{d\sim}$ on load (after the filter) to the rectified voltage U_{dL} on the load, i.e. $q_1 = U_{d\sim}/U_{dL}$.

The pulsation of voltage on the load is set by the work conditions of the consumer, and the pulsation of voltage at the input of the rectifier is known after selection of scheme of rectification and determination of its parameters *The ratio of the values of q and q_1 determines the degree of smoothing of the rectified voltage, and is called the* ***coefficient of the smoothing filter s***:

$$s = \frac{q}{q_1} = \frac{U_{\sim 1}}{U_d} : \frac{U_{d\sim}}{U_{dL}}. \qquad (13.43)$$

Along with the weakening of the variable component of the rectified voltage smoothing filter reduces the constant component ($U_{dL} = U_d - \Delta U_{PH}$). Obviously, the smaller the degree of reduction of the constant component (U_d/U_{dL}) at a constant weakening of the variable ($U_{d\sim}/U_{\sim 1}$), the better the filter will be. For filters of rectifiers of low-power ratio of constant component of voltages usually $U_d/U_{dL} = 1{,}05–1{,}1$, and for high power rectifiers $U_d/U_{dL} = 1{,}005–1{,}01$.

In practical calculations it can be considered that $U_d \approx U_{dL}$ and the smoothing coefficient, showing in this case the degree of attenuation of the variable component of the rectified voltage by the filter, is equal to $s=q/q_1 \approx U_{\sim 1}/U_{d\sim}$.

Let's consider the main types of smoothing filters.

Capacitive filter (fig. 13.17, *b*) is a condenser C_f included parallel to load resistance R_d. Shunting the load by low capacitive resistance $x_c = 1/\omega C << R_d$ for the variable of current component $i_d = i_c$, it creates in rectifier the additional voltage drop ΔU_a on R_a (fig. 13.17, *c*), which leads to a smoothing of the voltage U_d. In this case, we can assume that only the DC component current I_d passes through R_d, and the variable component of current $i_{d\sim}$ passes entirely through the condenser.

Capacitive filter is more effective in the rectifiers on small currents I_d (with large R_d), as in such a filter it is easier to get the inequality $\omega C_f >> 1/R_d$ for small values of capacitance C_f.

When calculating the capacitive filter source the required coefficient of pulsation q_1 and also the set values of the angular frequency $\omega_c = 2\pi f_1$ of power source and the load resistance R_d are considered to be initial values. The value C_f (in microfarad) can be determined from the expression:

$$C_f = \frac{1}{m \cdot \omega_c \cdot q_1} \frac{10^6}{R_d}. \tag{13.44}$$

Inductive filter (fig. 13.17, *d*) represents the throttle L_f included serially with the load and with a large reactance $X_L = \omega_c \cdot L_f$ for the variable component of the rectified current, which greatly decreases, and the voltage drop $\Delta U_{d\sim}$ from this component R_d becomes negligible (fig. 13.17, *e*).

For good smoothing the voltage on the load it is necessary

$$x_L = \omega_c \cdot L_f >> R_d . \tag{13.45}$$

For a given smoothing coefficient s, if the condition (13.45) is fulfilled, we can determine the desired value of inductance of filter L_f (in henry) from the expression:

$$L_f = \frac{s \cdot R_d}{2\pi \cdot f_c \cdot m} , \tag{13.46}$$

where: $\boldsymbol{f_c}$ – the frequency of the circuit voltage, Hz;
$\boldsymbol{m}$ – the number of phases of rectification.

Analysis of formula (13.46) shows that the same value of the coefficient s can be obtained by the smaller values of inductance L_f, the smaller the load resistance R_d. Thus, ***the inductive filter is advantageous to use in rectifiers of medium and high power, in which the load resistance is small***.

If it is necessary to have a very small value of the coefficient of pulsation q_1, then the capacity C_f or inductance L_f used as simple filters can be significant. In such cases more complex Γ-shaped or Π-shaped filters are used [17, 34].

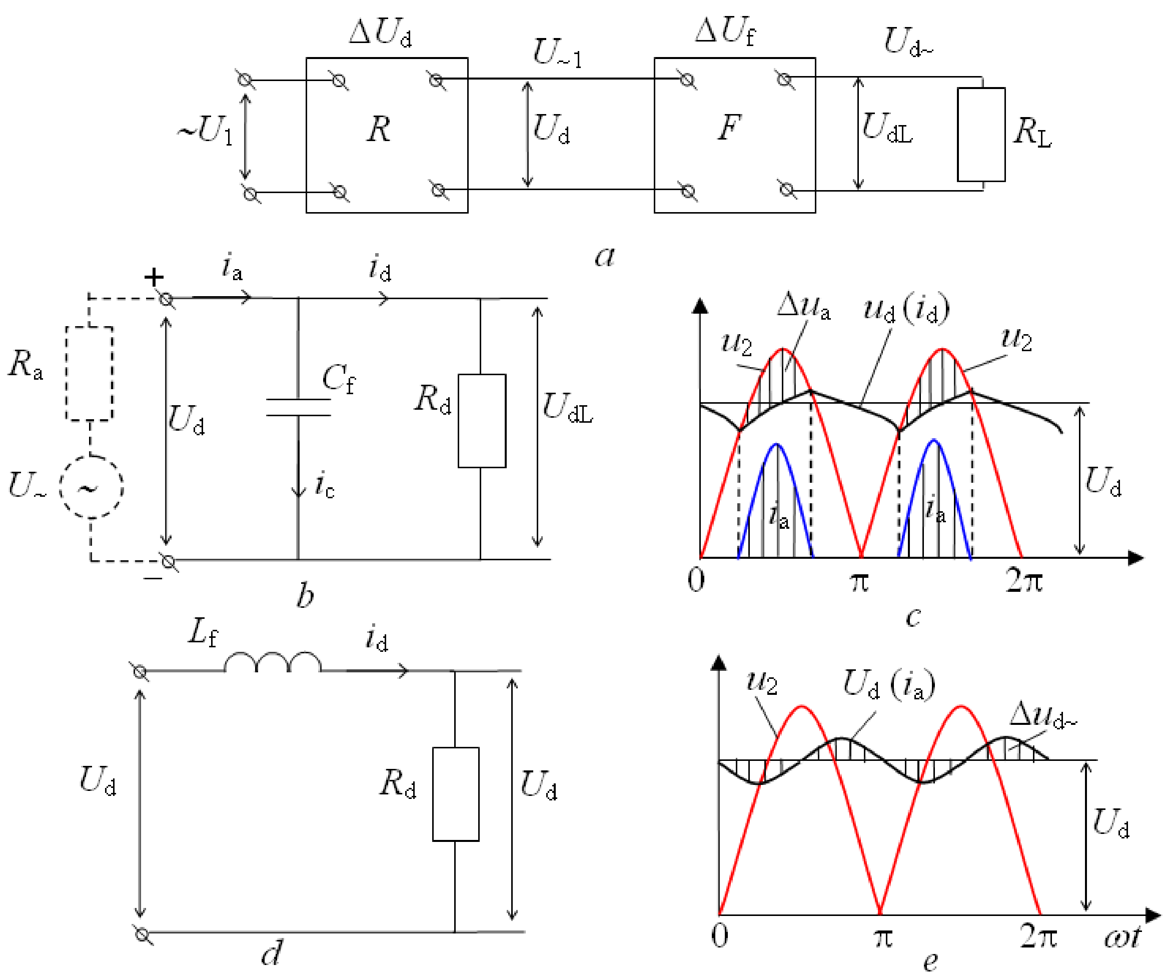


Figure 13.17: Circuits of inclusion of filter: *a* – block scheme of a rectifier with a filter; *b* – capacitive filter; *c* – inductive filter; *d* and *e* – curves of voltages and currents

13.4 Amplifiers

Amplifiers *are* ***devices that are designed to increase the values of the parameters of electrical signals at the expense of the energy of the included power source.*** Different amplifiers are used for the preferential amplification of the values of certain parameters of the signals. On this basis, they are divided into amplifiers of voltage, current and power.

There are linear and non-linear modes of operation of the amplifier. In amplifiers with almost linear mode of operation minimal distortion of the amplified signal is obtained. Signal distortion will be minimal if without distortion all its harmonic components increase. Property of amplifier to increase the amplitude of the harmonic signal components characterizes its amplitude-frequency characteristic (AFC). On type of AFC it is distinguished:

amplifiers of slowly changing voltages and currents, or amplifiers of DC (fig. 13.18, *a*) – the range of variation of the amplified signals from 0 to 10^3 Hz;

amplifiers of low frequencies (fig. 13.18, *b*) – the range of variation of the amplified signals from 20–50 Hz to $20 \cdot 10^3$ Hz;

amplifiers of high frequencies (fig. 13.18, c) – the range of variation of the amplified signals from 10^4–10^5 Hz to 10^7–10^8 Hz;

broadband amplifiers (fig. 13.18, *d*) – the range of variation of the amplified signals from 20–50 Hz to 10^7–10^8 Hz);

narrowband amplifiers (fig. 13.18, *e*).

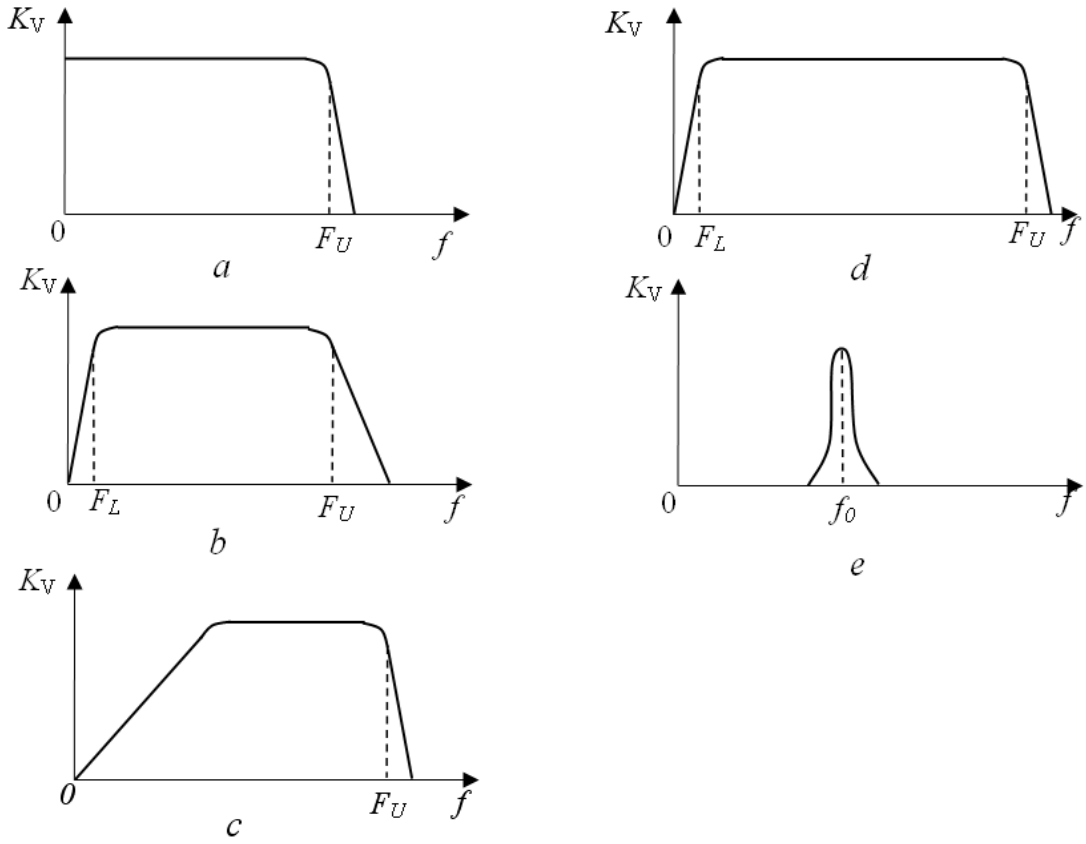


Figure 13.18: Amplitude-frequency characteristics of amplifiers

If in amplifiers with nonlinear mode of work the value of the input voltage is more than a certain boundary level, the change in the voltage at the output of the amplifier is practically absent. Such amplifiers are mainly used in devices of pulse technique, including logical.

Currently amplification technique is based on the wide use of amplifiers on integrated microcircuits, which allow them to apply for the implementation of various functional nodes of automation systems, control and measurement.

As noted in chapter 13.1.4, in practice, there are three inclusion schemes of transistors with a common emitter, common base and common collector. Accordingly, there are three schemes of transistor amplifiers.

Let's consider the principle of work of a typical amplifier cascade on bipolar transistor included on scheme in the common-emitter (fig. 13.19). The source of the amplified signal, shown within the dashed line, represents a source with an internal resistance R_{in} and EMF $e_c=u_c$.

The resistors R_1, R_2, $R_к$ in the circuit provide the necessary value of the constant voltage across the collector and emitter junctions when all circuits of the transistor are powered from a single common power source E_k. The resistor R_E provides temperature stabilization of the working point what for transistor amplifying schemes is significant. With increase of temperature the constant component of current of the emitter I_{E0} increases, in consequence of what the voltage drop $R_E I_{E0}$ increases on the resistor R_E, meanwhile the potential of the emitter relatively to the base is reduced, what reduces the DC component of the base current and limits the degree of increase of the rest current in the circuit of the collector. To eliminate this effect, when the variable components pass through the transistor circuits, the resistor R_E is shunted by the condenser C_E. The condensers C_1 and C_2 are designed to prevent the permanent component of current from the power source and the signal at the output and input of an amplifying cascade.

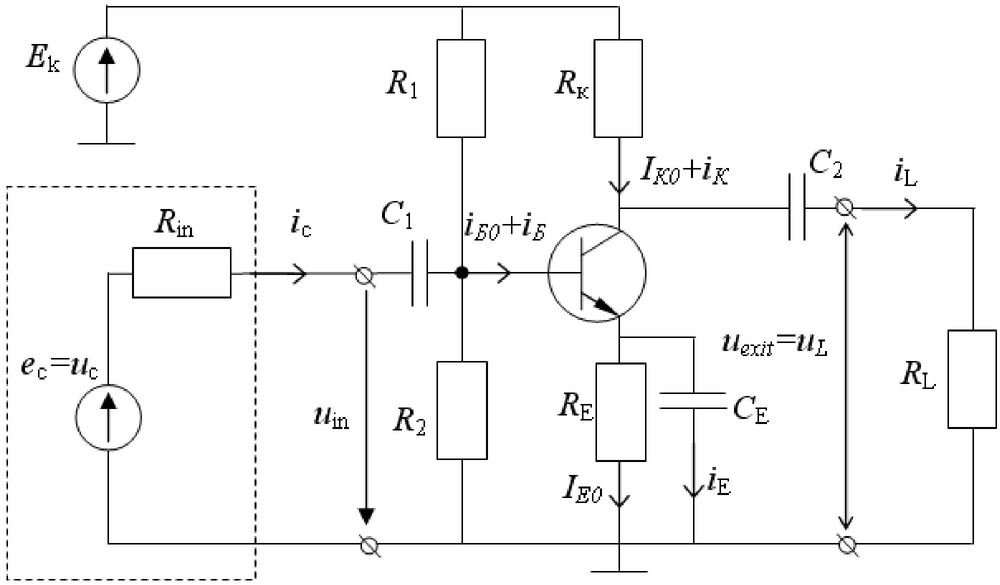


Figure 13.19: Scheme of an amplifying cascade with a common emitter

One of the most important indicators characterizing the properties of amplifiers is considered to be a complex coefficient of amplification which in general case can be represented as the ratio of the complex voltage at the amplifier output to the complex voltage at its input:

$$\dot{K} = \frac{\dot{U}_{\text{exit}} e^{j\psi_{\text{exit}}}}{\dot{U}_{\text{in}} e^{j\psi_{\text{in}}}} = K \cdot e^{j(\psi_{\text{exit}} - \psi_{\text{in}})} = K \cdot e^{j\varphi}\ , \tag{13.47}$$

where: $\boldsymbol{K} = \dfrac{\boldsymbol{U}_{\text{exit}}}{\boldsymbol{U}_{\text{in}}}$ – module of coefficient of amplification of amplifier; $\varphi = \psi_{\text{exit}} - \psi_{\text{in}}$ – the difference between the phase angles of the signal at the output and the input of the amplifier.

Amplifiers inevitably contain a combination of active and reactive elements, so the module of coefficient of amplification and the phase difference of the angles at the output and the input of the amplifier is frequency-dependent. When studying amplifier the dependence of the module of the amplification coefficient from the frequency $K(f)$ amplitude-frequency characteristics of the amplifier and the dependence of change of phase angle from frequency are usually considered separately. Frequency range (fig. 13.20) from *0* to f_L and from f_U to $f=\infty$ are characterized by the significant change of the amplification coefficient. *The scope of the change of frequency of the input signal from $f_н$ to $f_в$ is called* ***the pass band of the amplifier*** *and is characterized by a slight alteration of the coefficient of amplification from frequency.*

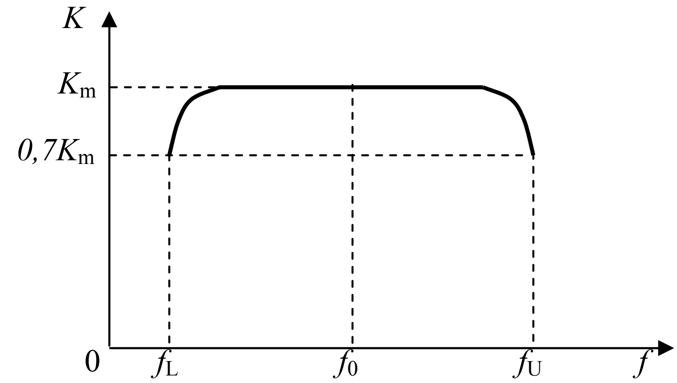


Figure 13.20 – AFC of the amplifier with the CE

In practice to receive larger values of amplification coefficient multistage schemes are used, example of which for the scheme with common emitter is shown in figure 13.21.

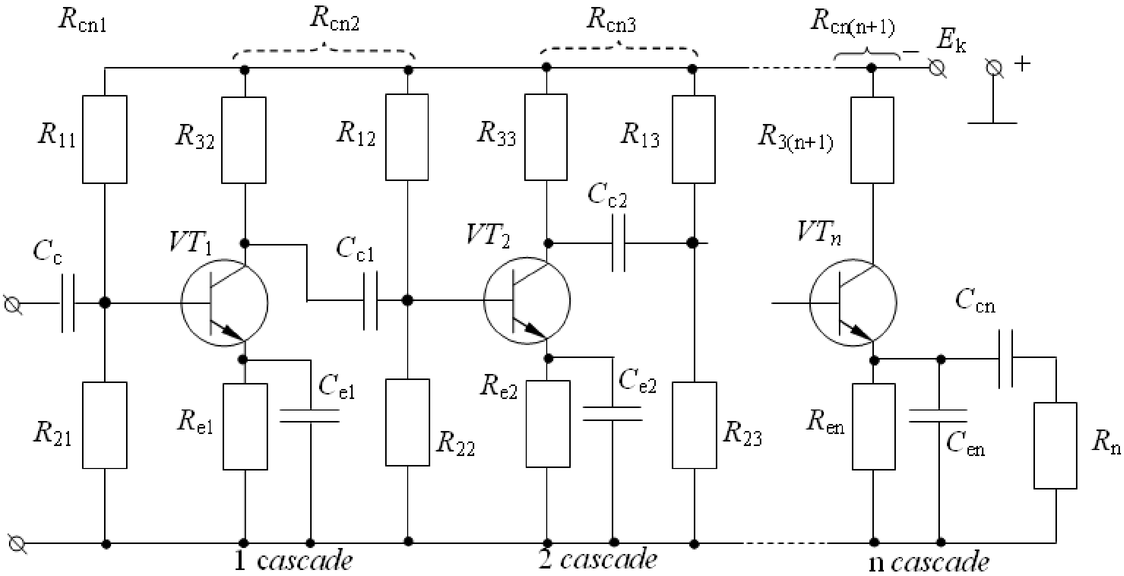


Figure 13.21: Multistage amplifier on scheme with common emitter

Key findings

1. Applied in semiconductor equipment p-n and n-p junctions have one-sided conductivity that provides the rectifier diode properties.

2. Managed diode (thyristor) allows you to adjust the amount of passing current through it, and, consequently, to adjust the magnitude of the rectified voltage.

3. The transfer of thyristor from the closed to the open state can be carried out:

by feeding to the anode of the thyristor direct voltage exceeding the voltage of switching;

by feeding to the control electrode the positive pulse at the direct voltage at the anode of the thyristor.

4. One of the main applications of transistors is the increase of electrical signals.

5. The pulsations of the rectified voltage depend on the circuit of the rectifier and the quality of the smoothing filter.

Control questions

1. Explain the phenomenon of one-sided conductivity of semiconductors?
2. Explain volt-ampere characteristics of *p-n* junction?
3. Explain the device of silicon diode?
4. Distinctive features of germanium diodes?
5. Distinctive features of selenium diodes?
6. Explain the thyristor device?
7. Explain the volt-ampere characteristic of a thyristor?
8. How is it possible to implement unblanking of a thyristor?
9. Explain the bipolar transistor device?
10. What schemes of inclusion of transistors are used in practice?
11. What are features of the work of the scheme with common emitter?
12. What are features of the work of the scheme with a common base?
13. What are features of the work of the scheme with a common collector?
14. Explain the principle of work of a single-phase half-wave rectifier?
15. The principle of work of a single-phase full-wave scheme?
16. The principle of work of a single-phase bridge scheme of a rectifier?
17. The principle of work of a three-phase rectifier scheme with a zero point?
18. The principle of operation of a three-phase bridge scheme?
19. The principle of work of a single phase controlled full wave rectifier?
20. The principle of operation of a three-phase controlled rectifier with a zero point?
21. The principle of work of the bridge scheme of the three-phase controlled rectifier?
22. The principle of work of a capacitive filter?
23. The principle of work of an inductive filter?
24. The principle of work of the amplifier on bipolar transistor?

14 GENERAL INFORMATION ABOUT THE ELECTRIC DRIVE

Key concepts: electric drive (ED), feedback, automated ED, unregulated ED (adjustable, tracking, software-controlled, adaptive), group ED (single, multimotor), static moment, resulting moment of inertia, reactive (active) moment of resistance, steady (transitional) mode of ED, mechanical characteristic of the motor (production mechanism), rigidity of mechanical characteristic, static stability, methods of equivalent values, long (short-term, intermittent) mode of the motor.

14.1 Basic concepts

Electric drive (ED) *is an electromechanical device designed to automate workflow.*

Let us consider the basic elements of ED (fig. 14.1). It consists of an electric motor *EM*, transfer mechanism *TM*, a transformer *T* and controlling device *CD*.

In ED, depending upon the requirements, the motor of direct current (DCM)of independent, parallel, serial, or mixed actuation is used, AM, step-by-step motors, etc. The main task of the electric motor in the drive is the conversion of electric power of power source *PS* (in particular, circuit) into mechanical energy of a rotating shaft (rotary motor) or the energy of the linearly moving mass (linear motor). In other words, the motor must develop driving forces: driving moment or mechanical driving force, transmitted to the working mechanism *WM.*

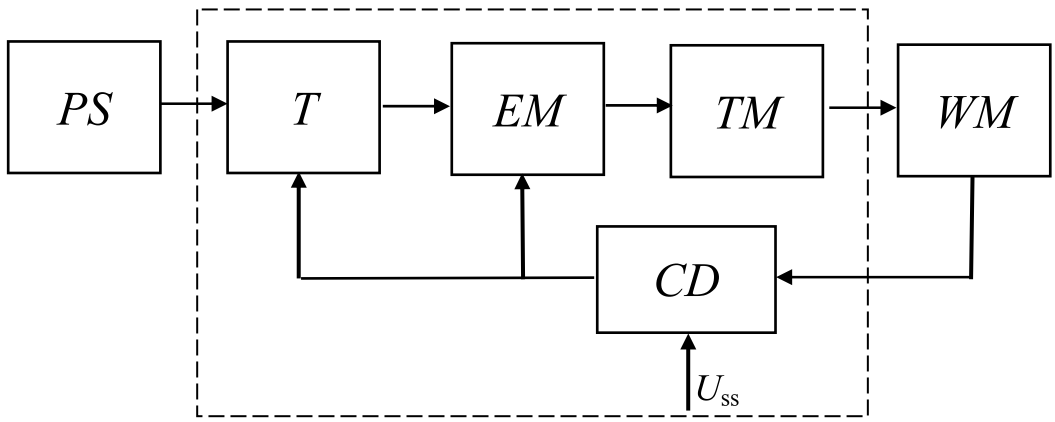


Figure 14.1: Structural scheme of the electric drive

In some cases, the electric motor performs the inverse transformation of mechanical energy of rotation or linear movement, coming from a working mechanism, into electric one. In this case, the motor creates a braking torque.

In modern ED motor operates working mechanism (*WM*) through the transfer device, lowering the rotation frequency (gearboxes, V-belt transmission, and so on) either enhancing or changing the movement type (rack dented, crank, and so on). In some drives, the electric motor is directly connected with the working mechanism.

An important element of the ED is the converter, the necessity of which is caused by the following reasons. First, the electric drives must have the property of changing the frequency of rotation. Typically, this is done by regulation the voltage and frequency of the current consumed by the motor. At the same time, the frequency and voltage of the power supplies are invariable. Secondly, for the work of the DC motor it is necessary to feed on its winding direct voltage, and industrial circuit has a variable voltage. Therefore, for its use as a power source it is necessary to convert the AC voltage into a DC voltage.

To regulate voltage and frequency, and to convert one type of voltage to another converters are used. In ED of large capacity converters of electric machines are used, in drives of medium and small power transistor and thyristor converters are used.

The controlling device ED is used to control the conversion of electrical energy into mechanical energy and ensure the required quality of the process. On execution it is a set of functionally interconnected electromagnetic, electromechanical, semiconductor and other elements. Buttons and other command devices, relays and contactors, blocks of non-contact automatics and others refer to these elements. Many modern high-precision ED contain in its control system computing systems and computers for special purpose.

The control of ED is carried out by the impact on *T* and *EM* control signals generated by *CD*. The control signal can be formed only subject to the setting signal U_{ss} or U_{ss} and signals incoming from other elements of the drive. Figure 14.1 shows a particular case, when on *CD* signals come only from *WM*.

In the case when the control is performed only by subject to the setting signal U_3, ED is open. If there are communications on control of parameters of working mechanism, ED is closed and is called ***automated electric drive***. Communications, providing entering of signals on the *CD* from the other elements are called ***feedbacks***.

As technological processes of modern production are complex, and the demands for technological discipline are constantly increasing, there is a need for wide use of achievement of modern science and technology at the creation of *CD*. Currently semiconductor devices, controlling computers, microprocessor techniques are widely used.

ED are divided into classes depending on their attributes. And in each class only those drives that have some common characteristic, such as classification depending on the engine used, type of power converter or type of movement, etc. are included.

Let us consider how drives are classified according to *the types of regulation*.

Unregulated ED operate working mechanisms with the same speed, i.e., provide simple operations (start, stop, sometimes the reversal of the motor). At a steady mode rotation frequency is determined by the natural mechanical characteristic and moment of static load.

In ***adjustable ED*** the frequency of rotation of motor can change under the influence of the control signal.

Tracking ED are able to provide automatic conversion of any input signal in the movement of the production mechanism not specified in advance. This input signal may be the angle of rotation of any of the input shaft, consequently of what the output shaft of the production mechanism must repeat its movement.

In ***software-controlled Ed*** the linear or rotational movement comes true through a given program. The program is a sequence of trajectories (or laws) of the movement of the production mechanism, reproducible by drive.

Adaptive ED is capable of carrying out automatic selection of the best mode of work of the motor by change of the structure and parameters of the control system.

Depending on the method of transferring of mechanical energy from the motor to the working organs of the production machines ED are divided into three classes: group, single and multimotor drive. ***Group ED*** is one in which one motor is operated using transmission or gear group of working machines or group of working organs of the one machine. ***Single ED*** is one in which the electric motor is operated only by one working organ of the machine. In some cases, the motor is structurally built into the mechanism so that it forms the whole unit with the working organ. Examples of electrified working organs can serve as an electric hand machine (see theme 18), electric hoist, electric spindle, etc. ***Multimotor ED*** is one in which the working organs of one machine are operated by multiple electric motors.

14.2 The mechanics of the electric drive

The machine, where this energy is implemented into useful work. Structural mechanical part of ED transmits mechanical energy from the electric motor to the production implementation of mechanical part of ED can be quite different; however, it contains certain links with functions common to different drives. The electric motor as part of the mechanical part of the drive is a source or consumer of mechanical energy. In the mechanical part of the drive only rotating element (rotor or armature) goes, which has a certain moment of inertia, can rotate with a certain speed and develop a driving moment or braking torque.

The transmission device converts the motion in the mechanical part of the electric drive. Using transfer devices the speed may increase or decrease, change the movement type, for example, convert rotary motion into forward etc. To transfer mechanisms gearboxes, helical, rack dented or belt transmission, drum with rope, crank mechanism, etc. are included. The transmission mechanism is characterized by a transmission coefficient representing the ratio of the speed on outlet to the velocity at the inlet, mechanical inertia and elasticity of its elements, gaps and friction in the gearing and couplings of device.

The working organ of the production machine brings mechanical energy into useful work. Most often it is the consumer of energy. This function of working organ is typical for mechanisms, carrying out materials handling, lifting or movement of load, etc. Meanwhile flow of mechanical power is directed from the motor to the working organ. Sometimes working organ can be a source of mechanical energy. In this case, it gives the mechanical energy stored by the mechanism, for example, when lifting a load, or received by the external mechanism, such as wind load on the surface of the crane. The flow of mechanical power meanwhile is directed away from the working organ to the motor. The working organ is characterized by certain inertia, working moment during its rotational movement, or working force at forward movement. In each mechanism it has its constructional implementation. In particular, on the crane the working organ is the hook, grab of mechanisms of lifting, trolley, bridge of mechanisms of movement, a rotary platform of mechanisms of rotation. On lift it is a cabin, cage, skip. On excavator it is a bucket of mechanisms of head, traction and lifting single-bucket excavators, impeller of rotary excavators, rotary platform of rotation mechanisms.

The transfer of mechanical energy from the motor shafts to the working organ or inversely related to the losses in the mechanical parts. Cause of loss is friction in the bearings, guiding elements, gearings etc. In the mechanical parts having elasticity there is additional loss due to viscous friction in the deformable elements. As a result, the power flux passing from the source to the consumer gradually decreases. It is obvious that mechanical energy loss is covered, by energy source, which is the motor at direct flow of energy and working organ in reverse.

The work performed by the motor or the working mechanism, is determined by the formula:

during rotation

$$W = \int_0^t M\omega dt\,, \tag{14.1}$$

and at forward motion

$$W = \int_0^t Fv dt\,, \tag{14.2}$$

where: $\boldsymbol{F}$ is force, newton;
$\boldsymbol{M}$ is moment, newton metre;
$\boldsymbol{\omega}$ is the angular speed radian/second;
$\boldsymbol{v}$ is linear speed metre/second.

Mechanical power is defined as the derivative of work on time, i.e.,

$$P = \frac{dW}{dt} = M\omega \tag{14.3}$$

for rotational motion, and

$$P = Fv \tag{14.4}$$

for the forward motion.

The task of ED is to perform the laws of motion of the working mechanism specified by the technological requirements. In this case, however it is preceded from the fact that the law of motion of the rotor of the motor is proportional to the indicated law for working organ.

If to assume that the mechanical part of ED consists of absolutely rigid not deformable elements and does not contain air gaps, then the movement of one element gives the full information about the movement of all the other elements, i.e., functional dependencies, corresponding to the laws of motion of all parts of the kinematic circuit of drive, proportional to each other and from the movement of one element it can move with a predetermined relationship between the coordinates to the movement of any other element. This allows considering the movement of ED on any mechanical element. Usually this element is taken from the motor shaft, and is brought to all the external torques or forces, as well as all the inertial mass of the mechanical links.

To bring the motor moment or amplification of load of the working organ of the production machine to the shaft the balance of power in the mechanical part of the drive is used

$$P_{\mathrm{m}} = P_{\mathrm{wm}} + \Delta P, \tag{14.5}$$

where: $\boldsymbol{P_{\mathrm{m}}}$ – the power of the motor on shaft;
$\boldsymbol{P_{\mathrm{wm}}}$ – the power on the working organ;
$\boldsymbol{\Delta P}$ – the power loss in the mechanical links.

If for mechanical part of the drive COP η is known, the equality (14.5) can be represented in the form

$$P_{\mathrm{m}} = P_{\mathrm{wm}}/\eta. \tag{14.6}$$

At the rotational movement of the working mechanism power at the working mechanism and on the shaft of the electric motor are defined as follows

$$P_{\mathrm{wm}} = M_{\mathrm{wm}}\cdot\omega_{\mathrm{wm}}, \qquad P_{\mathrm{m}} = M_{\mathrm{dt}}\cdot\omega,$$

where: $\boldsymbol{\omega_{\mathrm{wm}}}$ – the angular speed of the working mechanism;
$\boldsymbol{M_{\mathrm{wo}}}$ – the load moment on the working mechanism;
$\boldsymbol{\omega}$ – the angular speed of the motor shaft;
$\boldsymbol{M_{\mathrm{dt}}}$ – the drag torque on the motor shaft, which is also called static moment.

Then

$$M_{\mathrm{dt}}\cdot\omega = M_{\mathrm{wm}}\cdot\omega_{\mathrm{wm}}/\eta\ ,$$

or

$$M_{dt} = \frac{M_{wm}}{i_g \cdot \omega}, \tag{14.7}$$

where $\boldsymbol{i_{\mathrm{g}}} = \omega/\omega_{\mathrm{wm}}$ is the transmission ratio of the gearbox.

Similar equations can be obtained for the case of forward motion of the working mechanism. Power on the working mechanism

$$P_{\mathrm{wm}} = F_{\mathrm{wm}}v_{\mathrm{wm}}, \tag{14.8}$$

where: $\boldsymbol{F_{\mathrm{wm}}}$ is the load effort on the working mechanism;
$\boldsymbol{v_{\mathrm{wm}}}$ is the linear speed of movement of the working mechanism.

Then

$$M_{\mathrm{dt}}\cdot\omega = F_{\mathrm{wm}}\cdot v_{\mathrm{wm}}/\eta\ ,$$

or

$$M_{\mathrm{dt}} = F_{\mathrm{wm}}\cdot\rho/\eta, \tag{14.9}$$

where $\boldsymbol{\rho} = v/\omega$ is the radius of reduction of the effort of the load to the motor shaft.

The value M_r, defined by equation (14.9) is called ***the moment of resistance*** (or ***static moment***), reduced to the motor shaft. The values i_g and ρ are determined by constructional parameters of transmission mechanism.

The sense of reduction of the inertial masses and moments of inertia of mechanical parts to the shaft of the motor is that these masses and moments of inertia are replaced by one equivalent moment of inertia J on the motor shaft. Meanwhile condition of reduction is the equality of the kinetic energy, defined by the equivalent moment of inertia, to the sum of the kinetic energy of all moving elements of the mechanical part of the drive.

Equivalent moment of inertia J, converted to the motor shaft, is called the ***resultant or total moment of inertia of the electric drive***. Examples of rotating elements in a mechanical part of the drive can serve, in addition to the rotors of motors, coupling boxes, brake pulleys, drums, turntables of excavators and cranes. At steadily moving items like bridges, trolleys and lift of cranes; load of conveyors, etc. are included.

14.3 The equations of motion of the electric drive

Studying the motion of ED there is a need to identify the various mechanical quantities like way and angle of turn, speed, and acceleration, as well as moments and forces that cause movement and determine its nature.

Movement of ED is determined by action of two moments: the moment developed by the motor and drag torque. Depending on the reasons causing origin of drag torque, moments of resistance are divided into reactive and active ones.

The reactive drag torque appears only due to the movement being an *opposed reaction of the mechanical link in the movement*, for example, the moments of friction in the rotating elements; the moments on the impeller of centrifugal pumps, etc. The reactive torque is always directed against the movement, i.e., has the sign opposite to the sign of speed. The element that generates the reactive moment can only be a consumer of energy.

Active drag torque appears independently from the movement of the electric drive and *is created by extraneous sources of mechanical energy*. This is, for example, the moment caused by the weight of load moved vertically, the moment, created by the force of wind. The direction of the active moment does not depend on the direction of rotation, i.e. the sign of the active moment is not associated with the sign of the angular velocity. At the change of the direction of rotation sign of that moment is conserved. The source that generates active moment can both consume and give energy.

In systems of ED main work mode of the electric machine is a motor. Meanwhile drag torque has inhibitory character in relation to the movement of the rotor and acts towards the motor moment. Therefore, the positive direction of the moment of resistance takes the opposite to positive direction of moment of the motor and the basic equation of motion of ED has the following form

$$M - M_{dt} = J\frac{d\omega}{dt}. \tag{14.10}$$

In equation (14.10) moments are algebraic, and not vector quantities, since both of the moment M and M_{dt} act relatively the same axis of rotation.

The right part of equation (14.10) is called dynamic moment (M_{dyn}), i.e

$$M_{dyn} = J\frac{d\omega}{dt}, \tag{14.11}$$

where the moment of inertia J is defined as

$$J = \int_m r^2 m, \tag{14.12}$$

where: ***r*** is the distance from the axis of symmetry;
m is body weight.

From equation (14.11) it follows that at $M = M_{dt}$, the rotation speed of the electric drive will be invariable (ω=const), and dynamic moment is absent, as $\frac{d\omega}{dt} = 0$. This ***mode is called a steady one***.

When $M > M_{dt}$ $\frac{d\omega}{dt} > 0$, it corresponds to the acceleration of the motor. Dynamic moment in this case is opposite to the moment of the motor, limiting acceleration. If $M < M_{dt}$, then $\frac{d\omega}{dt} < 0$, and the motor slows down. Dynamic moment meanwhile acts in accordance with the motor moment.

Mode of work of ED *at changing speed of rotation (*$\frac{d\omega}{dt} \neq 0$*) is called* ***transitive***. The transitive regime occurs at start-up, braking, change of load, speed control, etc.

The duration of the transitional mode depends on the moment of inertia of the moving masses. On the basis of the equations of motion (14.11) important practical problem of the dependence of the speed from time in the transitional mode or about the time of the transitional mode of ED can be solved:

$$d\omega = \frac{M - M_{dt}}{J}dt, \qquad dt = \frac{J}{M - M_{dt}}d\omega. \tag{14.13}$$

However, for its solution it is necessary to know the dependence of the moment of the motor M and the drag torque M_{dt} from the angular speed of the motor shaft ω, which are determined by the mechanical characteristics of the motor (see chapter 10.7 and 11.9) and mechanical characteristics of the working mechanism, the nature of which is determined solely by the properties of the production mechanism (see chapter 14.4).

For the case of forward movement of the working mechanism (linear electric drive) the basic equation of motion of ED is:

$$\pm F \mp F_{st} = F_{dyn} = m \cdot a, \tag{14.14}$$

where: ***F*** is the force created by the electric motor;
$\boldsymbol{F_{st}}$ is force of static resistance;
$\boldsymbol{F_{dyn}}$ is dynamic force; *m* is mass of steadily moving bodies;
a is acceleration.

In the expression (14.14) dynamic force F_{dyn} depends on the mass m of the moving parts and the degree of change of the speed of their movement, which is expressed by the acceleration a.

14.4 The mechanical characteristics of the production mechanisms and electric motors

Studying the work of the electric motor, operating the production mechanism, it is necessary first of all to determine the conformity of the mechanical characteristics of the motor to characteristics of the production mechanism.

Mechanical characteristics of the production mechanism is *the dependence between speed and torque of mechanism reduced to the shaft of the motor by drag* $\omega = f(M_{dt})$.

The mechanical characteristics of the production mechanisms are divided into the following groups.

Mechanical characteristics, in which the drag torque M_{dt} does not depend on speed (line 1 in fig. 14.2). Cranes, winches, feed mechanisms of metal-cutting machines, piston pumps at a constant height of feed, conveyors with constant mass of relocatable material have such characteristics. All mechanisms can be referred to this group with a certain approximation, in which the main point of resistance is the friction torque, as usually within operating speed friction torque varies little.

Linearly increasing mechanical characteristics (line 2 in fig. 14.2). In this case, the moment of resistance is linearly dependent on speed ω, increasing with its increase.

Nonlinearly increasing (parabolic) mechanical characteristics (curve 3 in fig. 14.2). The drag torque M_{dt} here depends on the square of the speed. Mechanisms with this feature are sometimes called mechanisms with ventilation moment as the blasts the drag torque depends on the square of the speed. Centrifugal pumps, propellers, excavators, etc. are also referred to mechanisms having such mechanical characteristics.

As it has already been noted in chapter 10.7, under mechanical characteristic of the motor the dependence of its angular speed on the torque is understood, i.e., $\omega = f(M)$. For electric motors the decrease in speed of rotation at increase of load moment is typical. However, the rate of change of the speed with change of moment at different motor is different and is characterized by a measure called rigidity. ***The rigidity of the mechanical characteristic*** *is the ratio of the difference of the magnetic moments developed by electromotor device to corresponding difference of angular speed of the electric drive.* I.e., the rigidity β is determined by the ratio

$$\beta = \frac{M_2 - M_1}{\omega_2 - \omega_1} = \frac{\Delta M}{\Delta \omega}. \qquad (14.15)$$

Usually at working places the mechanical characteristics of the motors have a negative rigidity $\beta < 0$. Linear mechanical characteristics have constant rigidity.

In the case of nonlinear characteristics their rigidity is not constant and is defined at each point as derivative from the angular speed

$$\beta = \frac{\partial M}{\partial \omega}. \tag{14.16}$$

The notion of rigidity may be applied to the mechanical characteristics of the production mechanisms. These characteristics can be estimated by rigidity

$$\beta_{dt} = \frac{\partial M_{dt}}{\partial \omega}. \tag{14.17}$$

The mechanical characteristics of the electric motor can be divided into four main categories:

1. *Absolutely rigid mechanical characteristics* ($\beta = \infty$) is a characteristic in which the speed with change of moment remains unchanged. Synchronous motors have such characteristic (line 1 in fig. 14.3).

2. *Hard mechanical characteristic* is a characteristic in which the speed with change of moment is reduced to a small degree. DC motors with independent actuation have hard mechanical characteristics, as well as asynchronous motors within the working part of the mechanical characteristics (curve 2 in fig. 14.3).

3. *Soft mechanical characteristic* is a characteristic in which the change of moment, the speed varies greatly. Motors DC of serial actuation have such characteristic, especially in the area of small moments (curve 3 in fig. 14.3). For these engines, the rigidity does not remain constant for all points of characteristics.

4. *Absolutely soft mechanical characteristic* ($\beta = 0$) is a characteristic in which the moment of the motor with change of the angular speed remains constant. For example, DC motors with independent actuation have such characteristic at power them from a current source or at work in closed systems of electric drive in mode of stabilization of armature current (straight line 4 in fig. 14.3).

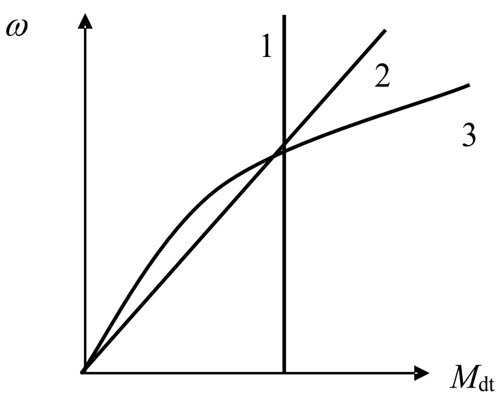


Figure 14.2: Mechanical characteristics of the production mechanisms

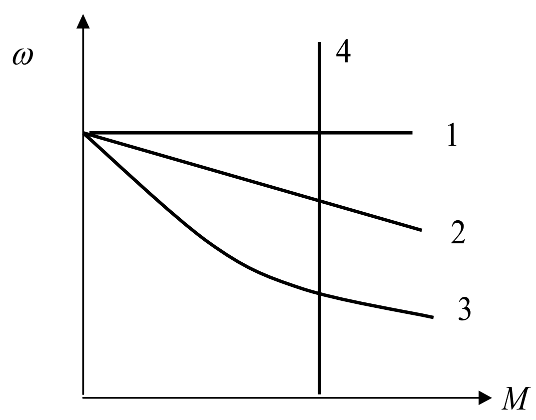


Figure 14.3: Mechanical characteristics of the motors

If the motor and the production mechanism have mechanical characteristics, it is easy to find the point (M, ω) characterizing the steady state. It is enough to add graphically two characteristics on moment. The resulting curve is called a ***joint characteristic of the electric motor and the production mechanism***. At a point where joint characteristic intersects the axis of rotation frequency ω, there is the point of steady regime, in which the sum of the moment of the motor and the mechanism are equal to zero. Therefore, in accordance with the expression (14.10) rotation speed will not change in time.

On figure 14.4 the mechanical characteristics of the motor 1 and the feed mechanism of the lathe 2 are presented as an example. Curve 3 of joint characteristics is obtained in the following way. An arbitrary rotation speed ω_1 is taken and determined without regard to sign the moments generated by the motor M_{mot1} and the production mechanism M_{pm1}. Then their difference M_{joint1} is determined graphically. The result is put aside in the direction of the larger of the moments M_{mot1} and M_{pm1} at the same speed ω_1. Then this procedure is repeated at a different rotation speed ω_2, and so on. Thus, the obtained point curve is drawn, which is a joint feature.

In the example (fig. 14.4) the joint characteristic intersects the axis ω, i.e., the moment in point with the rotation frequency ω' is equal to zero. Therefore, at this rotation frequency the condition (14.10) is satisfied and the steady mode is realized. Using the features 1 and 2, it is easy to determine the moment M_{mot}' developed by the motor, and of M_{pm} production mechanism in this mode. In practice, the coordinates of the point of the steady mode are determined somewhat differently.

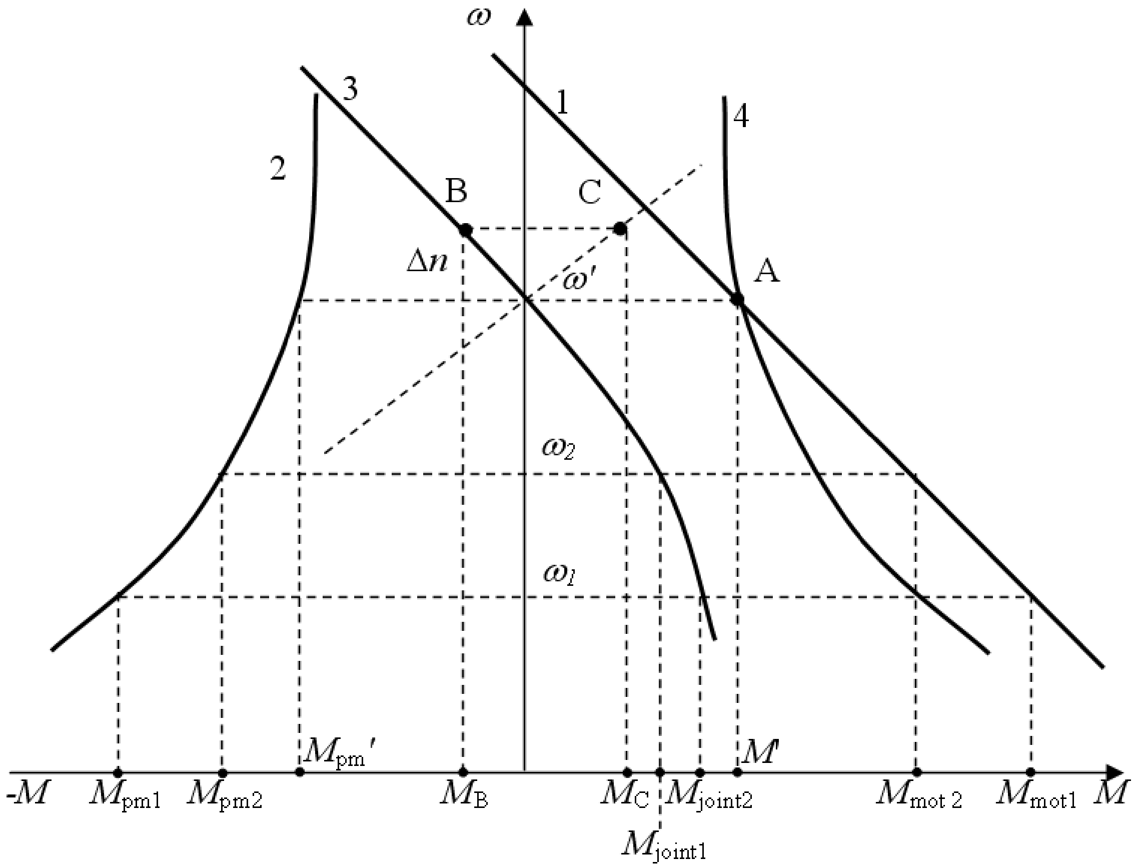


Figure 14.4: Joint characteristics of the motor and production mechanism

Condition (14.10) is performed, if the absolute value M is equal to M_{pm}. Therefore, if the mechanical characteristics of the production mechanism before moment to omit the minus sign, point of established mode will be there where newly acquired characteristic and mechanical characteristic of the motor will intersect. Graphically the transition from negative moments of static resistance to positive occurs in mirror of the curve *2* relatively to the axis ω in the first quadrant. In figure 14.4 this characteristic is the curve *4* and, consequently, the steady mode is shown by the point *A* with coordinates M', ω'.

The balance of drag torque mechanism and torque of the motor at a certain speed corresponds to the work of the electric motor and the production mechanism in the steady mode, i.e. $M = M_{dt}$.

The change of the drag torque on the shaft of the motor causes the motor speed and moment, which it develops, can be changed automatically, and the drive will continue work steadily at a different speed to the new value of moment. In electric motors the role of an automatic controller can perform EMF of the motor. Let $M = M_{mot1}$ and the motor work with speed ω_1. With increase of load, the motor is decelerated, and the speed is reduced due to EMF reduces. At reduction of EMF the current in the armature circuit of the motor increases and the moment delivered by the motor. The increase of the moment of motor continues until the equilibrium of moments $M = M_{mot2}$ happens, corresponding to the new speed ω_2. The examined conditions of work of electrical drive in steady mode characterize the *static stability of the drive* when the change with time of the speed and moment occurs relatively slowly in contrast to the dynamic stability that occurs during the transitional modes.

Static stability *is the state of steady mode of work of drive, when at accidentally arisen speed deviation from the steady value, the drive will return to point of steady mode.* At an unstable movement any, even the smallest speed deviation from the steady value leads to a change in the state drive, i.e. it does not return to the point of steady mode.

The drive is statically stable if in the point of steady mode the condition comes true

$$\frac{dM}{d\omega} - \frac{dM_{dt}}{d\omega} < 0, \tag{14.18}$$

or

$$\beta - \beta_{dt} < 0. \tag{14.19}$$

Condition (14.18) means that the drive is statically stable if at the positive increase of the angular speed the motor moment is less than the static moment (drag torque) and the drive as a result will slow down to a previous value of the speed. At a negative increase in the angular speed the motor moment is greater than the drag torque, and the drive speeds therefore to the previous value of the speed.

At constant moment of load (line 1 in fig. 14.2) static stability will be determined only by the rigidity of the mechanical characteristics of the motor as $\frac{dM_{dt}}{d\omega} = 0$. If it is negative, then the work in steady state is steady $\frac{dM}{d\omega} - \frac{dM_{dt}}{d\omega} = \frac{dM}{d\omega} < 0$.

Usually at design of electric drive mechanical characteristic of the production mechanism is already specified. Therefore, to obtain stable work at steady mode for certain speeds and drag torques of the production mechanisms it is necessary to choose the mechanical characteristic of the electric motor of corresponding form. This can be achieved by selection of the electric motor of corresponding type and by change of the electrical parameters of its circuits.

14.5 The choice of electric motor

The motor selection at design of ED is an important step. No elements of control system or feedbacks are able to provide the required torques, the desired speeds and acceleration of mechanism, if the motor construction, the main power node of drive does not create conditions for it.

The right choice of motor is determined by both economic and technical requirements to its parameters and indicators. First of all, it is preferable to choose the simplest, cheapest and most reliable motors, i.e. asynchronous and synchronous. If these machines are not able to satisfy the technical requirements, then DC motors must be chosen.

Requirements of the motor parameters are nominal voltage corresponding to the voltage of circuit; power, providing overcoming of drag torque at the required speeds and accelerations; overload capability that supports the work of drive at short-time loads; the range of the speed change at regulation, corresponding to the requirements of the technological process, etc.

The most important parameter used to select the motor is power. The machine is selected properly according to power if it executes the required function and does not overheat.

An essential part of construction of any electrical machine is insulating materials that have physical and chemical properties, allowing to isolate individual conductors of the windings from each other. If insulating materials lose their properties, shorting of individual sections of the windings happens, and the machine breaks down. Insulation materials lose their dielectric properties, if their temperature is above the maximum allowable temperature. *The ability of materials to maintain its properties at the maximum allowable temperature* is called ***the heat resistance***.

If during work of the motor with the isolation of the corresponding class of heat resistance temperature of its heating is less than (or equal to) the maximum allowable temperature of this class, the motor will work in normal conditions. If the motor temperature rises above the upper limit, the insulation begins to lose its dielectric properties and fail.

After connection the electric machine passes current to the power source in its windings, steel is switched and other physical processes happen in result of which part of the electrical and mechanical energy, called loss ΔP, is converted into heat. A certain amount of thermal energy is given to the environment, and the rest is spent on heating of the motor.

In the first moment of time after connection to power source the motor is intensively heated, then the process slows down. Finally there comes a period when a change in the motor temperature does not occur (see fig. 14.5).

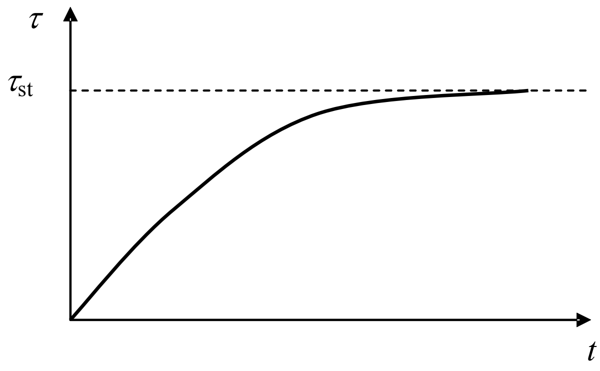


Figure 14.5: Feature of motor heating

It should be noted that when it comes to heating or cooling of electrical machines, the relative value is usually used instead of the actual temperature, then so called ***exceeding of temperature τ,*** *representing the temperature difference between the machine and the environment* occurs.

With some assumed characteristic of heating the electric machine has an exponential form (fig. 14.5). As it can be seen from the figure, over time, the temperature τ tends to a maximum value $\tau = \tau_{st}$.

The electric machine does not overheat for a long time, if its steady value of exceeding of temperature τ_{st} is less than (or equal to) a valid τ_{val} class of insulation of machine: $\tau_{st} \leq \tau_{val}$.

Still the case is considered when the load moment, and hence the power developed by the motor, do not change in time. If these values increase, the steady exceeding of temperature of the motor will increase, as the currents passing through the windings will increase therefore, the loss will increase.

When the load moment exceeds the permissible values for motor, the power consumed by the motor increases, and, as a result of increase of loss in the motor, τ_{st} may be more τ_{val} and the motor will begin to overheat. Limit of increase of the load is the rated power of the motor, i.e., if in the process of work the motor develops power, not exceeding the nominal, then the manufacturer guarantees its normal operation without overheating.

At the motor load it can orientate itself also on the *rated current* and *rated moment* of motor. The current and moment of the motor will not exceed its rated value. This is true, if the ambient temperature corresponds to 40° Celsius. Thermal calculations of the motor in the process of construction must orient at such a temperature of environment.

For a constant load it is enough to determine the power on shaft of production mechanism and choose the catalog motor of the same rated capacity or the nearest higher power to select the electric motor.

At variable load motor selection is complicated. In this case, ***load diagram*** is used, which defines *graphic dependence of power of resistance of the working mechanism on time*, as well as the load chart of power, loss of power and motor current. The choice of motor is as follows. Known on load diagram variable power *P(t)* of the mechanism (fig. 14.6) is replaced by the *constant average power* calculated for the cycle t_c by the formula

$$P_{av} = \frac{P_1 \cdot t_1 + P_2 \cdot t_2 + \ldots + P_6 \cdot t_6}{t_1 + t_2 + \ldots + t_6} \quad . \qquad (14.20)$$

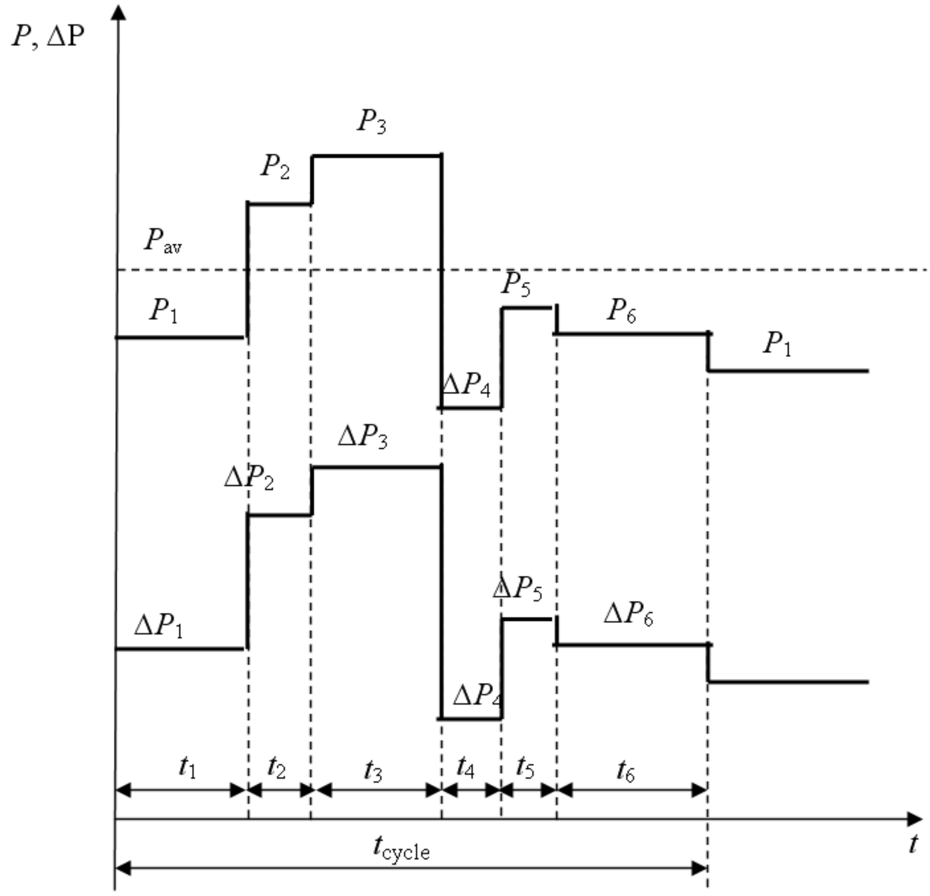


Figure 14.6: Stepped load chart of motor

Then P_{av} is multiplied by the ***coefficient of assurance*** k_s= 1,14 ÷1,3 $P'_{av} = P_{av}\, k_s$. Further on P'_{av} the motor is selected and one of the load charts is built for it and a calibration calculation is done.

For more precise selection of the motor the ***method of average loss*** is used. For this method, the graph of motor power is taken, which is different from the load chart of power of the production mechanism by appearance of dynamic moment at change of the drive speed. Indeed, in transitional conditions the power generated by the motor is spent not only on overcoming the static moment of resistance, but also overcoming the dynamic moment.

Usually this difference is neglected, and the chart of the production mechanism is used for the method for average losses. First, for each section of the load diagram with constant power with help of characteristic of COP $\eta(P)$ loss of the engine ΔP is determined, and then the average loss for whole load charts in equation is

$$\Delta P_{av} = \frac{\Delta P_1 \cdot t_1 + \Delta P_2 \cdot t_2 + ... + \Delta P_6 \cdot t_6}{t_1 + t_2 + ... + t_6}, \quad (14.21)$$

where: $\boldsymbol{\Delta P_1}$–$\boldsymbol{\Delta P_6}$ is loss on sections 1–6 of chart; t_1–t_6 is time of sections 1–6 of chart (see fig. 14.6).

Next, the nominal loss ΔP_{rat} at nominal power of motor and ηном in nominal mode and compared values ΔP_{rat} and ΔP_{av} are determined. If $\Delta P_{rat} \geq \Delta P_{av}$, then it is considered that $\tau_{st} \leq \tau_{val}$ and the motor is selected correctly. If $\Delta P_{rat} < \Delta P_{av}$, we must choose from the directory of the next closest motor with more power and repeat the calculation.

Methods of equivalent quantities: current, moment and power are less accurate but simpler, that is why they are more often used.

In each of these methods on constructed for the beforehand selected motor diagram, the value of equivalent value is determined (current, moment or power) by the following expressions:

$$I_{eq} = \sqrt{\frac{I_1^2 \cdot t_1 + I_2^2 \cdot t_2 + ... + I_n^2 \cdot t_n}{t_1 + t_2 + ... + t_n}}, \tag{14.22}$$

$$M_{eq} = \sqrt{\frac{M_1^2 \cdot t_1 + M_2^2 \cdot t_2 + ... + M_n^2 \cdot t_n}{t_1 + t_2 + ... + t_n}}, \tag{14.23}$$

$$P_e = \sqrt{\frac{P_1^2 \cdot t_1 + P_2^2 \cdot t_2 + ... + P_n^2 \cdot t_n}{t_1 + t_2 + ... + t_n}}. \tag{14.24}$$

The obtained values of equivalent values are compared with the corresponding nominal value. If the nominal value is not less than the equivalent one, the motor on power is selected correctly.

It must be remembered that correctly selected motor on power may be unsuitable for use in the drive, if its transmission capacity is unsatisfactory.

Check of motor on permissible overload in the method of the equivalent current is produced by equation

$$I_{max}/I_{rat} \le \lambda_i \tag{14.25}$$

where: $\boldsymbol{I_{max}}$ is the maximum value of current at variable load;
$\boldsymbol{\lambda_i}$ is a valid coefficient of motor overload on current (for DC motors of general purpose λ_i = 2–2,5; for special engines it may be higher).

If the condition (14.25) is not running, you should choose catalogue following motor with more power and test it only on overload capacity.

When choosing asynchronous motor it is necessary to check that its maximum moment is greater than the highest moment of load chart.

For DC motors with independent or parallel actuation, either asynchronous or synchronous method can be applied to each of the listed methods.

For DC motors of sequential and mixed actuation the only suitable methods of average loss and the method of equivalent current are used.

The choice of the motor during ***prolonged mode of work***, *when the motor temperature reaches steady value* has also been considered.

However, the motor can also work in the ***short-term mode*** when on the working period it has no time to get warm until steady value, and *during the time of disconnection it has time to cool to the ambient temperature*.

In momentary mode, it is necessary to load the motor higher than the nominal, so that it is completely used on the heat. Maximum dosage of load is carried out from condition $\tau_{max} \le \tau_{val}$.

It seems unreasonable to use motors of general purpose in intermittent mode because they have low overload capacity and require overrated power.

For short-term modes, the industry produces a special motor with *high overload capacity* and the indication of the nominal normalized duration of work (10, 30, 60 and 90 min). The choice of motor is performed as well as in long mode, using nominal data corresponding to the actual time of work. If the time of work differs from the standard, the real parameters of the motor (power, current, and moment) lead to the nearest selected normalized time.

There is another mode of motor ***repeatedly-short-term***, at which work (t_w) is performed in periods alternate with pauses (t_p); and in a work period the motor has not time to heat up to steady temperature, and during a period of pause it has no time to cool to ambient temperature.

Repeatedly-short-term mode is characterized by ***relative duration of inclusion***, which is defined as

$$ID = \frac{t_w}{t_w + t_p} \cdot 100\% \tag{14.26}$$

Special motors with higher starting torques are produced for repeatedly - short-term mode as well as for short-term. Normalized relative duration of the inclusion of such motors is 15, 25, 40, 60%. It is taken into account that the cycle time does not exceed 10 minutes; otherwise the mode is considered long. In the catalogs for motors of repeatedly - short-term mode their nominal data for each normalized (standard) values ID_{st} are indicated.

The choice of motor is the same for long mode, using the nominal data for the corresponding value ID_{st}. If valid duration of inclusion (DI) differs from the standard (ID_{st}), the motor is chosen on nominal data corresponding to the nearest ID_{st}. Meanwhile the actual parameters of motor (power, current, and moment) lead to the selected parameter ID_{st}.

Key findings

1. The main nodes of ED are the electric motor, transmission device, the control device, the transformer and the working organ.

2. The control of ED is carried out by the impact on the transformer and the electric motor control signals generated by the control device.

3. According to the type of regulation ED are divided into unregulated, regulated, tracking, software-controlled and adaptive.

4. The main task of ED is the fulfillment of the given technological requirements of the laws of motion of the working organ.

5. The main mode of ED is the steady mode.

6. When selecting an electric motor for ED it is necessary to correspond its mechanical characteristics to mechanical characteristics of the production mechanism.

7. There are four categories of mechanical characteristics of electric motors: absolutely rigid, hard, soft, and absolutely soft.

8. The drive is considered statically stable when at increase or decrease of angular speed the motor moment has a value that leads to the restoration of the former magnitude of the angular speed. The work of ED in the steady mode is stable, if the rigidity of the mechanical characteristics of the motor is negative.

9. The choice of motor for ED is carried out on its power and condition of heat resistance.

10. Depending on the nature of change in heating of the electric motor in the process there are short-term, repeatedly- short-term and long modes.

Control questions

1. Define electric drive. Name its main elements.
2. How are systems of electric drives classified?
3. How is the transmission ratio of the transmission mechanism determined?
4. What is the static moment? What does it depend on?
5. Describe the concepts of “reduced moment of inertia” and “dynamic moment”.
6. What is reduction of moments to one axis of the motor made for?
7. How is the equation of motion of ED written?
8. What do the modes of work of ED depend on?
9. What does the time of transitional mode of ED depend on?
10. Describe the concept of rigidity of the mechanical characteristics. How are mechanical properties of rigidity classified?
11. Define static stability of electric drive.
12. What does the heating of the electric motor depend on?
13. List possible modes of work of the electric motors. What are the conditions to choose their power?
14. How is power of motor chosen for mode of long constant load?
15. How is motor chosen by the method of average loss?
16. What is the concept of the method of equivalent current and the area of its usage?
17. What is the concept and scope of the method of equivalent moment?
18. How is motor chosen by the method of equivalent power?
19. In what case is the work mode of the motor considered prolonged?
20. What mode is called repeatedly-short-term, how is the relative duration of inclusion determined?
21. How is the power of motor chosen for repeatedly-short-term mode?
22. Which mode is called short-term?

15 SPEED REGULATION OF ELECTRIC DRIVES

Key concepts: speed regulation, regulation range of angular speed, regulation smoothness, efficiency of regulation, stability of angular speed, direction of regulation, allowable load of motor, porosity of control pulses.

15.1 The main indicators of the regulation of the angular speed of the electric drive

The industry uses a large number of production mechanisms working with a variety of changing speed. In particular, in the elevators, lifting and transport mechanisms speed should be reduced as far as the approach to the point of stopping to provide smooth deceleration and precise stopping in the right place. The speed at which the boiler must operate exhauster is determined by the fuel quality and its moisture, ash content, process conditions and the desired performance of the combustion boiler. In all of these mechanisms, as in many others, to achieve high productivity and quality of work it is necessary to regulate the speed.

A speed regulation is called *a forced change of speed of the electric drive depending on the requirements of the technological process.* The speed regulation is carried out by the controlling influence on the driving motor. The main indicators characterizing the different methods of speed regulation of electric drives are: regulation range, fluency, efficiency, stability of speed, direction of the speed control (decrease or increase of speed relatively to the main one), permissible load at various speeds.

The regulation range of the angular speed is *the ratio of possible steady speed: maximum ω_{max} to the minimum ω_{min} at given accuracy of regulation* (with a given static drop of speed of electric drive) for established limits of changes of the load moment and other disturbances:

$$D = \omega_{max} : \omega_{min} . \qquad (15.1)$$

Typically, the regulation range is expressed in numbers as a ratio, for example: 2:1, 4:1, 10:1, 20:1 and so on.

Smooth regulation *characterizes the speed jump at the transition from the given speed to the nearest possible.* The smaller is the jump, the higher is the smoothness. Number of speeds obtained in this range is determined by the smoothness of the regulation. It can be estimated by ***coefficient of smoothness of regulation***, which is defined as the ratio of the two neighboring values of angular speed at the speed regulation

$$\varphi_{sm} = \omega_i / \omega_{i-1} , \qquad (15.2)$$

where: $\boldsymbol{\omega_i}$ and $\boldsymbol{\omega_{i-1}}$ is angular speed, respectively, at the i-th and (i-1)-th stages of regulation.

At smooth regulation $\varphi_{sm}\to1$, and the number of speed $z\to\infty$. The speed number z, the range of regulation D and the coefficient of smooth regulation are linked by equation

$$D = \varphi_{sm}^{z-1}. \tag{15.3}$$

Efficiency of regulation *is characterized by the cost of manufacture and operation of the electric drive.* It should be noted that economically profitable is a regulated electric drive, which provides more performance driven by it in action mechanism at high quality of technological process and relatively quickly pays for itself. At evaluation of the efficiency of regulated electric drives it is also taken into account its reliability in exploitation and energy loss in process of regulation.

The stability of the angular speed is characterized by a change of angular speed for a given deviation of the moment of load and depends on the rigidity of the mechanical characteristics. The greater is the rigidity of characteristics, the higher is the stability of the angular speed.

The direction of the speed regulation *is the increase or decrease in the value of speed with respect to its main value.* There is one-zone regulation of base speed, one-zone regulation up from the main speed and dual-zone regulation, when it is possible to obtain the characteristics above and below the natural one.

Permissible load of the motor *is the maximum value of moment that the motor is capable to develop protractedly at work on regulation characteristics.* It is determined by heating of the motor and is different for ways of regulation. Change of the load moment depending on the speed at the different production mechanisms is also different. For example, many mechanisms require regulation at constant moment, such as cranes, winches, some rolling mills, etc. On the other hand, there are mechanisms in which the speed regulation is performed with constant power.

Fundamentally by selection of the appropriate power of motor any change in the load moment or power at the speed regulation can be satisfied. However, the regulation of the angular speed of the motor may be uneconomical, as at different angular speeds it will be used differently and on some of them will be underutilized. Underutilization of the motor leads to deterioration of operation indicators of the drive, as the COP of the motor is reduced, and at alternating current the power factor also decreases. Therefore, *it is desirable to use a control method in which the engine would be fully loaded with all angular velocities if possible.*

In the next section the main ways of regulation of the angular speed on the sample having the greatest usage in the construction industry, that is DC motor with separate actuation are considered.

15.2 Regulation of the angular speed of the DC motor independent actuation

The angular speed of the MDC with independent actuation is determined by the equation

$$\omega = \frac{U - IR}{k\Phi}. \tag{15.4}$$

From (15.4), it follows that the speed of a DC motor with independent actuation is possible to regulate in the three following ways:

1) by change in the actuation current I (magnetic flux Φ) of motor;

2) by change in the resistance of the armature circuit R through resistors (rheostat regulation of speed);

3) by change of the armature of the motor input voltage U.

15.2.1 Speed regulation by change of the actuation current (or magnetic flux Φ, which is determined by the actuation current) is one of the simplest and most economical way, as the power consumed by the actuation winding of the motor is about 2–2,5 % of motor power.

The actuation current is regulated with help of resistor (fig. 15.1) in the case of low power motors, or through a voltage regulator VR (fig. 15.2). Speed regulation in this case is made up from the basic one, and permissible moment of motor is changed according to the law of hyperbole and a valid power remains unchanged. Since the load capacity is determined by a current equal to the nominal, the power developed by the engine is constant ($P = P_{nom} = \text{const}$). When changing the excitation current electromechanical characteristics $\omega = f(I)$ correspond to different values of the angular velocity of the ideal idle defined by the formula

$$\omega = \frac{U}{k\Phi}. \tag{15.5}$$

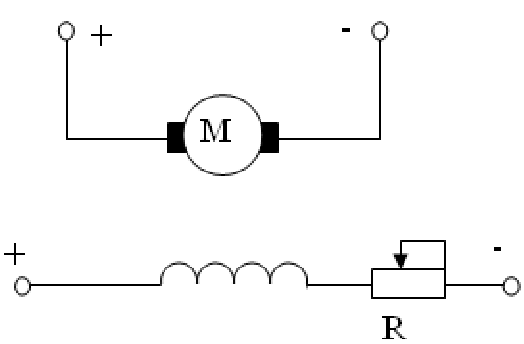


Figure 15.1: Resistor regulation of the actuation flux

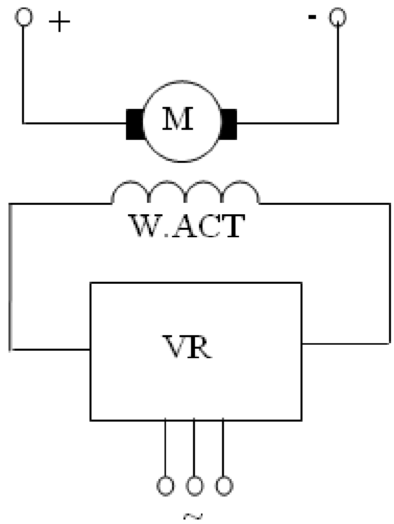


Figure 15.2: Flux regulation of the actuation by voltage regulator

Figure 15.3 shows the electromechanical characteristics of the motor at the speed regulation by flux actuation. The angular speed of the ideal idle ω_0 corresponds to the natural characteristics when the flux Φ = Φ_{rat}. Values of angular speeds of ideal idle at a weakened flux $\omega_0^{'}$ and $\omega_0^{''}$ lie above ω_0 and all features are intersected by the x-axis at one point. This follows from the fact that at $\omega = 0$; the equation for any electromechanical characteristics has the form:

$$0 = \frac{U - IR_a}{k\Phi}, \qquad (15.6)$$

where the current in the armature of the motor is determined as follows:

$$I = \frac{U}{R_a} = I_{shc}. \qquad (15.7)$$

Thus, at different actuation currents and at the angular speed of the motor it is equal to zero, the current in the armature circuit equal to a current of short circuit of the motor, this value of current is determined by the total point of intersection of the electromechanical characteristics.

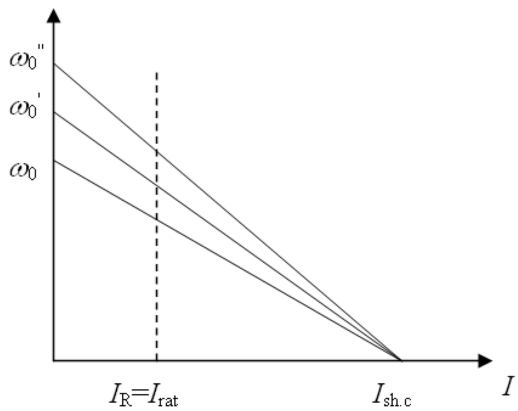


Figure 15.3: Electromechanical characteristics of the motor of independent actuation

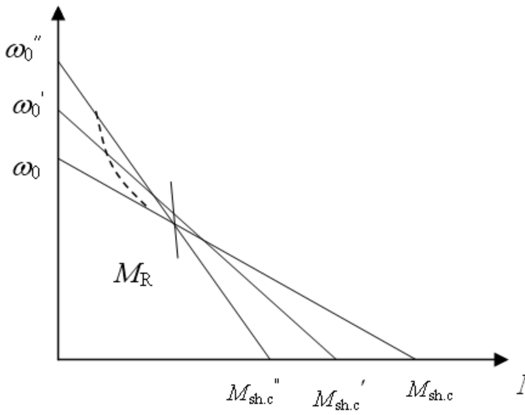


Figure 15.4: Mechanical characteristics of the motor of independent actuation

Mechanical characteristics (fig. 15.4) have the same values of angular speeds of ideal idle as electromechanical characteristics. However, these characteristics do not intersect at one point on the x-axis, so as on measure of reduction of the flux the moment of short circuit decreases is defined by the formula

$$M_{sh.c} = k{\cdot}I_{sh.c}{\cdot}\Phi. \qquad (15.8)$$

This method of regulation is economical at a constant power. Full use of the motor corresponds to points that lie on the line of the rated current I_R = I_{rat}. This corresponds to points that lie on the hyperbolic curve of drag torque M_R, as shown by the dotted line in figure 15.4. Meanwhile the power loss in the armature circuit at work on regulation characteristics will be the same as on the natural characteristics, and loss on actuation will be less. At work on angular speed corresponding to the points lying to the left of the specified curve of moment M_R, the motor will be underutilized. Work at speeds to the right of this curve will cause the motor overload.

Usually regulated motors have regulation range from 2:1 to 5:1, in some cases up to (8–10):1. The regulation range is limited by various factors. The main element among them is the deterioration of the conditions of commutation with increase of angular speed, as the reactive EMF that causes arcing at the collector, proportional to the current and angular speed, i.e., $E_p = c \cdot I \cdot \omega$. In addition, at high angular speeds it is required to increase the mechanical strength of the armature. The lower limit of the angular speed is limited by the degree of saturation of the machine and the heating of winding, i.e., the nominal angular speed.

Significant smooth regulation within a specified range can be obtained and it is determined by the number of steps of regulation rheostat or the same number of stages of special devices regulating the voltage bringing to the actuation winding.

In practice, it is often used pulsed parametric regulation of the actuation current at which it is possible to obtain smooth regulation of the angular speed in the range of (2–3):1 or more (see section 15.3).

15.2.2 Rheostatic regulation of the angular speed of MDC with independent actuation is carried out by changing the resistance of the armature circuit and requires no special explanations. In the subject 10 it was considered rheostat mechanical characteristics of MDC of independent actuation. The characteristics have similar views and at regulation of the angular speed by means of the rheostats in the armature circuit. Unlike starting rheostat regulation rheostat must be calculated in accordance with the work mode of the drive, it is included not only briefly during start-up, but also during work of the motor with a given angular speed.

In this method of regulation the rigidity of characteristics changes, and with it the stability of the angular speed, the angular speed is regulated down from the main, and full use on the motor current is achieved at regulation with constant rated moment if the motor has an independent ventilation.

Indeed, if we assume that for any angular speed rated current is permissible, i.e. $I_{per} = I_{a.rat}$, and the motor flux remains rated, then permissible moment of motor is equal to the rated one $M_{per} = k \cdot \Phi_{rat} \cdot I_{a.rat} = M_{rat}$; then on measure of decrease of angular speed permissible power of the engine decreases as $P_{per} = M_{rat} \cdot \omega$. Assuming the differential of angular speed 25% at change of the load moment at ±25% from rated, the regulation range will be approximately 2:1.

In most cases, rheostatic regulation of the angular speed is performed using contactors, locking separate stages of resistors, i.e., the drive speed is changed discretely, so this way at contactor control does not provide smooth regulation.

Another way to achieve smooth control with the bringing in of the resistors in the circuit of armature is the use of (at small motor power) pulsed parametric regulation of the angular speed.

Scheme of inclusion of DC motor with independent axtuation at pulse regulation of additional resistance in the armature circuit is shown in figure 15.5. Additional resistor R_{add} included in the circuit of armature or completely include into the scheme, or locked in a short circuit by the key K. Commutation of key K is carried out periodically. At locked in a short circuit R_{add} current in the armature circuit and the angular speed of the motor increase, and at the inclusion of R_{add} in the circuit when the key K is open, the current and the angular speed decrease. Fluctuations of the current and angular speed occur around some mean value. Average value of current is determined by the load moment on the motor shaft, and the average value of the angular speed depends on the ratio of the durations of closed t_1 and open t_2 state of the key K and from the load moment. The amplitude of the current oscillation and angular speed depend on the given parameters of the drive from frequency of the commutation of key, which should be high enough.

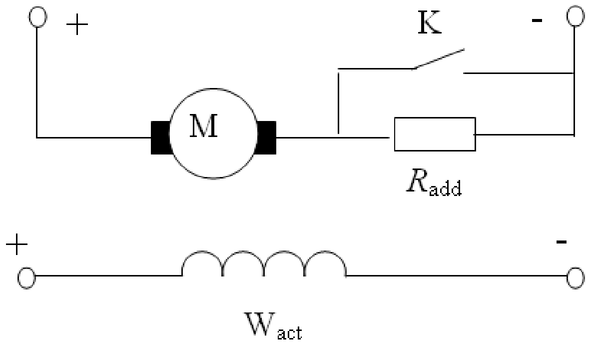


Figure 15.5: Scheme of inclusion of MDC with independent actuation at pulsed regulation of speed

The ratio of durations t_1 closed and t_2 open position of the key K is usually expressed as a relative value

$$\varepsilon = \frac{t_1}{t_1 + t_2}, \tag{15.9}$$

called ***the duty ratio of control pulses.*** It is obvious that with increase ε at constant load on the motor shaft its angular speed will increase, and at $\varepsilon = 1$, the motor will work at a natural feature (the key K is permanently closed). At $\varepsilon = 0$, the motor will work at rheostat characteristic corresponding to the permanently included resistor R_{add} (the key K is open). At other values ε equivalent (average) additional resistance in the armature circuit is determined by the ratio

$$R_{add.e} = R_{add}(1 - \varepsilon). \tag{15.10}$$

The mechanical characteristics of the motor (fig. 15.6), are defined by the formula (for medium values of angular speed and moment)

$$\omega_{av} = \frac{U}{k\Phi} - \frac{M_{av}}{k\Phi^2}\left[R_a + R_{add}(1 - \varepsilon)\right], \tag{15.11}$$

i.e. characteristics have the same form and the same properties as at purely rheostat regulation of the angular speed.

To implement the pulse method of regulation the resistance (and hence angular speed) contactless keys are used, made on the base of the transistor (for currents up to 15–20 A) or thyristors (for currents up to 100–150 A). Figure 15.7 shows the circuit of the thyristor key. The role of key performs the thyristor $VS1$, shunting resistor R_{add} at controlling pulse feed on it. The thyristor $VS1$ is turned off using auxiliary thyristor $VS2$, connecting to the thyristor $VS1$

commutating condenser C_k, beforehand charged through the thyristor *VD*4 and resistor R_s from low-power source U_s. Turning off of thyristor *VS*2 occurs at the end of the recharge of condenser C_k from voltage of the armature circuit (voltage drop on R_{add} at turned on thyristor *VS*1). At the next turning on of thyristor *VS*1 reverse oscillatory recharge of the condenser C_c is carried out via *VS*1, diode *VD*3 and the reactor L_k.

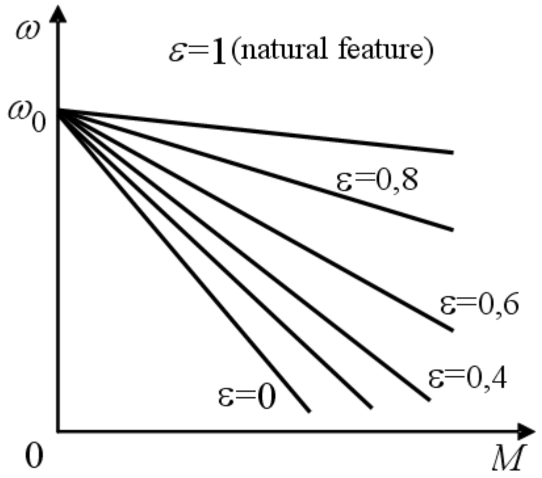


Figure 15.6: Characteristics of MDC at pulse regulation of speed control

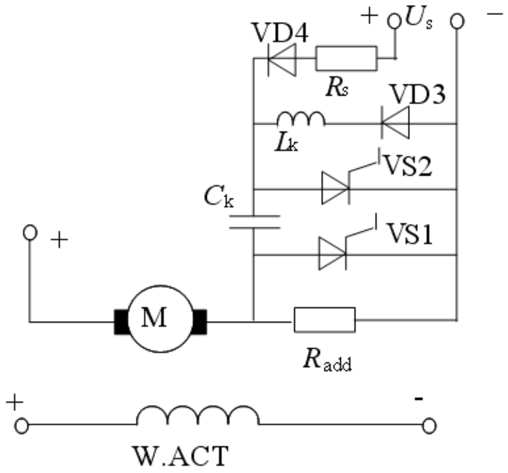


Figure 15.7: Scheme of thyristor key

At regulation of the angular speed by the bringing in of the resistors in the circuit of the armature of the DC motor power loss in this circuit are proportional to the consumed power and the differential of angular speed, expressed in relative units.

15.2.3 Speed regulation by change of the voltage on the armature of the motor. The regulation of the angular speed is carried out down from the main. With a decrease of the angular speed permissible moment remains constant, because the permissible armature current is equal to the nominal and flux with independent actuation remains constant (rated).

For different values of the voltage the angular speed of the motor is equal to:

$$\omega_1 = \frac{U_1 - IR_a}{k\Phi}; \quad \omega_2 = \frac{U_2 - IR_a}{k\Phi}.$$

Their attitude

$$\frac{\omega_1}{\omega_2} = \frac{U_1 - IR_a}{U_2 - IR_a}. \tag{15.12}$$

Hence it follows, that at change of the voltage regulation characteristics are parallel to each other, i.e. have the same rigidity (as shown in fig. 15.8), which determines the relatively high stability of the angular speed. Regulation range

$$D = \frac{U_{rat} - IR_a}{U_{\min} - IR_a}, \tag{15.13}$$

where: $\boldsymbol{U_{rat}}$ and $\boldsymbol{U_{min}}$ are rated and minimum voltage.

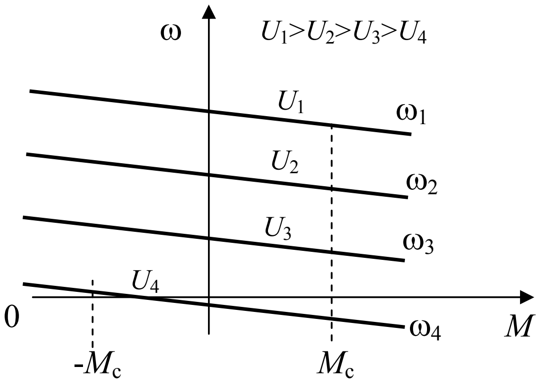


Figure 15.8: Characteristics of MDC in the regulation of the rate by change armature voltage

From (15.13) it follows that the relative differential of the angular speed increases with decrease of voltage. This limits the regulation range by the value D = (8–10): 1 in drive systems without feedback (in closed systems the range of regulation is significantly higher and can reach values of 1000:1 and more).

Smoothness of regulation is determined by the smoothness of the supply voltage variation and is usually characterized by a coefficient of smoothness φ_{sm}.

COP of motor in this case is equal to the ratio of the actual angular speed of the motor to the angular speed of the ideal idle at a given characteristic. The power loss in the armature circuit at a constant moment of load remains unchanged at regulation of the angular speed and is equal to the loss at work on natural characteristic. COP of the motor drops as useful power on measure of decrease of the angular speed decreases. Owing to the small power loss in the armature circuit this method of speed regulation is economical.

As the voltage of supply circuit is kept constant, the indicated method of regulation is possible by using the appropriate transformer with a regulated DC voltage at its output, for example, in a system controlled rectifier, which is the motor.

In practice, other systems of change of voltage on the armature MDC are used, among which controlled thyristor rectifiers and pulsed regulators of voltage (pulse width converters) can be named. More detailed information on them can be found in the specialized literature.

15.3 Regulation of the angular speed of asynchronous motors

Electric drives with AM are widely used at various technological plants because AM are simple in construction, reliable in use, cheaper, much easier and smaller in size than MDC of the same power. In addition, some means of regulation of the angular speed does not require special transforming devices.

Most often three-phase AM are regulated by change of additional resistance in the rotor circuit (rheostat regulation); by change in the voltage brought to the stator; by joint change of frequency and voltage (frequency method); by switching the number of poles of the stator winding of the motor. In addition, some other ways can be used to regulate the angular speed, such as impulse regulation, regulation by means of an electromagnetic clutch of slip and others.

15.3.1 Rheostat regulation of the angular speed of AM. Mechanical characteristics of the motor under the regulation by the inclusion of the resistance in the rotor circuit (rheostatic regulation) are shown in figure 15.9.

From the figure it follows that the greater the resistance ($R_1 < R_2 < R_3 < R_4$) is connected to the rotor circuit, the less speed the motor develops at the same moment M_{dt}.

In the rotor circuit there are large currents, for which it is difficult to create resistors with continuously changing resistance. So structurally, the resistors providing rheostatic regulation, are performed *stepped* and therefore the speed of rotation is regulated stepwise. This method has the same advantages and disadvantages as for MDC.

The regulation range is variable and depends on the load. *The rigidity of characteristics is significantly reduced with decrease of frequency of rotation*, which limits the regulation range up to D = (2–3) : 1.

Significant disadvantage of this method is significant energy loss, which is proportional to the slip: $\Delta P_2 = P_1 \cdot s$. Such regulation is only possible for AM with phase rotor.

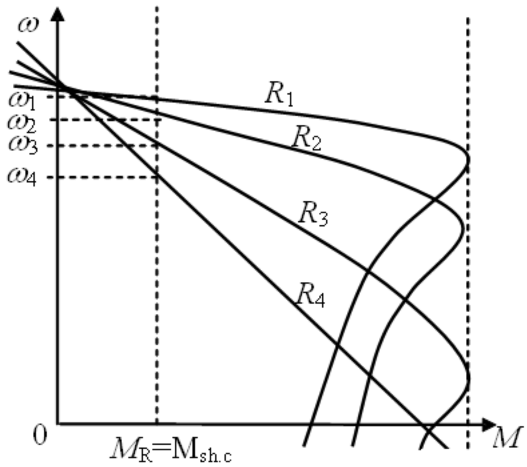


Figure 15.9: Mechanical rheostat characteristics of AM

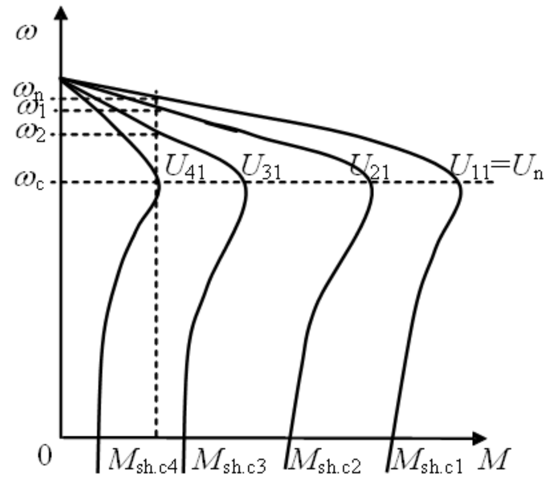


Figure 15.10: Mechanical characteristics of AM at change of the voltage on the stator

15.3.2 Regulation of AM by change of the voltage on the motor stator. Critical moment M_C is changed in direct proportion to the square of the motor brought to voltage U_1, and s_c does not depend on it. This determines the type of mechanical characteristics corresponding to different values U1 (fig. 15.10).

As a rule, regulation is carried out by reducing the voltage. Meanwhile as can be seen from figure 15.10 ($U_{11} < U_{21} < U_{31} < U_{41}$), the frequency of rotation (critical slip) remains constant, and the maximum moment decreases proportionally to the square of the voltage.

If $M_R > M_{sh.c}$, the motor will not move, it is need to run the engine at rated voltage or pre-shoot with its shaft load. The regulation range is small (up to ω_C).

To increase the regulation range in the rotor circuit the unregulated resistor whose resistance is enough to get the critical slip s_C = 3–4 is introduced. Such regulation (fig. 15.11) unlike rheostat allows smooth speed change, and deletes a contact circuit in a rotary apparatus.

To change the voltage at regulation autotransformers, semiconductor amplifiers, thyristor regulators of voltage are used.

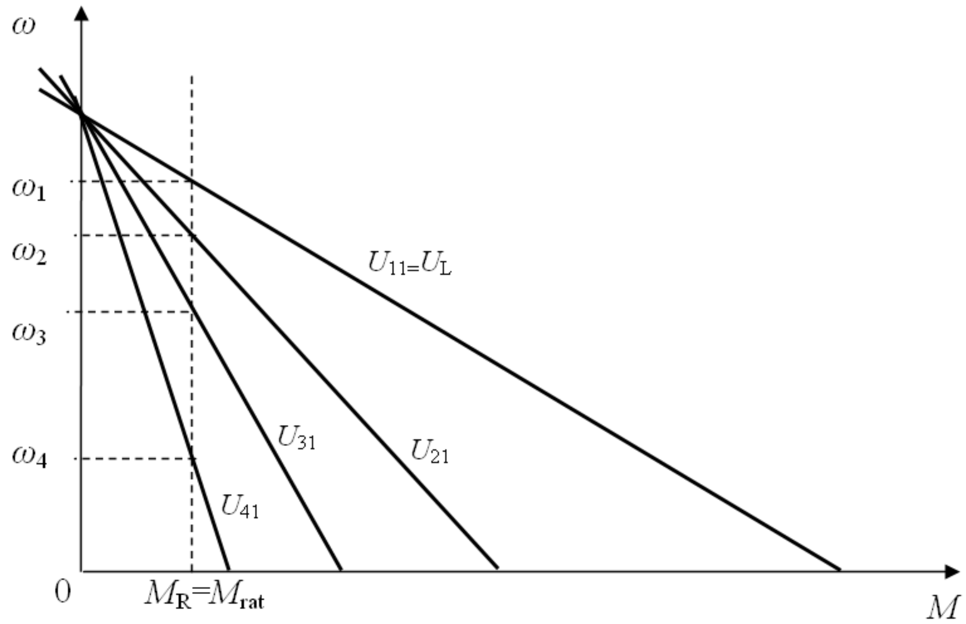


Figure 15.11: Mechanical characteristics of AM at change of the voltage on the stator and the inclusion of active resistance in the rotor circuit

15.3.3 Regulation of asynchronous motor by joint change of frequency and voltage (frequency regulation). The frequency method for smooth regulation of rotation frequency is of the greatest practical interest. For the best use of AM at frequency regulation it is necessary that with change of the frequency the voltage changes supplied to the stator windings. The law of the voltage variation depends on the frequency of the power and nature of the load.

So, if the static drag torque of load M_R does not depend on the rotation frequency, i.e. $M_R(\omega)$ = const, it is necessary at the regulation by change of frequency f_1 to change the voltage U_1

$$U_1 / f_1 = \text{const} . \qquad (15.14)$$

If the static drag torque is inversely proportional to the frequency of rotation, so that the load power $P_{st} = M_R \cdot \omega$ remains constant, then the ratio U_1 and f_1 must take the form:

$$U_1 / \sqrt{f_1} = const \ .$$

Figure 15.12 shows a set of mechanical characteristics at change of frequency of voltage in accordance with the expression (15.14). With decrease of frequency $f_{11} < f_{21} < f_{31}$ critical frequency of rotation is reduced, meanwhile in the area of the high and medium frequencies, the critical moment remains invariable, and in the region of small frequencies decreases a little.

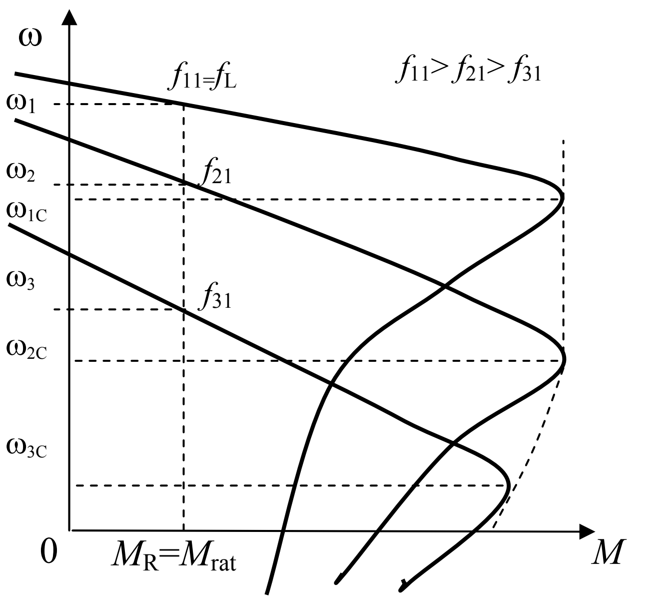


Figure 15.12: Mechanical characteristics of AM as the frequency of the voltage on the stator

Frequency method allows setting the frequency of rotation above and below the rated. It is admitted (mainly from strength conditions) that increase of frequency of rotation is 1.5–2 times more than rated, and the reduction ii 10–15 times less than the rated. The lower limit is limited by the fact that it is technically difficult to obtain a power source with low frequency, as well as to achieve a sufficiently uniform rotation of the motor shaft. Thus, the frequency regulation allows changing the frequency of rotation in the range up to D = (20–30) : 1. The lower limit of frequency of rotation can be reduced by using the feedback on the frequency of rotation, current and voltage.

Frequency control is one the most effective AM methods from the point of view of its technical and economic indicators. The working part of the mechanical characteristics has a high rigidity at any frequency of power f_1. *The power loss is low* because the motor works at low slips; smoothness of regulation can be almost any. Control can be accomplished using the simplest and most reliable motor with short-circuit rotor.

15.3.4 Speed regulation of asynchronous motor by switching the number of pairs of poles. From the equation

$$\omega_0 = 2\pi f_1/p \qquad (15.15)$$

it follows that at the number of pairs of poles p mechanical characteristics with different speed of rotation of ideal idle ω_o are obtained. As the p value is determined by the integers, then the transition from one characteristic to another in the regulation process has stepped character. There are *two methods of speed regulation by change of the number of pairs of poles.*

The first method. In the grooves of the stator two windings with different number of poles are placed. Depending on the required speed of rotation one or the other winding is connected to the power source.

The second method. The winding of each phase consists of two parts, which are serial-connected or parallel-connected in the process of regulation. The number of pairs of poles is changed 2 times.

The industry produces a special multispeed motors, the design of which allows changing the number of pairs of poles.

Figure 15.13 and 15.14 show the schemes of switching from single stars in double star and from triangle to the double star. Switching of a single star to the double one (fig. 15.13) is used when the *load moment* of the working mechanism *does not depend on the frequency of rotation* (fig. 15.15, *a*), i.e. $M_R(\omega)$ = const.

Switching from triangle to double star (see fig. 15.14) is used when the load power of working mechanism *does not depend on the frequency of rotation* (fig. 15.15, *b*), $P_{st}(\omega)$ = const, and the drag torque varies inversely proportionally to the frequency of rotation.

The trajectory of transition of the working point of the motor at switching of the stator winding is shown in figure 15.15, *a* by the dotted and solid lines with arrows. The transition from the highest frequency of rotation to the lowest is accompanied by work of the motor in generator mode with the energy output to the circuit.

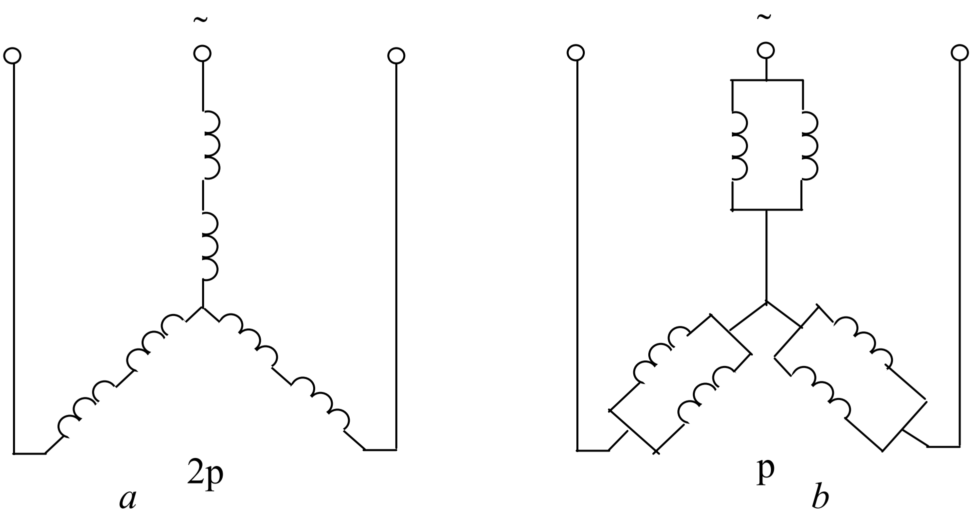


Figure 15.13: Scheme of switching of windings with a single star to a double

The main disadvantage of regulation by change of the number of pairs of poles is a *stepped character* of the rotation frequency change. At the same time, regulation is economical, it has a high stability of frequency of rotation and is mainly used for asynchronous short-circuit motor. In flexible automated production it is used for a number of metal-cutting lathes allowing reducing the number of mechanical transmission in gearboxes.

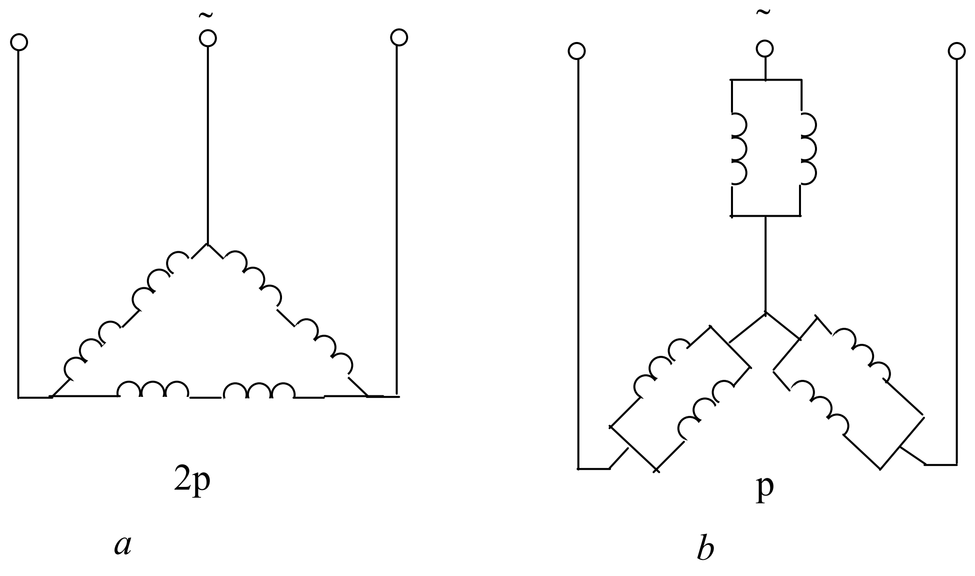


Figure 15.14: Scheme of switching of stator windings with triangle to double star

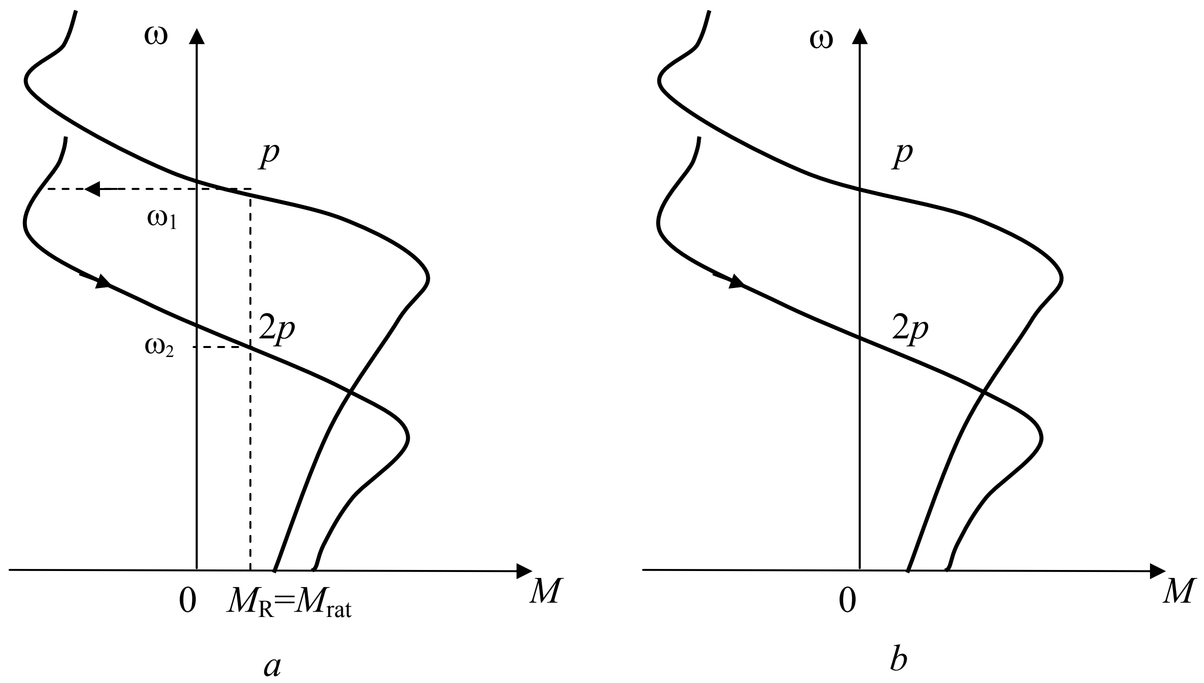


Figure 15.15: Mechanical characteristics of AM at switching of the stator winding with a single star to a double (*a*) and with the triangle to the double star (*b*)

Key findings

1. Speed regulation is a forced change of speed of the electric drive depending on the process requirements. The speed regulation is carried out by the controlling effect on driving motor.

2. The main indicators characterizing the speed regulation of electric drives are: regulation range, smoothness, efficiency, stability of speed, direction of speed regulation, permissible load.

3. The speed of a DC motor with independent actuation can be regulated by change of the actuation current, by the change in resistance of the armature circuit, by the change brought to the armature of the motor of voltage.

4. Speed of AM is regulated by change of additional resistance in the rotor circuit, by the change in the voltage brought to the stator, by joint change of frequency and supply voltage, switching of the number of poles of the stator winding of the motor. Most common three-phase AM are regulated by the change of additional resistance in the rotor circuit (rheostat regulation); by change in the voltage brought to the stator; by joint change of frequency and voltage (frequency method); by switching of the number of poles of the stator winding of the motor.

Control questions

1. What is the speed regulation of electric drive?
2. Describe the main indicators of the regulation of the angular speed of the electric drives.
3. What methods are used to regulate the angular speed of the DC motor with independent actuation?
4. In what direction is regulation of the speed of DC motor with independent actuation carried out by change of the actuation current? Why?
5. In what range is the speed of DC motor with independent actuation regulated at change of the actuation current? What is it limited by?
6. What direction is regulation of the speed of DC motor with independent actuation carried out by rheostatic regulation? Why?
7. What circuit is the additional resistance included in at the speed regulation by flux of actuation and at rheostatic regulation?
8. How is porosity of control pulses ε determined? What does it influence on?
9. How are regulation characteristics of DC motor with independent actuation at the voltage changes on armature placed?
10. What is the range of speed regulation of DC motor with independent actuation limited at the voltage changes on armature by?
11. What methods are used to regulate the angular speed of the asynchronous motor?
12. What is the range of speed regulation of asynchronous motor limited at the rheostatic regulation by?
13. What is the smoothness of speed regulation of asynchronous motor limited at the rheostatic regulation by?
14. Describe the features of speed regulation of asynchronous motor by change of the voltage on the stator.
15. Why is a constant resistor injected in the rotor circuit at the speed regulation of asynchronous motor by change of the voltage on the stator?

SECTION VI.
ELECTRICAL EQUIPMENT OF BUILDING GROUND, ENTERPRISES AND BUILDINGS

16 ELECTRICAL EQUIPMENT OF WELDING SETS

Key concepts: welding, electric arc welding, machine welding, welding generator, welding set, contact welding.

16.1 Types of electrical welding

Welding is the process of receiving of fixed connection of materials by local heating of the welded edges of the parts up to the plastic or molten state.

Electric welding is widely used in construction, because the welded seams are of high quality and have big strength. The strength of the welded connection is provided by atomic or molecular relations. Mutual diffusion of atoms of welded materials is also of great importance.

In electric welding an electric arc phenomenon is used, which is an electrical discharge, accompanied by a high temperature and a large current density, which can reach several thousand amperes per 1 cm^2. The voltage drop on the arc is small (10–20 V).

Modern welding technique offers a large variety of welding methods. Currently, ***two methods of electric welding: arc and contact*** are widely used.

Electric arc welding, at which melting of the metal of the welded edges of the parts and the electrode (or filler metal) is produced by heat generated by an electric arc, is performed manually, semi-automatically and automatically.

Manual arc welding can be done in two ways: by Benardos' way and Slavyanov's way.

Welding by the method of Russian inventor N. N. Benardos (fig. 16.1) is as follows. The positive pole of the DC source is connected to the welded parts, and negative is connected to the nonconsumable electrode E. Between the electrode E (coal, graphite or tungsten) and the product an electric arc is excited. Edge products are introduced in zone of arc filler M are heated up to melting and a basin of molten metal is received. Welded seam appears after solidification of basin. This method is used, as a rule, at welding nonferrous metals or their alloys, as well as at surfacing of hard alloys.

Welding according to N. H. Slavyanov's method is done (fig. 16.2) using a consumable electrode. The arc is ignited between the metal (consumable) electrode and the welded edges of the product. Obtained total molten metal bath, when cooled, forms a weld.

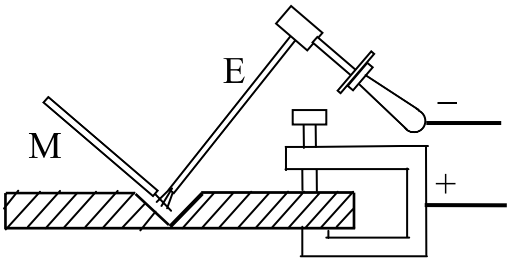


Figure 16.1: Electric welding by Benardos' method

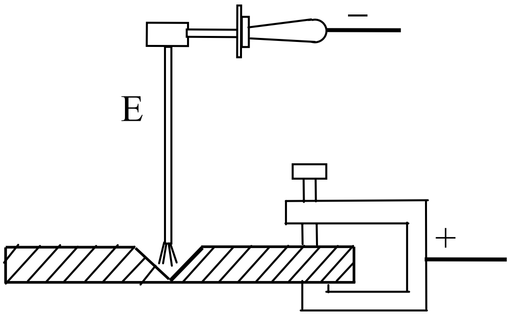


Figure 16.2: Electric welding by Slavyanov's method

By Slavyanov's method, which is widely used, it is possible to use AC current on the condition of coating of rod electrode with special coating. In the coating of the electrodes and in the composition of the fluxes, which cover the weld area, there is a large number of elements with low ionization temperature increasing the stability of the electric arc.

Automatic and semi-automatic welding under flux is carried out by mechanization of basic movements performed by the welder, feeding of the electrode along its axis in the arc zone and moving it along a welded seam.

At the semi-automatic welding feed of the electrode along its axis in the arc zone is mechanized, and movement of the electrode along a welded seam is produced manually by the welder. The automatic welding mechanizes all operations necessary for welding process.

Molten metal is protected from exposure of oxygen and nitrogen of the air by special granular flux. High performance and good quality of seams ensure a wide use of automatic and semi-automatic welding under flux.

Electric contact welding is performed by means of heat generated by current passing through the welded edges of the product. In the place of contact of the edges the greatest amount of heat is allocated, warming them up to welding state. Welding is completed by subsequent squeeze of the welded edges.

16.2 The basic requirements for power supplies of welding arc

Electric arc welding begins with a short circuit of welding circuit, which is the contact between the electrode and the part. Meanwhile the heat releases and the rapid warming of the contact area occurs. This initial phase requires higher voltage of welding current.

In the welding process during the transition of drops of electrode metal in the welding basin very frequent short circuit of the welding circuit happens. Along with this the length of welding arc is changed. At each short circuit voltage drops to zero. For subsequent recovery of the arc the voltage of order 25...30 V is needed. This voltage must be provided for time not more than 0.05 s to maintain the arc between short circuits.

Note that at short circuit of the welding circuit large currents develop short-circuit currents, which can cause overheating of the wiring and windings of the current source. These conditions of the welding process are mainly determined by requirements for power supplies of welding arc. To ensure stable welding process power supplies of the arc must satisfy the following requirements:

1. Idle voltage should be sufficient for easy actuation of arc and at the same time should not exceed the standards of safety. For single-operator welding generator idle voltage should not be more than 80 V, and for multiple-operator it should not be more than 60 V. For welding transformers maximum voltage of 70 V is installed at welding force of current more than 200 A and voltage of 100 V at welding force of current is less than 100 A.

2. Voltage of arcing (working voltage) should be set and vary depending on the length of the arc, providing stable combustion of the welding arc. With increase of length of the arc voltage quickly increases and with decrease it rapidly falls. The recovery time of working voltage from 0 to 30 V after each short circuit (at drop transfer of metal from the electrode to the welding part) must be less than 0.05 second.

3. Current value of short circuit should not exceed welding value of the current strength of more than 40...50%. In this case, the power supply must be capable of short circuit duration of the welding circuit. This condition is necessary to protect the windings of the current source from overheating and damage.

4. The power of the current source must be sufficient to perform welding works.

In addition, devices allowing regulating the value of the welding force of current within the required limits are needed. Welding equipment must satisfy the requirements of state standards.

16.3 Welding transformers of DC

Welding transformers of DC are divided into the following groups:

By the number of fed posts – single-operator, designed to power one of the welding arc; multioperator feeding simultaneously several welding arcs.

By the way of installation – stationary, fixed to the foundations; mobile, mounted on trucks.

By the type of the motors, causing the generator to rotate, – machines with electric drive; machines with internal combustion motor (gasoline or diesel).

By the way of implementation – single-hull, in which the generator and the motor are mounted in a single jar; separate, in which the generator and motor are mounted on a single frame, and the drive is via a special coupling box.

The most widely used in the construction are single-operator generators with split poles, working on the principle of using the magnetic flux of the armature to receive incident external characteristic.

Figure 16.3 shows a scheme of a welding generator of this type. The generator has four main and two additional poles. Meanwhile the same basic poles are placed closely, forming a forked pole.

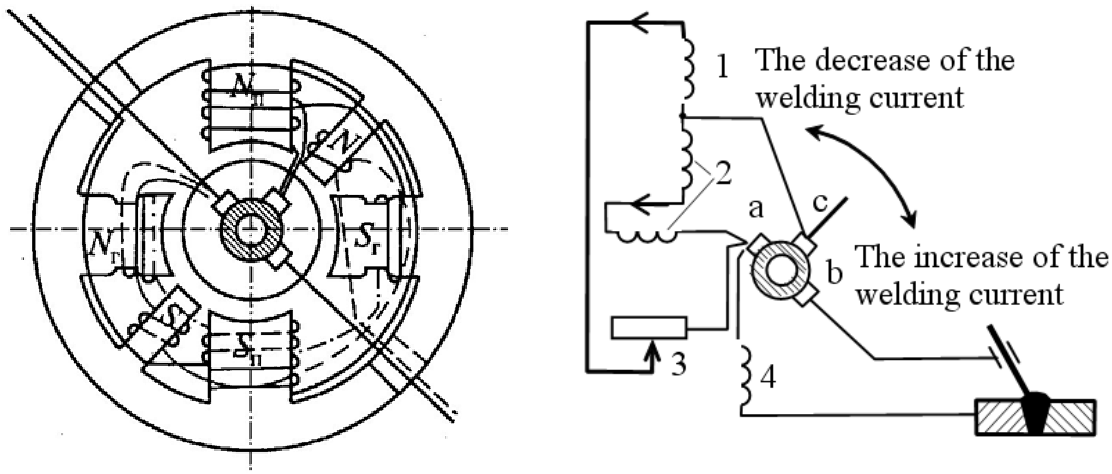


Figure 16.3: Scheme of generator with split windings: 1, 2 – respectively regulated and unregulated winding of actuation; 3 – rheostat; 4 – series winding; a, b, c – brushes

Actuation windings have two sections: unregulated 2 and regulated 1. Unregulated winding is located on all four main poles, and regulated one is placed only on the crosscut poles of the generator. In the circuit of regulated winding of actuation rheostat 3 is included. On additional poles series winding 4 is placed. On the neutral line of symmetry between opposite poles on the collector of the generator there are the main brushes a and b, to which welding circuit is connected. Additional brush *c* is used for the power of windings of actuation. The coarse regulation is made by shifting of the brush cross bar, on which all three brushes of generator are placed. By sliding the brush in the direction of rotation of the armature, the demagnetizing effect of the armature flux increases and the value of the welding current is reduced. At the reverse shift demagnetizing effect is reduced and the welding force of current increases. More smooth and precise regulation of the current is produced by a rheostat included in the circuit of the actuation winding. Increasing or decreasing the rheostat force excitation current in the winding cross-pole magnetic flux Φ_m change, thereby changing the voltage of the generator and the magnitude of the welding current.

In addition to generators with demagnetizing effect of armature reaction welding generators are used, whose drooping characteristics and limitations of the force short-circuit current are provided by the demagnetizing effect of coherent excitation winding included in the welding circuit. Principle scheme of this generator is presented in figure 16.4. The generator has two windings: winding of actuation and demagnetizing winding 2. The actuation winding is fed either from the primary and secondary brushes, or from a special one: power source with constant voltage. Therefore, the magnetic flux Φ_a generated by this winding, is constant and does not depend on the load generator. Demagnetizing winding is connected serially with the armature winding so that at the arcing welding current passing through the winding generates magnetic flux Φ_s, directed against the flux Φ_a.

Recently rectifier welding installations have got wide application in the welding industry. They convert AC to DC using selenium, germanium or silicon rectifiers.

Rectifier installations have higher COP. In addition, such important advantages as no rotating parts, low weight, small dimensions and low cost should be mentioned. An important advantage is their high dynamic properties due to lower electromagnetic inertia. Force of current and voltage change almost instantly when changing the welding circuit. Usage of three-phase bridge system of rectification provides less pulsation of the rectified current and a more uniform load of phases of the power circuit of AC.

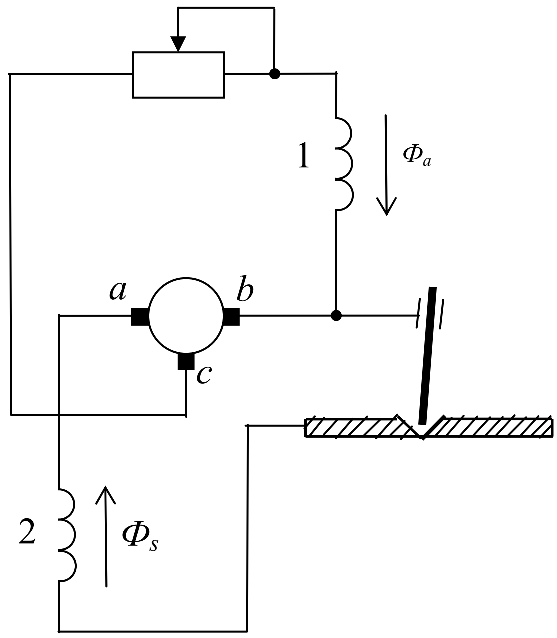


Figure 16.4: Welding generator with demagnetizing action of reaction of armature: 1 – actuation winding; 2 – demagnetizing winding; a, b, c – brushes

16.4 AC welding sets

AC welding sets are used in factories and in construction. They are divided into four main groups:

1. With a separate choke.
2. With built-in choke.
3. With movable magnetic shunt.
4. With increased magnetic scattering and the movable winding.

These groups differ in construction and in the electric scheme. Welding sets consist of a step-down transformer and a special device. The transformer provides power of arc of voltage 60...70 V by AC, and a special device is used to create the incident external characteristics and regulation of the magnitude of the welding current.

Welding sets with separate choke (fig. 16.5) consist of a step-down transformer and choke. The transformer T has a core (magnetic core) *2* of stamped plates, made of thin electrical steel of 0.5 mm thickness. At the core there are the primary 1 and secondary windings 3. The primary winding from insulated wire is connected to the AC voltage of 220 or 380 V. In the secondary winding made from copper bus bar, current of voltage 60...70 V is induced.

Small magnetic dispersion and the low ohmic resistance of the windings provide a small internal voltage drop and high COP of transformer.

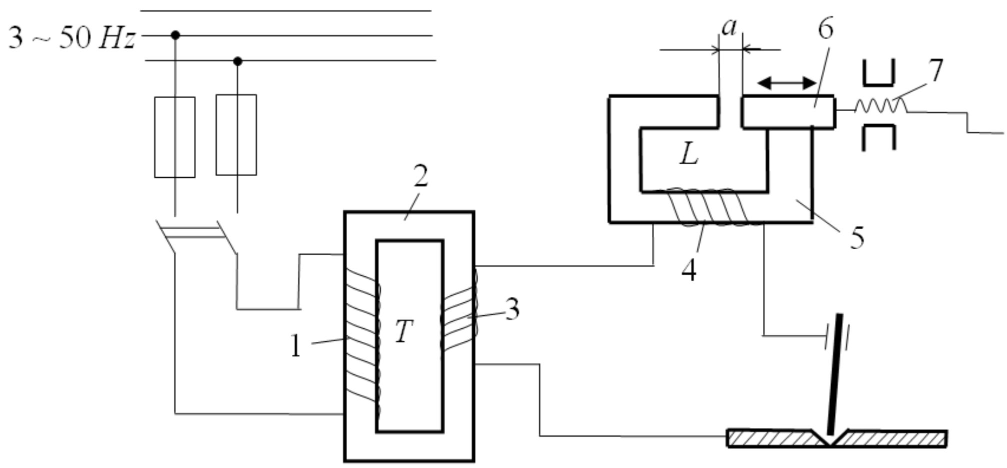


Figure 16.5: Circuit of the welding transformer with separate choke: 1, 3 – primary and secondary windings respectively; 2 – core; L – winding of choke; 4, 5 – stationary and movable part of the core respectively; 6 – screw device; *a* – gap

Consistently to the secondary winding of the welding circuit the winding 4 of the choke $Д_c$ (current regulator) is included. The core (magnetic circuit) of a choke is composed of plates of thin transformer steel and consists of two parts: a fixed 5 on which the winding of the choke is placed, and mobile 6 moved by means of screw devices 7.

The choke is used to regulate the welding force of current and the creation of incident external characteristics of the transformer on the arc. At actuation of the arc (at short circuit) high current passing through the winding of the choke, creates a powerful magnetic flux, bringing EMF of the choke directed against voltage transformer. Secondary voltage developed by the transformer, is fully absorbed by the voltage drop in the choke. The voltage in the welding circuit almost reaches zero.

At the origin of the arc welding force of current is reduced; the EMF of self-induction choke subsequently decreases, directed against the voltage of the transformer and in the welding circuit to the working voltage is set required for stable arcing, less than the idle voltage. Changing the size of the gap between the fixed and movable magnetic core, the inductive resistance of the inductor is changed and thereby the force of current in the welding circuit. At increase of gap magnetic resistance of the magnetic circuit of the choke increases, the magnetic flux weakens, EMF of self-induction of coil is reduced and its inductive resistance is reduced as well. This leads to an increase of welding force of current. Welding force of current decreases with decrease of gap.

Welding sets with separate choke are widely used in the construction and installation sites, factories and welding of the main pipelines.

Welding sets with built-in choke (fig. 16.6). The magnetic core of the transformer consists of a main core 7, on which there are the primary and secondary windings 6 of the transformer, and the additional core 4 with the winding of the choke 5 (regulator of current force). Additional magnetic core is located above the main one and consists of the fixed and movable parts between which using screw mechanism 3 the required air gap is set *a*.

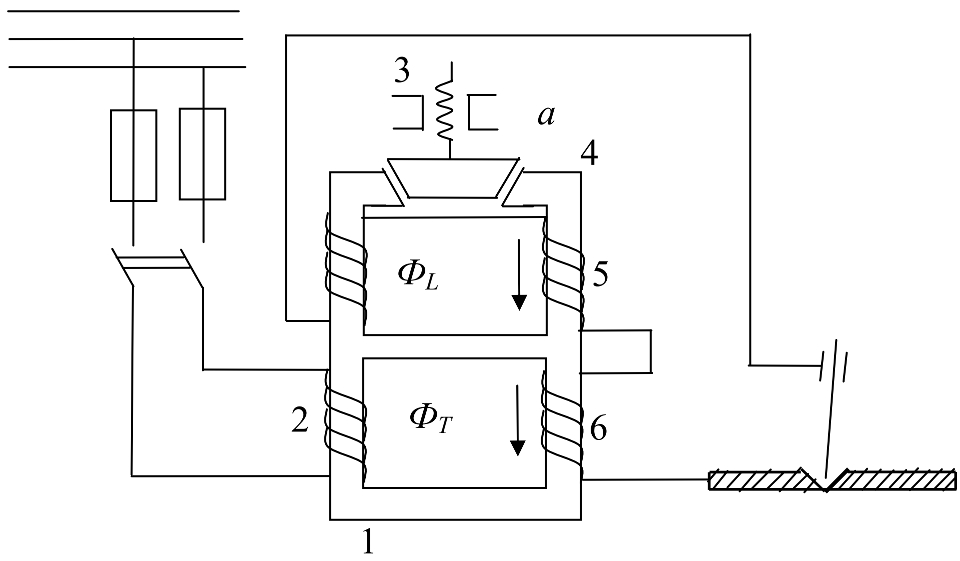


Figure 16.6: Scheme of the welding transformer with built-in choke: 1 – core; 2, 6 – windings of transformer accordingly the primary and secondary; 3 – screw mechanism; 4 – additional core; 5 – winding of choke; *a* – gap

Regulation of welding force of current is made by changing of the air gap *a*: the bigger is the gap *a*, the more is the welding force of current.

Welding sets with a movable magnetic shunt (fig. 16.7) have a closed magnetic core, at which on one rod the primary 4 and secondary winding 3 are placed, and on the other one there is reactive winding 1. Between them there is the core – magnetic shunt 2. The shunt closes the magnetic fluxes generated by the primary and reactive windings. At it magnetic fluxes of scattering are formed, which create significant inductive reactance. Thus the incident external characteristics of the transformer are ensured.

Welding sets with increased magnetic scattering and the movable winding without choke. The transformer has a magnetic core in the form of rods, which are two coils, one with the primary winding and the other with the secondary one. Coils of windings are parallel-connected. The primary coil is fastened fixedly. Coil of secondary winding is moved by a screw mechanism manually. Regulation of welding force of current is carried out by change of the distance between the coils of the primary and secondary windings of the transformer. The smaller is the distance between the coils of the windings, the greater is the welding force of the current.

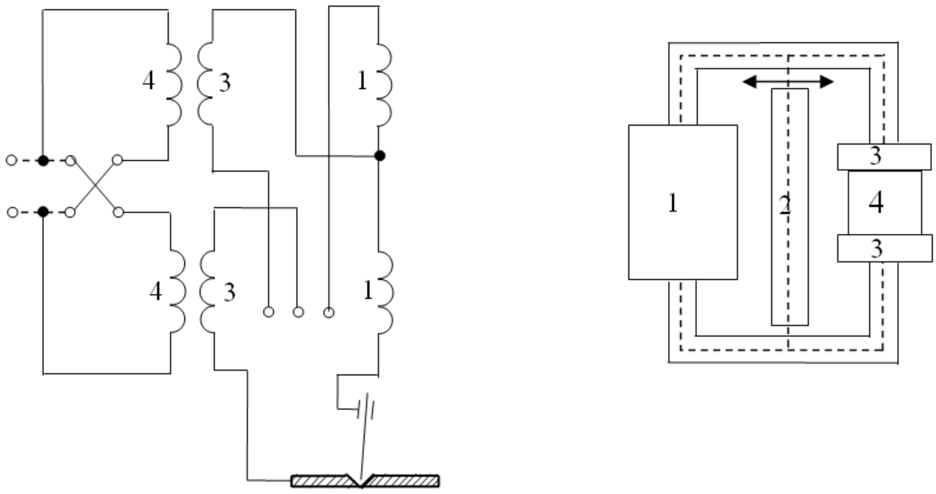


Figure 16.7: Scheme of a welding set with a movable magnetic shunt:
1 – reactive winding; 2 – magnetic shunt; 3, 4 – the secondary and primary windings respectively

Three-phase welding transformers are used in the welding of three-phase arc by paired electrodes. The use of three-phase welding sets has great economic importance, as they provide high performance, power economy (COP reaches 0.9) and uniform loading of the phases of the network with high power coefficient ($\cos\varphi \leq 0,8$). However, the welding of three-phase current has been limited in use due to the complexity of welding equipment and unfitness for welding in the overhead and vertical positions.

16.5 Installations of contact welding

Contact welding, or welding under pressure, is a method of welding parts, in which the concentrated release of heat in the area of the junction is caused by significant excess of active resistance in this point over the resistance of the parts. The amount of released heat at the point of junction is determined by a well-known formula

$$Q = 0,24\, I^2 \cdot R_t \cdot t, \tag{16.1}$$

where: $\boldsymbol{R_t}$ is transitional resistance at the point of junction.

The devices of butt, point, and roll welding refer to the installations of contact welding. Their schemes are shown in figure 16.8. They are widely used in construction, in particular for welding of reinforcement and metal constructions.

The greatest power of the machines of industrial manufacture, intended for contact welding, reaches 750 kVA. These machines allow welding of the work piece section up to 3500 mm^2 and details of thickness up to 32 mm.

For contact welding both direct and alternating current can be used. However, in practice alternating current is mainly used, because the current in thousands of amperes and a voltage of a few volts is required for welding, just without significant capital costs can be obtained on alternating current with the help of transformers.

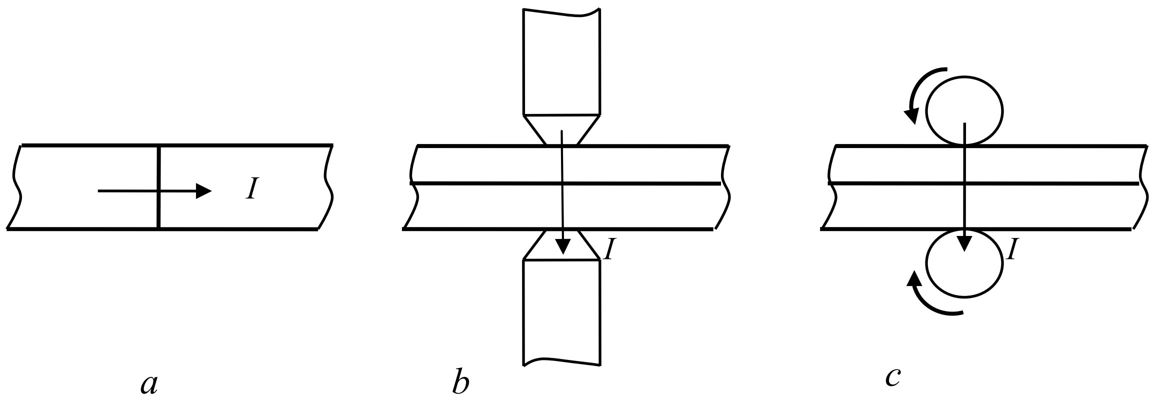


Figure 16.8: Basic methods of contact welding

The feed of welding current can be either continuous (in some cases at the roll welding), or discontinuous.

A significant duration of current is supported by time relay, while turning on and off of the welding current is performed by thyristors.

Key findings

1. Welding sets of AC and DC are used in the construction.
2. Advantages of welding rectifiers on DC are higher over COP, no rotating parts, low weight and dimensions, low cost.
3. Three-phase welding machines provide power economy, uniform loading of the phases of the circuit, high power coefficient cosφ.

Control questions

1. What is welding?
2. What are automatic and semiautomatic welding characterized by?
3. Specify the basic requirements for power supplies of welding arc.
4. Explain the structure of the welding generator of DC.
5. Explain the principle of work of the welding transformer with separate choke.
6. How does welding transformer with built-in choke work?
7. What are the installations of contact welding characterized by?

17 ELECTRICAL EQUIPMENT
OF LOAD-LIFTING MACHINES

Key concepts: load-lifting machine, crane electric motors, control equipment, controller, bracket, contactor, magnetic actuator, time relay, intermediate relay, relay of minimum current, overcurrent relay, thermal relay

17.1 General information about load-lifting machines

Load-lifting machines, are commonly used in construction and on enterprises of the construction industry jib, tower, gantry, bridge and other cranes and lifts for various purposes. Electrical equipment of these machines has similar design features and purposes.

The work of electrical equipment of load-lifting machines is characterized by the following features:

- mode of work repeatedly – short-term;
- frequent changes of direction of rotation (reverse);
- the need of regulation of frequency of rotation of the drive;
- significant overloads, vibration;
- difficult access for maintenance and repair;
- work in conditions of pollution, humidity, significant temperature difference.

To support listed work conditions electrical equipment of load-lifting machines shall meet the requirements of high strength, high insulation and reliable protection from the actions of the environment. Machines and equipment of special cranes execution meet these requirements. ***Electrical equipment of load-lifting machines*** *is divided into the* ***basic equipment*** (equipment of the electric drive) and ***auxiliary equipment*** (equipment of working and repair lighting and heating).

To the main electrical equipment includes*:*

- electric motors;
- devices of control of electric motors, memory controllers, contactors, magnetic actuators, control relays;
- apparatuses of regulation of frequency of electric motors rotation - starting and control rheostats, brake machines;
- apparatuses of brake control – brake electromagnets and electro-hydraulic tappets;
- apparatuses of electrical protection – protective panels, automatic switches, and maximum thermal relays, fuses, distribution boxes and equipment, ensuring maximum and zero protection for electric motors;
- devices of mechanical protection – finite switches and limiters of lading capacity ensured the protection of the crane and its mechanisms of transition of extreme positions and overload;
- semiconductor rectifiers supplying power of windings of actuation of the brake machines, the windings of the magnetic amplifiers, power circuits and control circuits of some types of cranes;

- generators AC and DC motors are used on some types of tower cranes as power supplies for all the electrical equipment or electrical equipment of drives of separate mechanisms;
- apparatuses and devices which are used for various switching and control of power circuits and control circuits such as buttons, knife switches, switches, changers, and measuring devices.

Auxiliary equipment includes:

- lighting equipment (lamps, projectors);
- appliances of electric heating (electric furnaces, heaters);
- sound signaling equipment (bells, sirens);
- the apparatuses of control and protection (transformers, switches, fuses, and so on), installed in circuits of lighting and heating.

17.2 Electric motors of load-lifting machines

On load-lifting machines electric machines, both direct and alternating current are used. The device and principle of work of electrical machines of DC were discussed in chapter 10.1, AC machines – in chapter 11.2. In this section we consider the characteristics of electric motors of load-lifting machines.

17.2.1 Crane electric motors. Electric motors of special crane type are intended to work both indoors and outdoors. Therefore, they are made closed with self-ventilation (asynchronous motors) or with independent ventilation (DC motors) and with moisture-resistant insulation. Because the motors are designed for heavy working conditions, they produce increased strength. All crane electric motors are characterized by high overload capacity, large starting moments at relatively low starting currents and short duration of the acceleration. The ratio of starting moment to rated ranges in limits 2.3 ÷ 3.2.

Crane electric motors with contact rings of single series MTF, MTH, 4MTH and with short-circuit rings, 4MTKF are intended for drive of mechanisms, the work of which is characterized by short-term and repeatedly short-term modes. Series of electric motors of the 4th development in comparison with MTF and MTKF are calculated for high temperature and differ from them in smaller dimensions and weight.

Crane asynchronous electric motors have the designation, consisting of letters and numbers: MT – with phase rotor, MTK – with short-circuit rotor. The class of heat resistance of the motor is designated by the letter. Motors with index B (MTB and MTKB) have high-temperature insulation of class B with an allowable temperature of heat resistance 130°C. Motors with index P (MTP and MTKP) have high-temperature insulation of class P with temperature of heat resistance 155°C. The motors MT and MTK comply with insulation of class F with a permissible temperature of heat resistance 120°C.

The first digit of a three-digit number (0...7) after the letter designation characterizes conditional outside diameter of the stator pack, the second number - is the sequence number of series, the third number - is the relative length of the stator core. The numeral after the hyphen indicates the number of poles of the machine. For example, the designation MTKF 412-8 means crane sort circuit motor of the fourth dimension, the first series, the second length, eight-terminal circuit.

For example, MTP-411-8 – crane electric motor with phase rotor, 4-th value, 1-t length eight-terminal circuit, with insulation of class P.

To the jar of the electric motor the table is attached with the main parameters characterizing the motor and the name of the factory-manufacturer. In the table the power of the motor in kW at rated load is indicated, the power coefficient cosφ, frequency of turns, voltage, at which motor is calculated in case of connection of its windings "star" or "triangle", force of rotor current at rated voltage.

On jib cranes motors with phase rotor are mainly used, because they include rotor circuit, it is possible to regulate the magnitude of the starting current and starting moment.

Starting moment at a some starting resistance can be maximum. The maximum moment corresponds to a critical slip and is determined by the rated moment and the coefficient of ratio of the maximum moment. The starting and maximum moments influences the ability of the electric motor to overcome the inertial efforts of load and gears of actuating devices.

In the case of the steady mode of work the moment delivered to the motor shaft must always be greater than the moment of lifted load. If the moment transmitted from the load on the motor shaft is greater than its maximum moment, then the motor stops, because it cannot overcome the static moment. The moment developed at this engine will be critical. If in proper time the motor is not turned off, which is under a heavy load, it overheats and can be burned down. In this regard, you should not overload the motor.

Overload capacity of crane motors with phase rotor at ПВ=25% consists 2.5–3.4.

Crane motors work in repeatedly- short-term mode: periods of short-term work alternate with long periods of switched off state in which the engine is cooled. In this mode the motor is heated less than with long-term continuous work, so it is possible to load it more.

Permissible load of the motor depends on the RID and is defined by the standard: 15, 25, 40, 60 and 100% of load specified for continuous mode of work.

Electric motors of load-lifting machines are placed under untight bonnets. Therefore, they are exposed to dust, humidity, high and low temperatures. In connection with it, on load-lifting machines motors in a secure execution are used.

17.2.2 Start of asynchronous electric motors of cargo lifting machines. Start of electric motors with short-circuit rotor are carried out with magnetic actuators. This way is possible at condition, that the power of engine is not higher than 20% of power source of external electrical circuit. Start of powerful short-circuit motors is carried out switching the stator winding with the "stars" on the "triangle" at a voltage of 220 V.

Electric motors with phase rotor are included with help of controllers and starting rheostats included in the rotor circuit of the motor. During startup resistance of the rheostat is gradually reduced, increasing the starting moment and the frequency of rotation of motor.

The frequency of asynchronous motors with phase rotor is regulated by changing the resistance of the rotor, for what turns on and off starting rheostats. Inclusion in the rotor circuit is reduced the frequency of rotation of the rotor, and turning off of rheostat – increases it. Bridging (removal of the circuit) of part of the rheostat is performed by the controller. To regulate the frequency of rotation in this way is possible only in the case of overcoming by motor high load drag torque (lifting of heavy load, the rotation with load on the big departure). At idle with a slight load, the frequency of rotation of motor almost does not depend on the resistance in the rotor circuit and close to synchronous. The method of regulation of frequency of rotation by change of the resistance in the rotor circuit is the simplest, but also the most uneconomic, due to large loss in starting rheostat.

Reverse of asynchronous motors is achieved by change of the direction of rotation of the magnetic field. For a one-time reversal on the flap of the leads of the stator any two phases are switched. In the case of necessity of periodically change of the direction of rotation of the rotor reversing magnetic starters, controllers or switching cutouts.

17.2.3 DC electric motors. Crane electric motors of DC of types ДК, П and 2П are manufactured for a rated voltage of 220 and 440 V. In the designation of the motor (for example, ДК-309Б) the letters indicate the series, and numbers – conditional sizes. The first number after the name of the series (1...8) – the value of the motor, which characterizes the external diameter of steel armature package; the second number – is the length of the packet for a given value; the third – is the length of the stator core; letter after numbers – insulation class.

Other parameters and design features (voltage, power, frequency of rotation, cooling method) are characterized by catalog number.

17.2.4 Generators of AC and DC. As already noted in theme 4 generators convert the mechanical energy of rotation into electrical. Construction cranes use generators of AC (synchronous) and DC by power from 50 to 100 kW.

The generator is a part of the power installation of crane, receiving rotation from the diesel engine through the clutch. Electric cranes of DC in case of power from an external circuit of AC generator is driven by an electric motor included in an external electric circuit.

17.3 The elements of the control equipment

Controllers. Controllers are used to control the work of the motor, i.e. its switching on, regulation of frequency of rotation, stop and change direction (speed reversal). The controllers used to control the electric motors of the crane mechanisms, on the principle of work and it is divided into two types:

direct control, or power, locking or opening power circuits of the motor by means of the contact devices of controller with hand drive;

remote control, or magnetic driven by brackets, switching control circuits.

Power controllers used on tower cranes serve the cam controllers AC KKT (fig. 17.1).

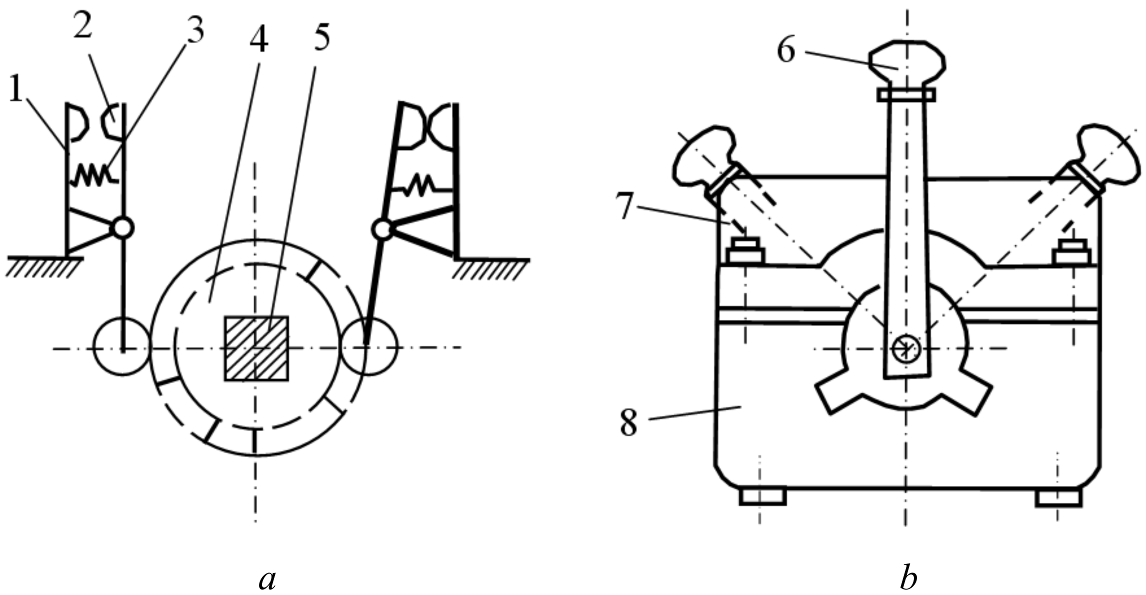


Figure 17.1: Cam controller KKT-61: *a* – contact system; *b* – general view; 1 – base of contact element; 2 – live lever with roller and a movable contact; 3 – driving spring; 4 – cam washer; 5 – shaft; 6 – arm; 7 – cover; 8 – jar

The main components of a cam controller are contact elements and the shaft 5 with cam washers 4. Each contact element consist of a base 1, a live lever 2 with the roller and a movable contact, and a driving spring 3, which provides a closing of movable and fixed contacts. The contact elements are anchored to the jar 8 of the controller. The shaft with cam washers (cam drum) rotates in bearings fixed in the jar of the controller. The rotation of the cam drum is carried out by means of the handle 6 mounted on the corbelled end of the shaft.

Controllers are produced of two types: controllers to control one or two motors.

Magnetic controllers represent the panel in the open or protected execution, which includes contactors, relays of control, fuses, and other control devices and electrical protection.

To control the coils of contactors and relay of magnetic controller usually a ***bracket*** is served. Works of the bracket is similar to the work of a cam controller KKT, but the number of switchable circuits has less, and the contacts silver, bridge type.

Magnetic controllers have a number of advantages in comparison to the power:

- magnetic controller of any power is controlled by means of a small device – bracket without using considerable muscular effort of the driver;
- magnetic controllers can be installed outside the cabin, anywhere on the crane;
- contactors of magnetic controllers more wear-resistant than the contacts of cam controllers.

The use of magnetic controllers allows to automate the operations of startup and braking of the motor, which simplifies the control of the actuator and protects the motor from overloads.

However, magnetic controllers have a much more complex scheme, and a greater number of devices than the power, and therefore require more careful maintenance.

Contactors and magnetic actuators. ***The contactor is called an electrical apparatus intended to turn on and off power collectors.***

The principle of work of the contactor is in the following. At feed of voltage to the coil of an electromagnet under its action are closed power contacts of the contactor and is carried out inclusion of collector. By removing the voltage from the coil of the electromagnet, under its action open the power contacts of the contactor and the collector switch off.

Depending on the kind of current ***there are contactors of AC and DC***. The number of simultaneously switched circuits contactors are divided into unipolar and multipolar. DC contactors are produced in one - and two-pole and contactors of AC - two-, three - and four-pole.

Main contacts make massive, designed for high force of current, but the auxiliary contacts - are small, as in the control circuit force of current does not exceed 5–10 A.

After the break of electric circuits under load, between power contacts of the contactor, an electric arc appears, which causes accelerated wear of the contacts and even their destruction. To reduce the burning time of the arc different systems of forced arc control are used.

Contactors are used in magnetic controllers of tower cranes as linear contactors in circuit of protection and reversers.

Magnetic actuator is called small contactor of special version intended for start, stop and speed reversal of asynchronous short-circuit motors, and also for commutation (closing and break) of other electrical circuits. Magnetic starter may have built-in thermal relay for protection an electrical circuit from overloads.

On tower cranes starters are used for control of short-circuit motors, in magnetic controllers and for commutation of other power circuits.

Relay of control and protection. For control and protection of electric motors time relay, intermediate relay, relay of minimum current, relay of maximal current, thermal relay are used.

Time relay is used in a magnetic controllers of cranes for automatic closing and break of circuits of control with a given delay of time.

Figure 17.2 shows the device of the electromagnetic relay of time of DC. The relay coil is fixed on the yoke. To the yoke on swinging prismatic bearing anchor is fixed, which in the switched off state keeps by the return spring.

The relay work of time based on the fact that, in consequence of the phenomenon of self-induction at turning off the coil current gradually decreases. When the coil is turned on in the magnetic relay system magnetic flux arises, under which the armature quickly, without time delay is taken up to the yoke. If the coil is short-circuit on or turn off, the current which gradually decreases in the winding will maintain the magnetic flux of relay.

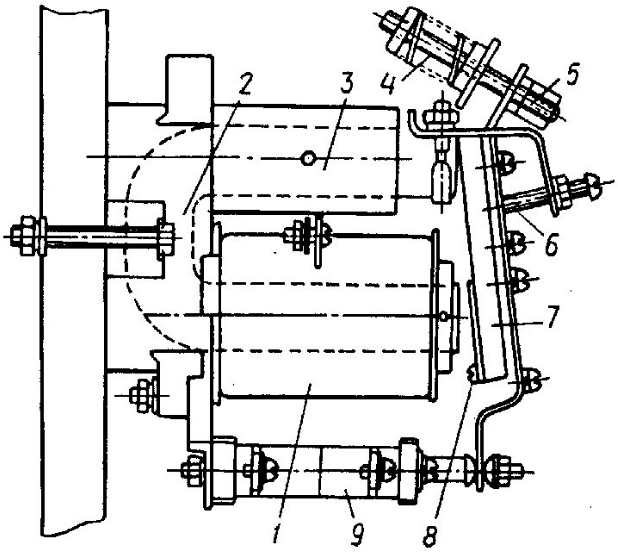


Figure 17.2: Time relay of DC: 1 – coil; 2 – yoke; 3 – cartridge; 4 – return spring; 5 – adjusting nut; 6 – thrust screw; 7 – armature; 8 – non-magnetic cushion; 9 – contact system

Thereby, the armature remains for some time attached to the yoke. When the force of attraction of the armature to the yoke will be less effort of return spring, the armature of the relay under its action will move away from the coil. The time during which the armature remains attracted after turning off the coil, is called the time of aging of the relay. Because the armature is associated with the moving contact of the contact system, the contacts are opened (or closed) with time delay. The exposure time depends on the type of relay, way of turning off the coil and is within 0,2 ÷ 0,3 s.

Intermediate relay is used in the crane schemes as an auxiliary device, if the main unit does not have a sufficient number of contacts required for scheme work, and if the power of contacts of the basic device is insufficient for break or closing the control circuit.

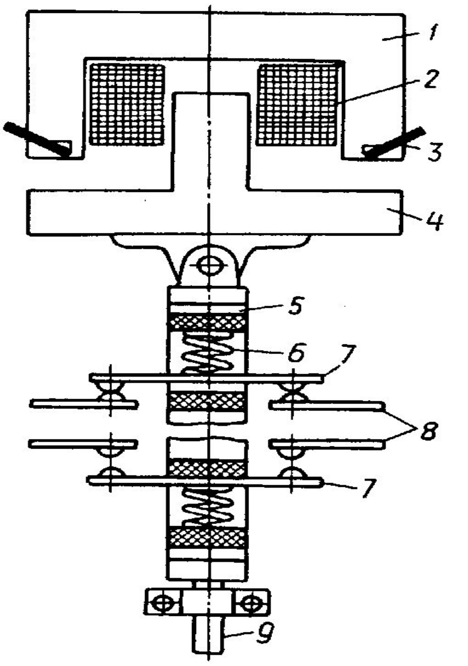


Figure 17.3: Intermediate AC relays:
1 – yoke; 2 – coil; 3 – short-circuit coil;
4 – armature; 5 – insulating batten;
6 – contact spring; 7 – contact bridge;
8 – fixed contacts; 9 – rod

Device relay is shown in figure 17.3.

Intermediate relays are available in coils of direct and alternating current. The relay consists of three to six contacts. The movable contacts of the relay – of bridge type. They are fixed on the rod connected to the armature. When the coil is energized, the armature is attracted to the yoke, and the associated with it bridging contacts are closed or opened the fixed contacts, performing the required switch in the control scheme. The intermediate contacts of the relay are calculated for high current (up to 20 A) and can be included only in the control circuit.

Relay of minimum current. The relay is used in the scheme of the drive of cargo winch with brake machine to control the force of current of the actuation winding.

Device relay of minimum current is shown in figure 17.4.

The relay coil is included in the actuation circuit of a brake machine. When the force of current in the circuit reaches the value of act of the relay, whereby the attraction of the armature to the pole bit 2 will be more opposed force of the spring 12, the relay will be turned on. The upper contact 6 will lock, and the lower 11 will open.

The value of current of act of the relay can be regulated by change of the force of tension of the return spring using castellated nut 5 and changing the air gap in the electromagnet by with a screw 4. At the weakening of the spring tension, or a decrease in air gap relay is turned on at a smaller force of current in the coil.

Relay of maximal current – an electromagnetic current relay of instantaneous action. Relays are used to protect electric motors from damage caused by sharp increase of force of current, for example, when a large overload, sharp turning on, short circuit.

The device of relay of maximal current shown in figure 17.5.

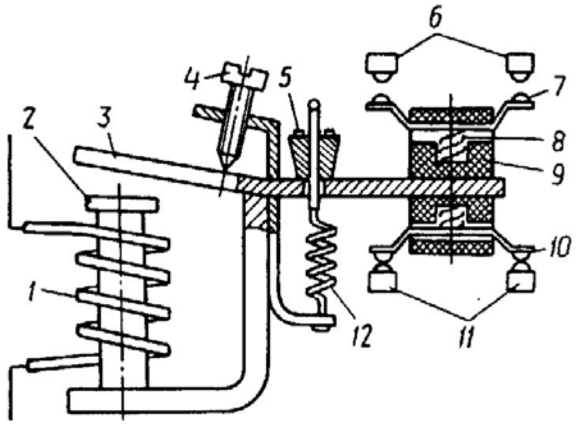


Figure 17.4: Relay of minimum current:
1 – coil; 2 – yoke 3 – armature; 4 – screw; 5 – nut; 6, 7, 10, 11 – contacts; 8, 12 – springs; 9 – blocks

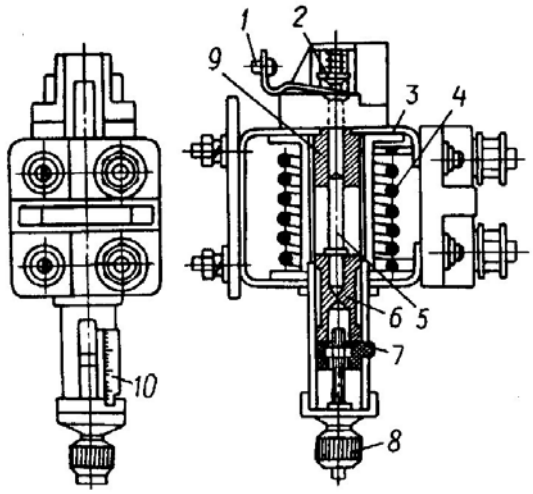


Figure 17.5: Relay of maximal current:
1 – stationary contact; 2 – contact bridge; 3 – magnetic core; 4 – coil; 5 – pusher; 6, 7, 9 – bushing, 8 – regulation screw; 10 – scale

The relay coil is joined sequentially to the phase of the power circuit of the motor, and the contacts 1 – in circuit of control of apparatus, which provides automatic shutdown of the power source circuit of the motor. At passing of current in the coil a magnetic field is excited, which increases with force of current. This field locks on the magnetic core and acts on the pusher, mounted in the sleeve 6. Under the action of magnetic forces the pusher together with the sleeve is pulled upwards and, if the force of current is greater than the specified value, on which relay is configured, influences the contact bridge, breaking the contacts. A separate relay in this case do not have their contacts, and are installed in the device with one contact for all relays. This group relay may consist of four block-relay.

The relay is configured to force of current of actuation by rotation of the screw 8 in accordance with the scale pointer connected with this screw. The lower the anchor of the sleeve lowered, the larger force of current is required to actuation of the relay.

The thermal relay is used to protect electric motor from a small but prolonged overloads, at which the force of current of the motor current is 30% higher than the rated value. The thermal relay is actuated at a certain value of the force of current during some time interval.

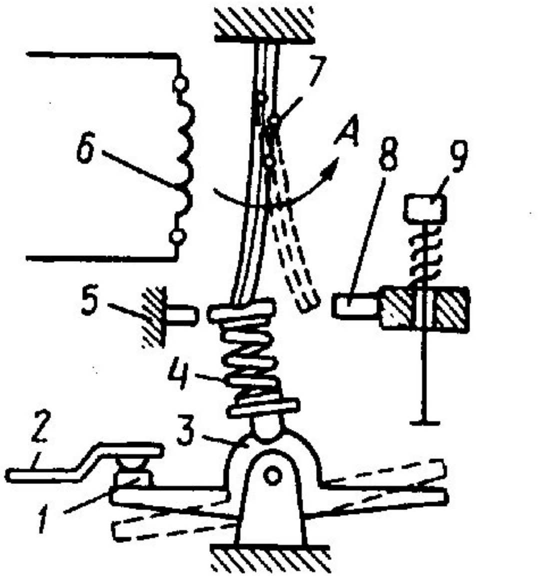


Figure 17.6: Device of thermal relay:
1 – movable contact; 2 – stationary contact; 3 – block; 4 – spring; 5, 8 – prop; 6 – heating element; 7 – bimetallic plate; 9 – return device

The main element of the relay (fig. 17.6) is a bimetallic plate is made from two metals with different coefficients of linear expansion. When the plate is heated by working current flowing through the heating element (or directly on the plate) then it bends in the direction of metal with a smaller coefficient of linear expansion.

In thermal relays bimetallic plate rests on the upper end of the spring. The lower end of the spring presses on the ledge of plastic blocks, hinge fasten on the axis. In the position shown in figure 17.6 movement of the plate and the upper end of the spring is limited by the stop 5. The spring influences on the ledge of blocks so, that it turns out rotated clockwise, and fixed on it the movable contact – closed with fixed contact. When current carries influence heating element, the bimetallic plate is heated and its lower end is moved in the direction of the arrow A. As a result, the upper end of the spring moves to the right and the plastic block is rotated counterclockwise (as shown by the dashed line), and the contacts 1 and 2 open. The stops 5 and 8 limit the position of the lower end of plate.

In the initial position the relay is returned spontaneously, when the bimetallic plate get cold (relay with self-reset). Stop 8 may be removed; then the relay returns to its original position by return device. The relay is actuated with a time delay that is inversely proportional to the force of the current. The more current in the heating element, the less the time during which the bimetallic plate is heated up to act of relay.

The thermal relay is not actuated in the case of instantaneous growth of force of current, therefore, cannot serve as a reliable protection from short circuits. The thermal relay is used in schemes of load-lifting machines for the protection of short-circuit asynchronous motor and installed in the magnetic contactors or in automatic switch with thermal or combined releases.

The resistors. Resistors are used in electrical equipment of tower cranes and they are divided into starting and control. They are included in the power circuit of electric motors and are used in the circuits of the control and alarm.

Starting and control resistors (rheostats) are included in the rotor circuit of the electric motor and serve to smooth acceleration, braking and regulation of frequency of rotation of the electric motor, and also for the braking it in mode of opposition circuit.

Elements of ***belt resistors*** (fig. 17.7) are satisfied from the wound on the rib the tape 3, mounted on a steel holder with porcelain insulators 1. These elements come in a box similar to a wire resistors.

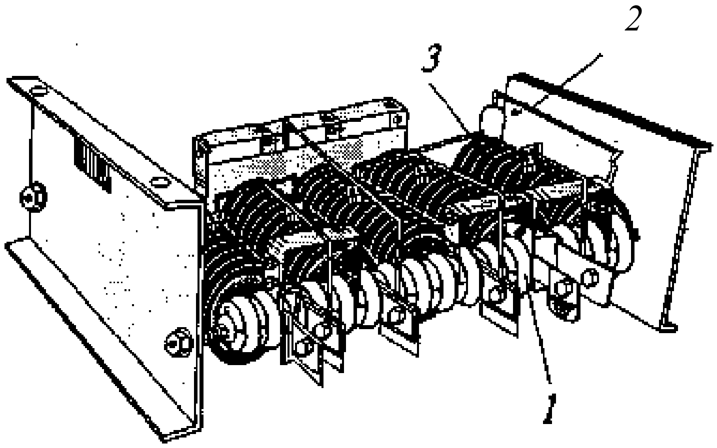


Figure 17.7: The box with belt resistors:
1 – insulators; 2 – bridge; 3 – FeCrAl alloy tape

Starting and control rheostat depending on the power and purpose of the electric motor consists of one or more boxes of resistors.

Rheostats in the rotor circuit of the motor are included in the process of work with controllers. The resistors are calculated as a rule, only on the momentary switching on at startup or braking of the motor. Prolonged work of the electric motor with the included resistors (arm of controller is not set to the extreme position) is invalid, as in this case the resistors strongly overheat.

The braking device. ***Braking machines*** are used in electric drive of hoisting winches to obtain low speeds of movement of load.

On tower cranes brake machine of alternating current TM-4A is installed, which represents a short circuit asynchronous electric motor of special implementation, having small frequency of rotation.

Braking machine is calculated for short-term work with ПВ = 15 % and should be used only for small movements of loads.

Braking electromagnets and electrohydraulic pushers. Braking electromagnets and electrohydraulic pushers are used for block brakes in the mechanisms of the crane.

Braking electromagnets. Braking electromagnets have two main parts: the magnetic core and the winding of actuation (coil). The magnetic circuit consists of a fixed yoke and a movable armature. At passing of current through fixed on the yoke coil, a magnetic field arises, under the action of which the armature is attracted to the yoke and through a system of levers unbrakes brake.

Braking electromagnets are divided by the nature of power to the electromagnets of AC and DC.

Electrohydraulic pushers. Electrohydraulic pushers – are machines that converts electrical energy to mechanical energy and having in straight lines moving the actuator (rod).

Compared with the brake electromagnets electro-hydraulic pushers have a number of advantages:

- dimensions and their weight is less compared to the same ones on working parameters of the electromagnets, the power consumption is also several times smaller;
- the amount of forcing efforts of hydropushers does not depend on the position of the piston, while at the electromagnet force varies sharply depending on the size of the air gap between the yoke and armature;
- with increase of external load to maximum thrust force of the pusher piston stops. In this case there is no motor overload or mechanical damage of the pusher elements.

Semiconductor rectifiers. Semiconductor rectifiers are used for rectification of AC to DC, which is used on tower cranes for power of actuation windings of braking machines and braking electromagnets, control circuits of the coils of contactors and control circuits of magnetic amplifiers, for dynamic braking of asynchronous motors, and also for power of circuit of restrictors of carrying capacity and anemometers.

End switches. End switches serve to limit the action of mechanisms of the crane, switching on of the alarm circuits, and are used as switches of block.

According to the principle of work switches are subdivided:

- on the lever (fig. 17.8), going off at the action on them breakaway devices;
- driving (spindle), which are rigidly connected with the shaft of the mechanism and go off after the turn of shaft of switch at a certain angle.

The fuses. Fuses are designed to protect electrical equipment and electric circuits from large currents, resulting from short circuits, and a significant (50% or more) overloads.

In interlock the conductor with a low temperature of melting (fuse) is placed, a current of the protected circuit passes through it. At the increase of force of current the large amount of heat exudes, which makes the conductor melt and it opens the circuit. On tower cranes tubular cartridge fuses without filling ПР-2 and with filling ПН-2, НПР, НПН are used.

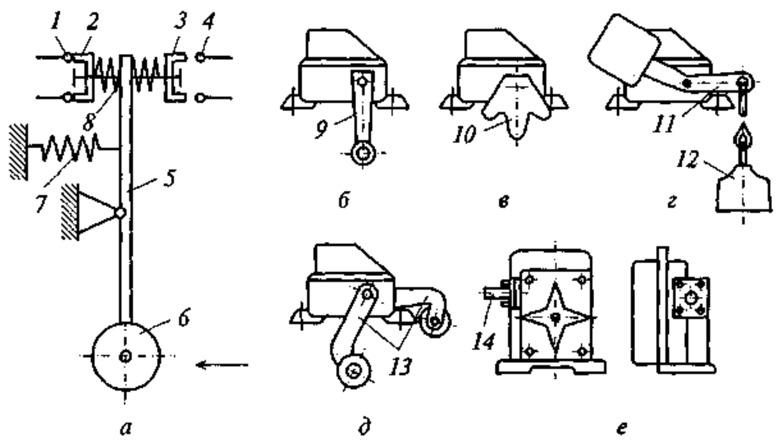


Figure 17.8: End switches: *a* – lever; *b* – КУ-701; *c* – КУ-04; *d* – КУ-703; *e* – КУ-706; *f* – ВУ-250; 1, 4 – stationary contacts; 2,3 – contact bridges; 5, 9, 13 – levers; 6 – roller; 7 – return spring; 8 – spring of contact bridges; 10 – sector; 11 – lever with a counterweight; 12 – load; 14 – input shaft

The circuit breakers and power distribution boxes. The circuit breakers and power distribution boxes serve for infrequent commutation (closing and break) of electrical circuits of AC and DC voltage up to 500 V. For tower cranes, the circuit breakers are used in the protective panels and power distribution boxes. Power distribution boxes are used on tower cranes as an introductory (gantry) circuit breakers installed in the lower part of the metalware of crane, on portal or on track chassis.

Circuit breaker (fig. 17.9, *a*) has one or more movable knives *1*, hinge in contact column *6.* The knives are connected with the traverse *3* of an insulating material. At switching on of circuit breaker the knives are connected in the contact jaws *2.* The wires from the power source are attached to jaws, and to contact column of knives - wires switching on by the circuit breakers circuit. The breaker control (enable or disable) using arm. The circuit breaker is controlled (enable or disable) using arm *4.*

On the number of opening circuits there are one-, two - and three-pole circuit breakers.

Power distribution box (fig. 17.9, *b*) is a cabinet 7 with built-in in it circuit breaker 8 and interlocks 10. The circuit breaker is managed with help of a lever drive by side grip 9. The handle has a locking device whereby it cannot open the lid of the cabinet at the switching on the circuit breaker and switch off the circuit breaker when the cover is open. In some constructions of power distribution boxes instead of separately positionable circuit breakers and fuses embedded block interlock-switch is used (fig. 17.9, *c*). The unit consists of contact jaws 11, mounted on an insulating panel, and movable knives 14 made together with interlocks. The block is turned on and off by a lever (13) associated with knives using a lever system 12.

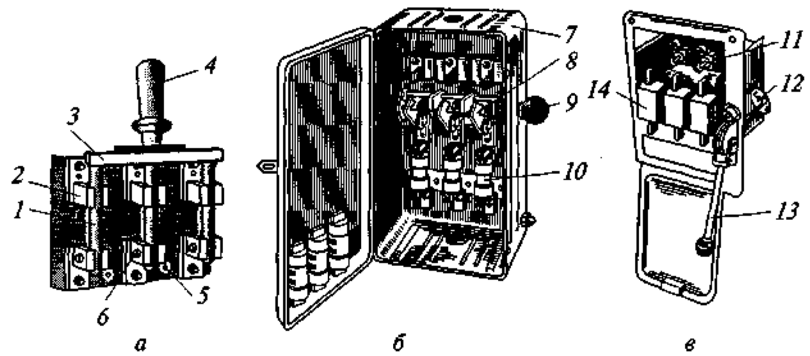


Figure 17.9: Apparatus for infrequent commutation of electric circuits:
a – circuit breakers; *b* – power distribution box; c – block of interlock-switch;
1 – knife; 2, 11 – contact jaws; 3 – traverse; 4, 9, 13 – arm; 5 – insulation board;
6 – contact column; 7 – cabinet; 8 – built-in circuit breaker; 10 – interlocks;
12 – lever system; 14 – movable knife-interlock

At all distribution boxes that are installed as a portal circuit breaker on the crane or as circuit breaker on connecting post at crane way, a device is provided for locking of the box with the arm installed in the "Turn off" position. It must be carried out so that in the locked position it was impossible to turn on the arm and at turned on arm – lock device.

Automatic circuit breaker. Automatic circuit breakers (machines) are intended to automatically disconnect the electrical circuits in the event of a breach of the normal conditions of their work (for example, when the overload or short-circuit), and also for infrequent commutation.

Automatic machine (fig. 17.10, a) consists of a jar (base 1 and cover 2), commutating devices (fixed contacts 3 and movable contacts 4), arc-suppressing chambers 16, the control mechanism and releases of maximal current.

Parts of the control mechanism includes: arm 7, figures detail 6, springs 8 and 10, levers 9 and 15.

Automatic switch off (opens its power contacts) at actuation of releases of maximal current.

According to the principle the releases are: thermal, electromagnetic and combined, consisting of a serially thermal and electromagnetic releases. The main element of thermal release is a bimetallic plate.

Electromagnetic release consists of a coil *14* and core *13.* At the origin of current of the short circuit core instantly sag into the coil. The lever 11 is rotated, free from engagement with the tooth figured part 6 and the automatic machine is turned off without time delay.

Equipment of manual control. For infrequent switching of control circuits and lighting in schemes of tower cranes control buttons, control switches, packet switches and universal switches are used.

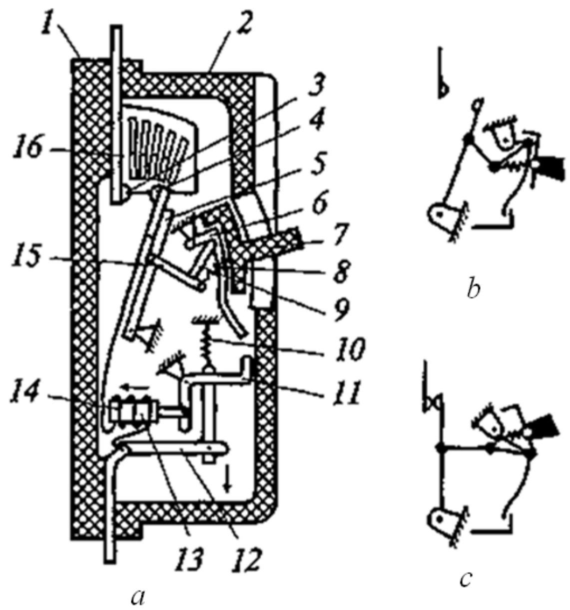


Figure 17.10: Automatic switch A-3100: *a* – structural scheme;
b – lever system of the automatic machine before turning it on;
c – the lever system of the automatic machine after turning it on.

Control buttons (fig. 17.11, a) serve for closing and break of the circuits of the coils of contactors, magnetic starters and relays, as well as for switching on the audio signal. A set of buttons that are embedded in a common casing, called a button *station.*

The control switches are hand operated and pedal (foot). Switches with manual drive are used to disconnect the line contactor, usually called the emergency switches. Pedal switches (fig. 17.11, b) are used for switching on of control circuits, for example to control of the alighting speed of loading winches in the scheme of opposition circuit. The contacts of the control switches is calculated for a force of current up to 10 A.

Packet switches (fig. 17.11, c) are used in the scheme of cranes for switching on of control circuits and lighting. Using packet switches working lights and heaters are switched on.

Packet switch consists of two nodes: the contact system and the switching mechanism.

Packet switches are produced in open and protected execution for the amount of current from 10 to 60 A.

Universal switches (fig. 17.11, *d*) is multiple circuit electrical apparatus used for infrequent switching of electrical circuits. On tower cranes, which are provided for the control mechanisms from the cab or from a portable erecting panel, universal switches are used for switching scheme of the crane on panel or cabin. On some cranes changers are used as controllers to control the magnetic controllers.

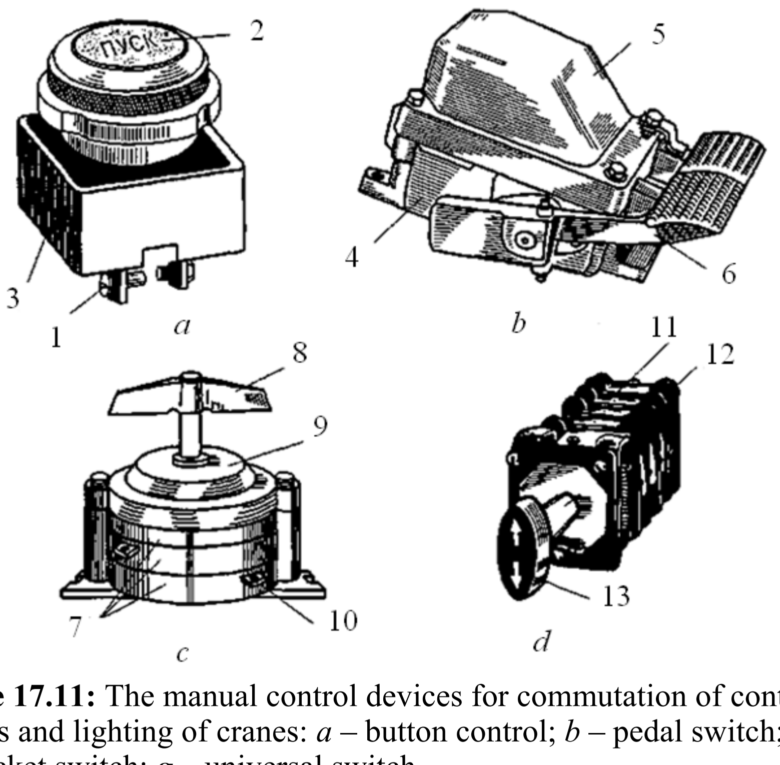


Figure 17.11: The manual control devices for commutation of control circuits and lighting of cranes: *a* – button control; *b* – pedal switch; the packet switch; *g* – universal switch

Current collectors. Using a current collector electrical equipment of rotating part of the crane is connected to the external circuit and electrical equipment that are installed on the antirotating part of the crane.

According to the principle of work current collectors of tower cranes are divided into ring and without ring.

Without ring current collector is a loop of flexible conductors binding the terminals of a circuit for rotary and unturning parts of the crane. The length of the wires is chosen sufficient for two complete turns of the crane (720°) in both directions from the initial position.

Without ring current collector is used on most tower cranes, as it is much easier and more reliable then ring. At exploitation of crane with and without ring current collector it should systematically monitor the work of the rotation limiter, as its malfunction can lead to twisting and breakage of the wires of the flexible loop.

Wires and cables. For connection of electrical equipment to an external circuit, and an electric connection between the electric motors and electric devices on a tower crane wires and cables are used.

Cables and wires cables of all circuits of crane electrical scheme must be visible alphanumeric marking.

According to the rules of electrical installation, cranes can be run by wires and cables with copper conductors. The wire cut and current-carrying conductors of cables is selected on permissible continuous current to the load depending on the power consumed by the receiver. However, on conditions of the mechanical strength of the cut of copper wires shall be not less than 2.5 mm^2. In the control circuits for connection of controllers, and also in circuits of telecontrol and connection flexible cables with copper conductors with cut less than 2.5 mm2 are permitted to use, provided that these wires do not bear mechanical loads.

External wiring on the crane are performed by flexible cable with copper conductors in rubber or equivalent insulation, intended for work in the temperature range from -40 to +40°C. For wiring in cabinets of magnetic controllers and in cabins single-conductor or multi-cored (ПР, ПРГ, ПВ-ХЛ, ПГВ-ХЛ) or cables for external wiring are used.

Cable drums. Electric power move from the external circuit to the electrical equipment of the crane on cable. The length of the cable that connects the lead-in cutout on the track chassis (portal) of tower crane with connected point at the crane path, usually equal to 50 m.

To protect the cable from wear and breakages tripping if touch inequality of craneway different tools are used. If length of way more than 50 m connected item is placed at the middle of the craneway, and for cable wooden tray is arranged, on which the cable is draged by a crane. When the path length is 50 m or less along the craneway the wire or rope are pulled on post, and to them with wire rings cable is attached.

The use of the cable drum eliminates the need to perform these complex and unreliable devices. Cable drum is intended for winding (or reel) cable at movement of the crane along the track. The drum is a hollow cylinder, within which is placed an ring current collector connecting winding cable with lead-in cutout.

The cable is winded on the outer cylindrical surface of the drum. Cable drum is fixed on the metalware of crane and has a drive unit, with help of which the winding of the cable onto the drum during movement of the crane to connected item takes place. The cable windes from the drum due to its own tension or as a result of change the direction of rotation of the drive of drum.

17.4 Electric drive of construction cranes

Electric drive of actuating devices of LLM is used for crawler and pneumowheel self-propelled cranes, and also on car and tower cranes.

On construction cranes the most widespread are multi-motor electric drive of AC and DC. Moreover, the electric current is often generated by their own power plant and comes from the synchronous generator to the electric motor of actuating devices. The electric schemes of the drive of construction cranes provide the possibilities of power of electric motors not only from the generator, but also from the external circuit of three-phase current by voltage 380 V. Power from the external circuit is performed via a cable, which makes it possible to transmit electricity through the ring current collector on the control panel in the cranes with AC drive. In cranes with DC drive from the external circuit current is supplied to the AC motor, which rotates synchronous generator of DC.

As an example let's consider the electrical scheme of the crane КБ-401А [45]. Figure 17.12 shows the electrical scheme of the power circuits of the crane, figure 17.13 – scheme of the control circuits, and figure 17.14 – scheme of the lighting circuits, heating and alarm.

Electric drive of tower crane КВ-А (fig. 17.12) are calculated on power by an external three-phase electric AC network with a line voltage of 380 V and neutral occasion. Electric scheme of the control circuits (fig. 17.13) operates on alternating current by voltage of 220 V and a direct current from a rectifier *V2.* Auxiliary devices (lighting, heating, alarm) are fed by an alternating current voltage of 220V (fig. 17.14). The electrical circuit of repair lighting operates on alternating current by voltage 12V from a step-down transformer T2 (see fig. 17.14).

The power of the electric motors (see fig. 17.12) is carried out via the lead-in cutout Q ,automatic switch *F1*, the line contactor contacts *КЛ* and the contacts of the contactors of reverse.

The frequency of rotation of all motors during startup is regulated by the resistance change of starting and control rheostats. The frequency of rotation of the electric drive of rotary mechanism additionally is regulated through the auxiliary brake with electromagnet *V2*, which brake mechanism in the first position of the handle of the bracket. To obtain low frequency of rotation of mechanism of the lifting of load, the electric drive with brake machine of AC and dynamic braking of the drive motor is used.

In the electric drive of mechanism of lifting of load of crane КБ-401А protection of silicon rectifiers with electromagnet *V7* of DC is provided.

Protection of rectifiers from overstress is provided by three chains, each of which contains serially included resistors (*R4, R5, R6*) and condensers (*C4, C5, C6*), connected by a triangle and the included in the three phase rectifier bridge *VI.*

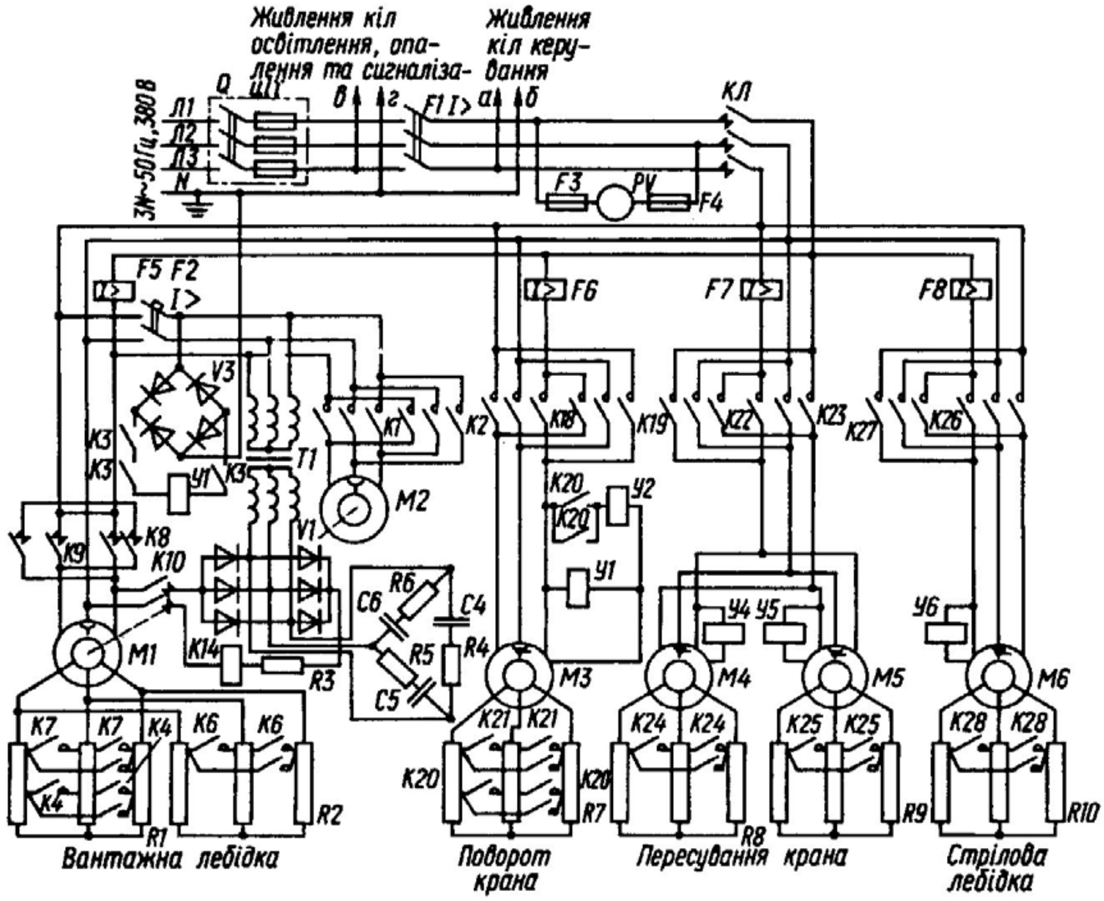


Figure 17.12: Scheme of the force circuits of the crane КБ-401А.

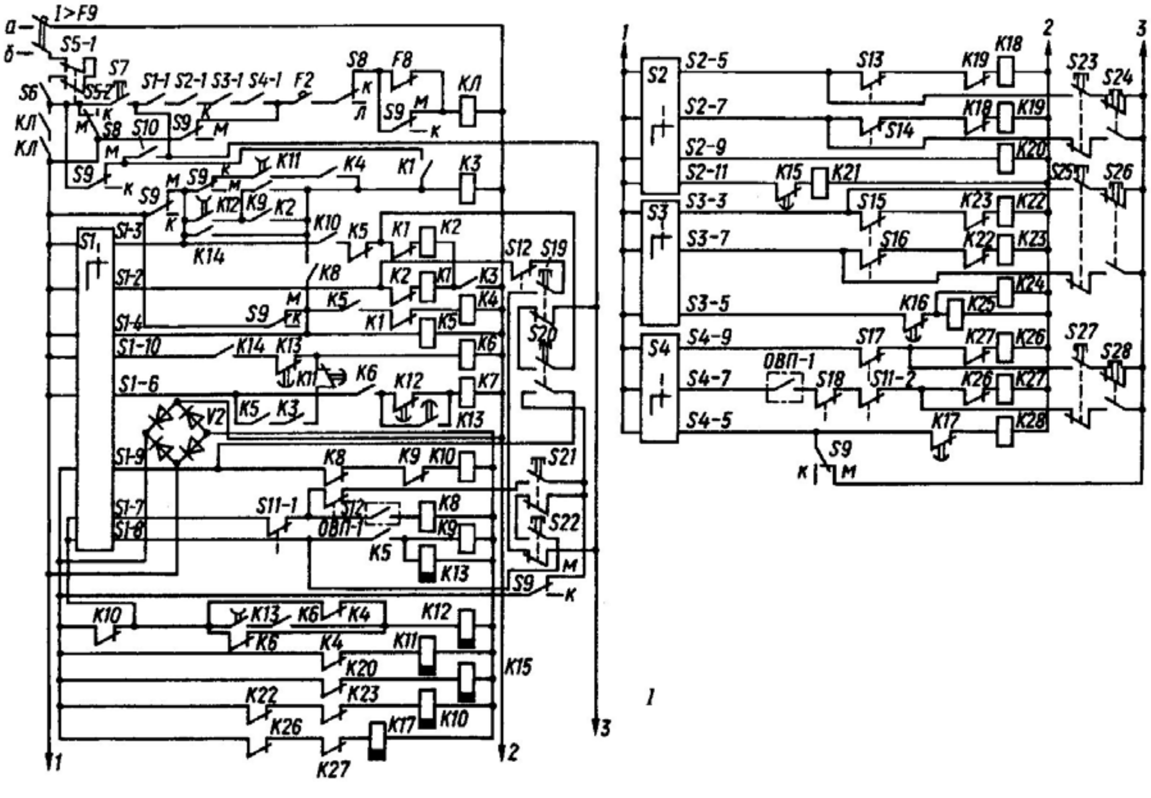


Figure 17.13: Scheme of the control circuits of crane КБ-401А.

The main electromagnet of DC is fed from the power circuit by special circuit through rectifier *V3* and the contacts of the contactor *K3.*

In electric drives of all mechanisms magnetic controllers are used, which can be controlled either from the cab of the crane, either from the pendant.

During the implementation of the work control is carried out from the cab by means of brackets *S1, S2, S3, S4*. The sequence of closing of their contacts are shown in tables 17.1...17.3, where a closed contact is marked "X".

In the case of mounting of the crane and its test when the driver can't be in the cab, the mechanisms are controlled with pendant by buttons *S19...S28*. Transfer of control to the cabin or pendant is a universal switch S9, the handle of which is installed in the position K (control of crane from the cab) or in the position M (control from pendant).

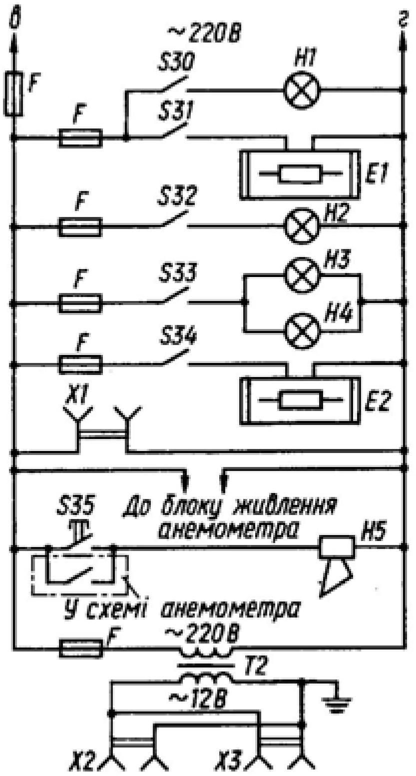


Figure 17.14: Scheme of the lighting circuits, heating and alarm system of the crane КБ-401А

Table 17.1: Closing of the contacts of the controller of cargo winch of crane КБ-401А

Contact	**The position of the handle**						
	Lifting			**0**	**Cast**		
	3	**2**	**1**		**1**	**2**	**3**
S1-1				X			
S1-2			X				
S1-3					X	X	X
S1-4	X	X	X				X
S1-6	X						X
S1-7	X	X	X				
S1-8							X
S1-9					X	X	
S1-10	X				X		

Table 17.2: Closing of the contacts of the controller of the turntable of the crane КБ-401А

Contact	The position of the handle						
	Lifting			0	Cast		
	3	2	1		1	2	3
S2-2				X			
S2-5	X	X	X				
S2-7					X	X	X
S2-9	X	X				X	X
S2-11	X						X

Table 17.3: Closing of the contacts of the controller of movement of the crane and lifting of boom of crane КБ-401А

Contacts of bracket		The position of the handle				
		Forward Lifting		0	Backwards Cast	
S3	S4	2	1		1	2
S3-1	S4-1			X		
S3-5	S4-5	X				X
S3-7	S4-7				X	X
S3-9	S4-9	X	X			

In the schemes of electric drives of crane mechanisms a stepped acceleration of motor under the control of the time relay is provided At these steps of starting and control rheostats is short-circuited in accordance with the delay of time relay. For example, if you set the handle of on the boom bracket *S4* right away in the second position of lifting (cast), first turn on the contactor of reverse *K26* (*K27*) and the motor will start to work with the full resistance of the rheostat. At the same time turn off time relay *K17*. When the time delay are finished, the relay *K17* is triggered and its contacts will close the circuit of the coil *K28*. Contactor *K28* will turn on and short-circuit the rheostat, leaving in the rotor circuit of the motor *M6* small resistance, which cannot be turned off.

Electric motors, electric devices and crane mechanisms are protected by automatic machines, relay of maximal current, fuses and end switches (see fig. 17.12).

Zero protection is performed in contacts of brackets *S1-1, S2-1, S3-1, S4-1*, closed only in the zero position of the handles. These contacts are connected serially with the button *S7* in the circuit of the coil of the line contactor CL.

The electric motors of the crane mechanisms are protected from overvoltage by the relay of the maximum current. The relay coil *F5, F6, F7* and *F8* are included in a one phase of power of electric drive of each mechanism. Relay combined in one unit and operate on a common contact *F8* included in the circuit of the coil of the line contactor CL. Actuation of any of the relay causes the break of circuit of the coil and disconnection of the crane circuit from the feeding circuit.

One phase of brake machine *M2* and the power circuit of rectifier *VI* also are protected by relay F5, and the other two phase - three-pole automatic machine *F2*. The third pole of the machine is included in the circuit of the coil of the line contactor CL. Therefore, in case of actuation of the automatic machine line contactor is switched off.

Defense of common power of circuit from short circuits is carried out by automatic circuit breaker and fuses of force input box Q. Ultimate protection from passage by crane mechanisms of extreme positions end switches, opening contacts which are included in the circuit of the coils of the respective contactors.

End switch *S11* is opened when the hook assembly is suitable to the boom. Limit switch *S13* is opened in the extreme right position of the rotary platform, and *S14* – in its extreme left position. Contact *S15* of end switch of limiter of movement of crane is opened in the end position during the movement of the crane forward, and the *S16* of the same switch – is in its extreme position during the backward motion. End switches *S17* and *S18* switch off the electric motor of boom winch, respectively, in the extreme upper and extreme lower positions of the boom.

Work of cargo winch is controlled the restrictor of weight-carrying ability ОВП-1, exit contact of which is included in the circuit of the coil *K8*. In case of contact break (as a result of excess of weight-carrying ability) the contactor of load lifting goes off and electric circuit give possibility to execute the operation of load cast.

If it is necessary urgently to stop all crane mechanisms, line contactor can be turned off by the emergency switch *S6* in the control cabin or *S10* – on pendant.

In lighting circuits, heating and alarm system (see fig. 17.13) *H1* bulb of lighting of cab of control and lamp *H2, H3, H4* of headlights of lighting the working area are managed by packet switches *S30...S34*. The control cabin is heated by heating devices *E2*. Tubular heaters *E1* are used for heating of the lantern of cab.

Sound siren *H5* is switched on by button *S35* and openning contact of the exit relay of anemometer. Under normal wind load and the operable circuit of anemometer its exit relay is turned on and the contact in the circuit of the siren is opened. In case of switching off the exit relay of the anemometer (due to increased wind or malfunction in the circuit of the anemometer) the contact is locked and turn on the siren.

For lighting circuits the power module of the anemometer and the transformer T2 are connected with rosaces *X1* and *X2* for turning on the lamp of repair lighting. Chain of lighting, heating and alarm are protected by fuses.

Key findings

1. Electrical equipment of LLM on their purpose is divided into the basic (equipment of the electric drive) and auxiliary (equipment of working and repair lighting, heating).
2. For drive of LLM motors of special crane type are used.
3. Switching on and off of motors of LLM are performed by a special apparatus: starters and contactors.
4. To protect electrical equipment from emergency modes interlocks with cutout fuses, thermal relays and relay of maximal current are used.

Control questions

1. What equipment relates to load-lifting machines and what features are characterized its work?
2. Give a description of the main electrical equipment of LLM.
3. Give examples of auxiliary electrical equipment of LLM.
4. Give a description of crane asynchronous electric motors.
5. How to carry out startup of asynchronous motors of LLM?
6. Give a description of crane electric motors of DC.
7. What are the basic elements of the control equipment?
8. Explain the principle of work of the relay of DC.
9. Explain the principle of work of the relay of minimum current.
10. Explain the principle of work of the relay of maximal current.
11. Explain the principle of work of the thermal relay.
12. Explain the purpose of the main elements of power circuits of the crane КБ-401А.
13. Explain the purpose of the main elements of the control circuits by crane КБ-4001А.

18 ELECTRIC MANUAL MACHINES

Key concepts: electric manual machine, insulation class, combined electromechanism, vibrator.

18.1 General information

In the construction industry a variety of mechanisms and manual machines with an electric drive are used. In applying a variety of standard or special working tools or specialized nozzles by the same machine different technological operations can be performed and it is possible to process a variety of materials. Therefore an electric manual machine can be divided according to the main, corresponding to the name of the machine purpose into the following groups: drilling machines; grinding machines; machines for sawing wood; wrenches and screwdrivers; machine of percussion; vibrators.

Electric manual machines (EMM) are driven by an electric motor or an electromagnet, composed with the machine a whole. As motors it is used:

asynchronous three-phase electric machines with short-circuit rotor, normal and high frequency of current;

single-phase asynchronous electric machines with short-circuit rotor, normal and high frequency of current;

inverted (i.e., the stator rotates and the rotor is anchored stationary) three-phase asynchronous electric machines with short-circuit rotor, normal and high frequency of current;

universal collector electric machines;

electric machines of the alternate motion (electromagnetic).

In EMM as a rule the generally made for them motors are used specially, working at a voltage of 36 or 220 V. In mobile machines motors of general purpose are used on voltage 380/220 V.

The following symbolic notations of motors are used by specialized factories for the production of manual machines:

КН – collector normal frequency of current;

КНД – the collector normal frequency of the current with double insulation;

АН – asynchronous normal frequency of current;

АП – asynchronous heightened frequency of current.

Following the letters numbers mark the gabarit of the motors (the diameter and the length of the active steel stator or inductor).

In force of manual use of electrical mechanisms by this group special attention in their design is given to the questions of electrical insulation.

18.2 Isolation of electrical manual machines

EMM are produced by three classes of performance on voltage and isolation:

Class I – for a rated voltage 220 V, in which at least one metal item, available for touch, is separated from the parts only under working isolation;

Class II – rated voltage 220 V, in which all metal parts available for touch, are separated from the parts under voltage, double or reinforced isolation;

Class III – for a rated voltage 36 V.

The EMM of the first class are dangerous in relation to the defeat of a worker by electric current. At work they must be reliably grounded, rubber mats and dielectric gloves must be used, but even with this in construction conditions they are not everywhere allowed to exploit. Complete electrical safety work with the machines of the I-st class can be achieved if they are connected to the circuit through the protective breakaway device, which ensures disconnection of the machine from the circuit in case of leakage of current and short circuit in the motor windings. The time of actuation of protection is not more than 0.05 s.

The EMM of the second class (double isolation) is the most progressive, as they can feed from the lighting circuit, they do not require grounding, and at it a complete electrical safety of work is provided in compliance with the operating rules. Double isolation of machines is carried out in two main ways:

the stator (inductor with coils) of motor, brush mechanism, a switch and all conducting (connecting) wires are placed in the jar and the handle of insulating material (high strength plastic); and the shaft of the rotor (armature) has an electrical insulating sleeve, insulating it from the rotor (armature) and the collector (fig. 18.1);

the stator (inductor with coils) of motor, brush mechanism and all conducting (connecting) wires are placed in a plastic or aluminum jar, which is mounted in the jar from plastic. To the jar the arm is anchored in which electric switch is installed and conducting cable is anchored (as a variant the plastic sleeve may be placed between the stator and the external metal jar). The motor shaft doesn't have an intermediate isolating sleeve, instead of the sleeve a driven pinion from electrical isolating material (plastic, textolite) serves as the second isolation. Pinion can only have a nave from electrical isolating material, and the row – steel.

Machines of class II (with double isolation) on the jar or on factory board have a special character (see fig. 18.1).

Machines of class III are safe for work and must obtain power from autonomous sources of current or from the circuit via transformers or converters of frequency, if the car has a built motor of heightened frequency of current.

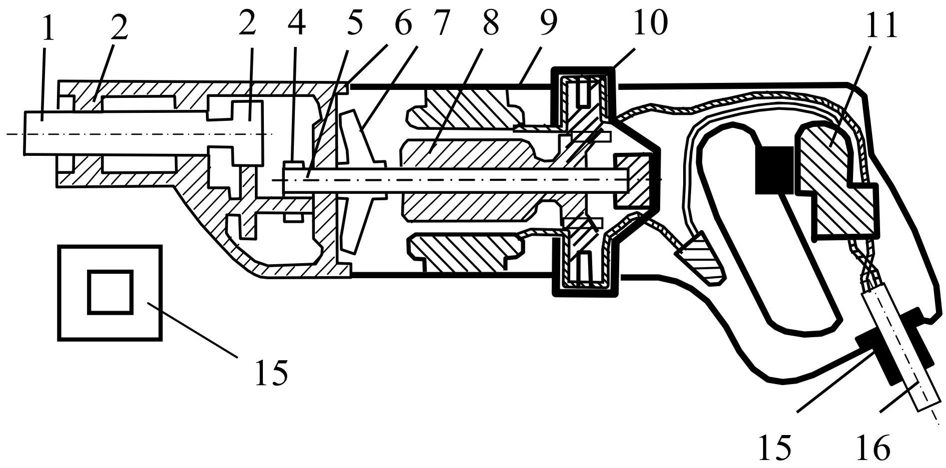


Figure 18.1: Electric manual drilling machine: 1 – spindle; 2 – metal jar of reducer; 3 – reducer; 4 – insulating pinion; 5 – shaft; 6 – intermediate plastic board; 7 – plastic blast; 8 – armature; 9 – plastic external jar; 10 – cap of brush holder; 11 – switch; 12 – conducting cable; 13 – protective flexible tube; 14 – symbolic notation on the jar of machines with double isolation

18.3 Examples of construction of electric manual machines

There are various constructions of drills, electric saws, electric planes, machines of electrolytic grinding, electromagnetic perforators and other electrified manual tools but power equipment of them are always built only on the basis of the motors of rotational motion, or only on the basis of the reciprocating motor devices, or represents itself a combined electromechanism with a rotating intensified element of reciprocating action.

18.3.1 Combined electromechanism. An electromagnetic perforator, for instance type ИЭ-4709 Б (fig. 18.2) is an example of a manual electric tool, which is used as an electric drill and electric hammer. This perforator is connected by flexible portable wire to the circuit by voltage of 220 V and frequency 50 Hz, consumed force of current in nominal mode is 3.2 A, consumed power is 650 W.

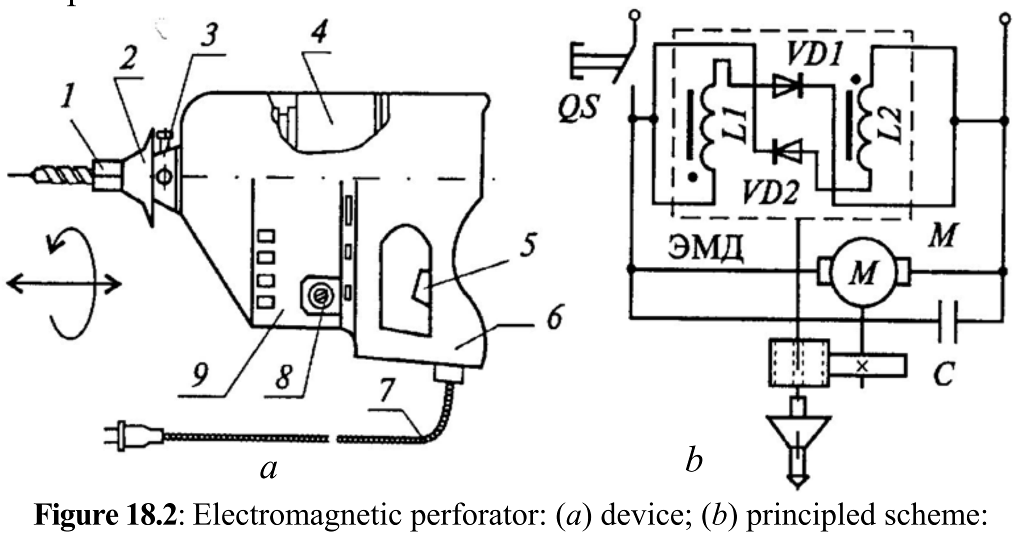


Figure 18.2: Electromagnetic perforator: (*a*) device; (*b*) principled scheme: 1 – working organ; 2 – rubber apron; 3 – box; 4 – motor; 5 – switch with self-reset; 6 – handle; 7 – wire; 8 – hole; 9 – jar

18.3.2 Vibrators. Vibrators are called simple vibrating machines intended for actuation of mechanical oscillations. They are machines that convert mechanical, electrical or chemical energy into mechanical vibrations and transmit them to the materials or devices. Oscillations are characterized by the amplitude A, i.e., the greatest deviation from the average position, measured in millimeters, and the frequency n, i.e. the number of periods of oscillations per unit of time, measured by the number of oscillations in 1 s.

Effective compacting of concrete mix by the vibration is achieved only for certain values of the amplitude and frequency at which accelerations are appeared, reducing the force of internal friction between the particles of the mixture so that they begin to move relative to each other under the action of gravity. Usually vibrators with vibration frequency n = 25...250 s^{-1} and an amplitude of oscillations 0.1...3 mm are used (large values of the amplitude for smaller values of frequency).

By the type of the drive vibrators are divided into electro-mechanical, electromagnetic, pneumatic, hydraulic and motor driven in action by internal combustion motors. The most spread electromechanical inertial vibrators with rotating unbalanced loads are obtained, mounted on the shaft of the rotor of the motor, or on a separate shaft receiving rotation from the electric motor via a coupling or V-belt transmission.

Surface and external vibrators. Electromechanical vibrators of centrifugal type are the most widely used. In these vibrators the inertial element in the form of unbalance or runner rotates and transmits the resulting centrifugal forcing force on the bearings of the shaft of unbalance or a support of runner.

Electromechanical unbalance vibrator ИВ -70 (fig. 18.3) consists of a jar, an electric motor and unbalance vibration exciter. In the aluminum jar 1 with the bearing boards 4 a three-phase asynchronous motor is placed, to the windings of the stator 3 which the current flows through the terminal-block box 2 and the rotor 5 is fixed on the shaft 6. The shaft leans on the bearings 7 and the unbalances 8 are fixed, closed by lids 9 on the console parts of the shaft. The covers are tightened by the studs 10 and closely join to the jar, in the bottom of which the installation legs with holes for the bolts of anchoring of the vibrator to the trough-shaped base, casing or other elements of construction through which the vibrations are transferred to the particles of the concrete mix are found.

At surface compaction of the concrete mix the base of vibrator transmits effective oscillations to a depth of 20 cm. Surface vibrator fixed on the batten, can serve for floating and surface sealing of concrete mix on a large area. Vibrator, detached from the batten and the base, can be used as an external vibrator for messages of oscillations to the casing, the chute, the hopper wall. It has two double unbalance, which constitute steel cylindrical parts, an eccentrically fixed on the shaft. As the center of mass of unbalance is shifted from the axis of the shaft then at the rotation of the shaft and unbalances centrifugal force of inertia occurs, which imparts forced oscillations to the vibrator.

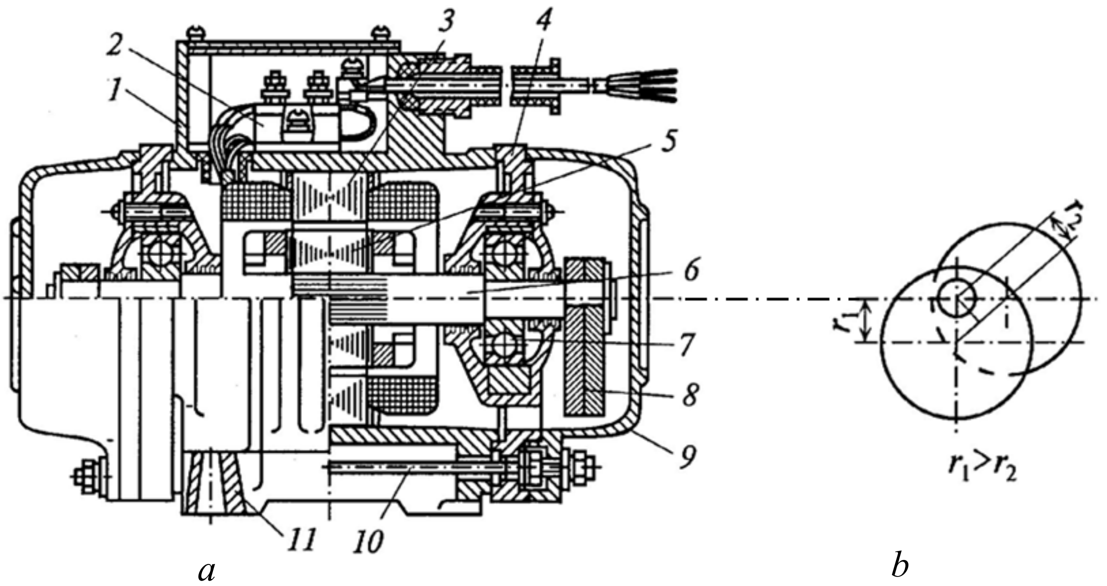


Figure 18.3: Electromechanical unbalance single-shaft vibrator ИВ-70 with a circular oscillations: *a* – general view; *b* – regulation scheme of unbalance; 1 – jar; 2 – terminal-block box; 3 – stator winding; 4 – boards; 5 – rotor; 6 – shaft; 7 – bearing; 8 – unbalances; 9 – lids; 10 – stud; 11 – installation palms

The oscillation frequency is equal to the rotation frequency of unbalances, and the amplitude of oscillation depends on the mass of the oscillating parts and static moment of the mass of unbalances, under which the product of the mass of unbalances on the eccentricity of the mass is implied, i.e. on the distance from the rotation axis to the center of mass of the unbalances.

Since the external unbalance in each pair has four spline chases, it can be set at different angles relative to the internal, changing the overall eccentricity of the mass of double unbalance. When the axis of the unbalances are the same, the eccentricity of the mass is the greatest, but while increasing the angle the eccentricity of the mass decreases as the common centre of mass for separated unbalances lies in the middle of the line connecting the centers of mass of each unbalance, and is separated from the axis of rotation at a shorter distance, as a leg of a right triangle is less than the hypotenuse. Accordingly, the static moment of mass of unbalances and the resulting by them forcing force decrease.

Vibrator ИВ -70 at a frequency of 2800 min^{-1} and the respective setting of external unbalances generates a forcing force equal to 2; 2.5; 3.15 and 4 kilo newton. The power of electric motor is carried out from the circuit of alternating three-phase current by voltage 220/380 V and frequency 50 Hz. Electric motor power is 0.4 kW, weight of the vibrator is 20 kg. With the direct service of the vibrator, for example, at the surface compaction of the concrete mix, the voltage 220/380 V represents a great danger for staff. In this case, the same device vibrator IV-68 is used, developing at a frequency of 1400 min^{-1} forcing force of 5 kilo newton and having an electric motor, which is powered by the voltage of 36 V from a step-down transformer. External vibrators are attached to the casing, chutes, hoppers. Their electric motors are fed directly from the circuit by voltage 220 / 380V and do not require step-down transformers, which is especially comfortable at use of a large number of vibrators.

Internal (deep) vibrators are used for compaction of the concrete mix at the manufacturing of large collapsible construction elements, saturated with reinforcement, as well as in the construction of monolithic ferroconcrete structures. Their work is very effective, because the jar of the vibrator influences directly on the concrete mix. Internal vibrators are manufactured with an integrated electric motor which rotates the unbalanced shaft in the jar, and with carried out electric motor sending rotation to vibro element by flexible shaft.

18.4 Exploitation and maintenance of manual electric machines

Safety engineering rules of electrical machines, as well as maintenance and test after repair are general for all kinds of machines and equipment with electric drive. However, there are additional requirements for manual electric machines, particularly to machines with double insulation (class II):

manual machines (outside working hours) should be stored in a dry heated rooms;

calculation of working time must be organized;

before introducing into work the machine must be inspected, it is necessary to check on idle the clearness work of the switch, and good condition (resistance) of isolation by megohmmeter on 500 V at the switched on switch;

it is forbidden to introduce machine into work, and also it is necessary to stop work in case of detection of cracks on the jar parts and the arm; damage of the brush holder lids, indistinct work of switch; damage of the plug connection, cable or protective tube; circular fire on the collector; smoke or odour character for burning isolation;

it is forbidden to work in the rooms of highly explosive or chemically active environment which destroy isolation, as well as in open areas during precipitation (rain, snow);

the operator must comply with the maximum allowable duration of work and to prevent overload, in excess of specified in the passport, and must not expose the machine by blows. You should keep in mind that at increase of the load (reinforcement of feed) in excess of the rated on the machine with an asynchronous motor having a hard characteristic, it will tip over (stop), which will cause ultimately premature combustion of the winding. Collector motor has a "soft" characteristic, so it will reduce the turns. At it the consumed power increases, as a result the motor will overheat in excess of permissible limits, and the performance will decrease as the spindle turns will not be optimal;

it is necessary to monitor the temperature of the motor jar, which should not exceed 60°C (practically, if the palm of the hand cannot bear touch to the motor jar, it overheated in excess of norm);

every day after termination of work the machine must be cleaned from dirtying and if it is necessary to pull fixing details.

During exploitation of manual machines with double isolation it is necessary to remember that:

It is not possible to ground them;

the use of personal protective equipment (rubber mats, dielectric gloves) is not required;

it is possible to make work indoors and outdoors with dirt, concrete, asphalt, metal, wood and other floors, as well as on steel structures, in boilers, pipes, etc.;

the machine can be used at temperatures from -35 °C to + 35 °C and relative humidity up to 90% at a temperature +20 °C;

every 50 hours of work it is recommended to clean the collector and the brush mechanism from the accumulated coal dust; blow through the car clean compressed air under pressure up to 0.15 mPa;

it is necessary to control machines periodically. Control of manual machines with double isolation is necessary to take after every 100 hours of work, but not less than once in three months. Control is also required at each change of brushes.

while controlling the machine with double isolation is disassembled and:

remove accumulated conducting dust; check by megohmmeter working and extra isolation (resistance each of them must be less than 2 mega ohm); conducting dust is removed with compressed air at a pressure up to 0.15 mPa and wipe the insulating surfaces by technical cloth soaked in gasoline;

inspect jar parts, a conducting cable and plug connection;

after assembling the machine the test for electric strength of isolation of the machine at the switched on switch by voltage 2500 V is taken, frequency 50 Hz for 1 min on a high-voltage installation, for example the instrument УПУ-1М (when the test electrodes are applied to one of the contact of male plugs and to the metal parts of the machine, accessible to the touch during work);

if at controlling the machine shows any defects, it should be put in repair.

Repair of machines is carried out only in specialized workshop by trained for it personnel. After repair each machine is subjected to tests under laboratory conditions.

Key findings

1. EMM is brought into motion with a specially made for it an electric motor or an electromagnet constituting with the machine a whole.
2. The industry produces EMM of three classes of implementation on voltage and isolation.

Control questions

1. What electric motors are used in EMM?
2. What kind of isolation is used in EMM?
3. Explain the device of a manual electric drilling machine?
4. Explain the scheme of the electromagnetic perforator?
5. Explain the structure and principle of action of electromechanical unbalanced vibrator ИВ-70.
6. What are safety regulations at work with EMM?
7. What are safety regulations at the EMM maintenance?

19 ELECTRICAL CURING OF CONCRETE AND SOIL

Key concepts: electrical curing of concrete (electrode, induction, infrared, indirect), electro steaming, electrical curing of the soil.

For thermal treatment of concrete about 70% of the time required for the manufacture of ferroconcrete products is lost. Therefore, in practice there are various methods of heat treatment of concrete, providing time reduction for this operation. Such methods include electric heating in the molding process of the concrete mixture and at a location of its forms. With preliminary electrical curing of the concrete mixture and form the time of heat treatment of concrete can be significantly reduced to several hours.

19.1 Electrical curing of concrete

There are several methods of concrete electrothermotreatment.

Electrode through – electrodes are placed vertically in the layer of concrete. It is used for collapsible and monolithic foundations, walls, blocks.

Electrode peripheral – electrodes are fixed in the casing in special boards or thermosetting layer of sawdust moistened with a solution of sodium chloride ($NaCl$). It is used for one-sided heating of the structures of a thickness exceeding 20 cm or double-sided – up to 20 cm.

Induction – the product is placed in an alternating magnetic field formed by the electric winding and is heated by the eddy currents. It is used at warming up collapsible and monolithic structures: columns, beams, frames, bores, pipes, etc.

Infrared heating hearths using incandescent bulbs, tube, wire and other heaters. It is used for warming up the monolithic structures of complex configuration and at drying of products.

Indirect warming up of the low temperature heater with tubular, flat, strings and other heaters mounted in the casing, or mats. It is used for all kinds of products.

Infrared warming up in the cells with emitting surfaces. It is used to manufacture plates and panels.

Electro warming up of the concrete mix outside the form in which the mixture in the hot condition is laid in the shape. It is used for the construction of monolithic structures and in the manufacture of products in the factory.

Heating by the electrode way may be made only by an alternating current as a constant current causes irreversible chemical reaction that alters the structure of the concrete. The resistance of concrete depends on its resistivity, the surfaces of contact with the concrete and the distance between the electrodes. The electro conductivity of concrete, depending on the content of moisture on measure of the curing of concrete is reduced. To maintain estimated heat various admixtures are placed in the concrete. These include CaC1, and NaC1 accelerating the curing and reducing the resistance of concrete.

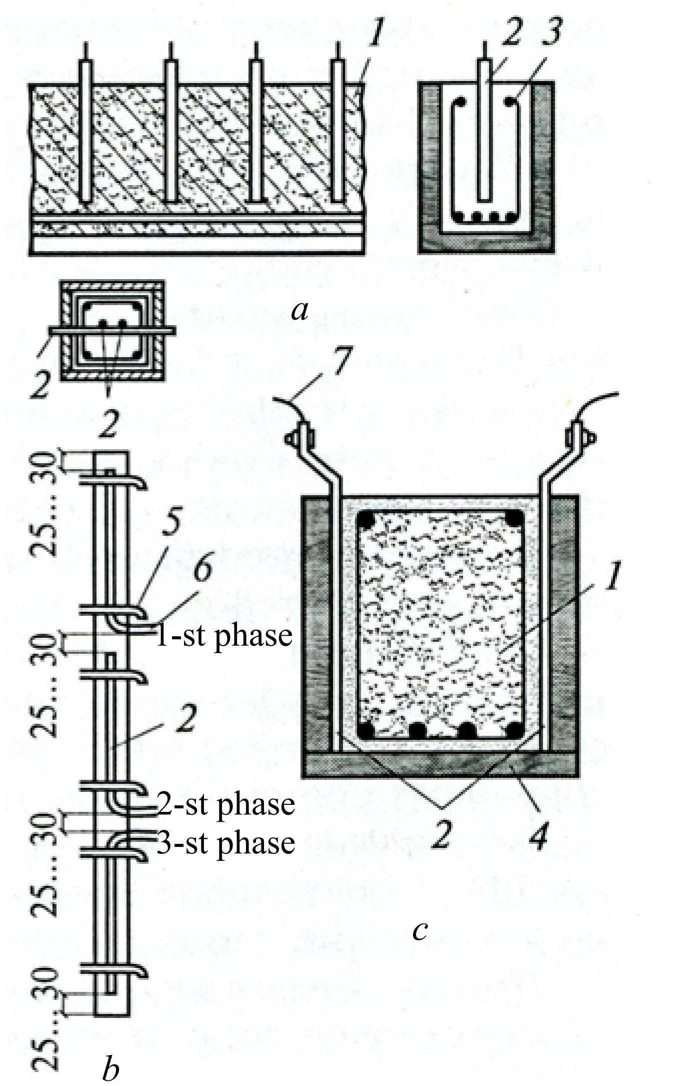


Figure 19.1: Electrode method of electrical curing of the concrete: a – with the help of rod electrodes; b – string; lamellar; 1 – concrete; 2 – electrodes; 3 – armature; 4 – casing; 5 – hooks; 6 – leads of electrodes for connection to the supply circuit; 7 – wire

Electrodes used at warming up are divided into lamellar, flat, rod and string ones (fig. 19.1). For the first two types roofing steel is used, for the others bars with a diameter of 5–12 mm are used. Lamellar electrodes have the form of plates, wholly or partially closing the opposite plane by the thickness of the product. String electrodes are fixed along the axis of long-length construction. The distance between the electrodes is taken in accordance with figure 19.1.

Especially important electrical curing is in the winter. The freezing of the concrete during process of curing reduces its strength more than it has been frozen before. Upon reaching the concrete 50–60% of the strength the freezing does not affect the final strength of the concrete. Warming up mode is chosen on this basis.

Long mode is used for massive structures; accelerated mode is used for light-weight constructions, and the intermediate mode is used for the other constructions.

In addition, there is stepped mode with multiple isothermal steps (it is used for monolithic and collapsible beforehand loaded structures), mode “isothermal warm and cooling-down”, at which warming up is done by the method of electro warming up out of forms (it is used for monolithic structures), self-regulating mode with constant voltage of current of warm (for massive structures), pulse mode with alternating disconnection of current. The maximum temperature of process is limited to the maximum allowable for a certain brand of concrete (usually 40–80°C).

The electrical curing of concrete is produced using special three-phase transformers with oil cooling with change of voltage by steps.

Along with the three-phase single-phase ones can be used, including welding, transformers, connected by three-phase groups. Welding transformers are calculated on repeatedly short-term mode, and their continuous load during warming up should be 60–70% of nominal.

The electric wiring from step-down transformers to place of electrical curing is performed only by isolated wires with anchoring on wooden supports, on insulators or special portable supports in the form of a trestle. In order to avoid loss in lines transformers should be located as close as possible to the electrodes in place of warming up of the concrete. Contacts of connecting wires with electrodes and other conductors are provided with screws or removable connection terminals.

Before switching on of the secondary circuit transformer is checked in the idle mode, at this the possibility of regulation of secondary voltage is checked also. During work it should be monitored using ammeters or measuring benders for uniform load on phases.

As the concrete hardens, its resistance decreases. To maintain the current the voltage on output of the transformer should be decreased.

Measuring of the temperature of the concrete during electrical curing is performed by thermometers in wells, prepared in advance, at least three in each structural element. In the first 5–6 hours the temperature is measured every hour, in the subsequent 18 hours it is measured each 2 hours and at other times it is 2 times per shift.

For electrical curing of concrete, brickwork, and plastered surfaces external sources of heat are used.

The electrical curing of products using external heat sources, unlike the electrode warming up is due to the heat that is released beyond construction and is transferred to the concrete through intermediate materials (sawdust, water, air, steam, metal wall) or by radiation. Since the external electrical curing is lower than the electrode one, it is used only for products of a complex configuration.

Warming up of the concrete by electric furnaces of resistance. In electric resistance furnaces used for indirect heating of concrete, the heating element is a wire with a high resistivity, for example, nichrome or fechral wire. The simplest reverberatory furnace designed for electrowarming concrete products of small thickness, is a wooden chute of parabolic shape of the grooved planks 40 mm thick.

For direct electrical curing inventory electroboards are used. The electrical board is a frame from the corners, inside which there is a heating steel or nichrome wire on the steel sheet with a thickness of 1 mm on a layer with thin isolation. On the top the wire is isolated by sheet asbestos and mineral wool of thickness of 20...30 mm, protected by a sheet of roofing iron. At warming up several of these boards are connected serially. Concrete temperature is regulated by inclusion of different numbers of electroboards in circuit.

For warming up of ferroconcrete pipes and rings cylindrical furnaces with a heating coil wound on a piece of asbestos-cement pipe are used.

Electrical curing using thermosetting layer. Warmed structure is covered with a layer of sawdust moistened to increase the electrical conductivity of a weak solution of salt (3–5 %). The electrodes of the round or flat steel included in the circuit are laid in sawdust. At turning on of the current sawdust is heated and heat is transferred to construction. To increase the electroconductivity of sawdust they are slightly pressed after filling up. The sawdust of temperature is maintained at level 80–90°C. The necessary power during the period of the rise of temperature is 7–8 kW per 1 m^3 of concrete and the electric energy consumption on warming up of the same amount of concrete reaches 120–160 kW·h.

Warming up with thermal form with heating elements. At electrical curing of collapsible ferroconcrete products panels of conducting rubber are used. The electroconductivity of such rubber is created due to the large content of carbon-black. Heating panels have an average conducting layer with a thickness of 2 mm, in which electrodes are made from brass mesh or strips and two outer layers of usual rubber 0.5 mm thick.

An important advantage of this method is sealing of the product during its warming up, eliminating the evaporation of moisture from the concrete.

Electrosteaming. Steam environment in the curing room is created by using electric heating elements like coils or electrodes installed in the bottom of the chamber. The power of the heating device is determined by calculating the 7–8 kW per 1 m^3 of heated products. The heater is supplied with line voltage. To accelerate the heating of the product it is recommended to use 0.5% solution of common salt instead of water.

Method of electrosteaming of ferroconcrete products is used for products of complex configuration.

Electrical curing by infrared. At infrared warming up, unlike other methods of external heating of the concrete, a direct transfer of thermal energy from the radiation source to the heated product is provided. As sources of infrared radiation incandescent lamp of type ЗН with a power of 300 and 500 W at a voltage of 127 V and 220 V are used. A regular incandescent lamp by power: 200–500 W are also used.

The power required for electrical curing of the concrete, which is one of the main factors determining the choice of electrical equipment and the calculation of the supply circuit, depends on the module's surface of warmed construction, temperature of warming up, the outdoor temperature, the initial temperature of the concrete, the construction of casing, the effectiveness of the heat insulation and particularly on the speed of warming up of the concrete.

As a rule, transformers are used as power sources for electrical curing. At electrothermal treatment of concrete auto-transformers and induction regulators are used in order to maintain a given mode of transformers with step voltage regulation. Transformers are chosen by power and voltage.

Complete transformer substation of outdoor installation КТП-ОБ-63У1 are designed for electrical curing of the soil and concrete. In complete transformer substation transformer ТМОБ-63 is installed by rated power of 63 kVA.

Approximate calculation of electric energy consumption (*W*) and the required power (*P*) for electrical curing of the concrete is produced respectively by the formulas:

$$W = W_{SP}\,V; \qquad P = \rho\,V, \tag{19.1}$$

where: $\boldsymbol{W_{SP}}$ – specific electric energy consumption, kW·h/m^3;
$\boldsymbol{\rho}$ – the specific power 1 m^3 of concrete, kW/m^3;
$\boldsymbol{V}$ – the volume of concrete, m^3.

Specific electric energy consumption W_{SP} (kW h/m^3) at warming of the concrete is presented in a variety of ways:

electrode method of warming up 80–120;
induction - // - 120–150;
infrared - // - ....100–200.

Table 19.1: Specific power for electrical curing of concrete structures, kW/m^3

The air temperature, °C	Temperature of warming up, °C	
	40	80
0	7,7...9,3/15,6...18	8,3...10,4/16,2... 19,2
-5 -30	8,2...10,1/16,1...18,9 8,6...10,9/16,5...19,7	8,7...11,2/16,6...20 9,1...12/15...25

Note. Up to line the limits of specific power at the speed of increase of temperature during heating 10°C/h, after a line – 20°C/h.

19.2 Electrical curing of the soil

The electrical curing of the soil is used in areas with a free electric power (for example, near a powerful hydraulic units).

There are several ways of electrical curing of the soil. The electrode method with direct connection of installations of electrical curing to the existing power supply circuit of voltage up to 380 V is the most convenient, cheap and safe way.

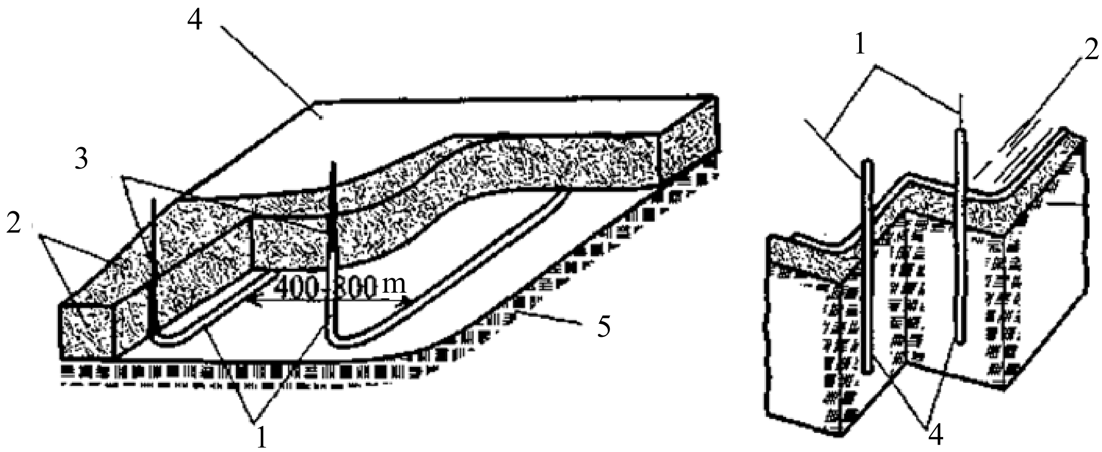
The electrode method: the electrical current by voltage of 220 V or 380 V passes through the soil. The electrical conductivity of a soil depends on its moisture degree and temperature, the presence in the soil of solutions of salts, acids, from the structure of the soil, etc. The complexity of the structure of the soil and physical phenomena and the changes associated with thermal processes, significantly affect its electrical resistance.

The surface of warmed area of the soil is filled in 15...25 cm by layer of sawdust soaked in an aqueous solution of salt (common salt, calcium chloride) or hydrochloric acid, with the purpose to initially conduct current and to make the soil warmer; even at a voltage of 380 V current practically does not pass through the frozen soil.

At electrical curing of the soil by horizontal electrodes (fig. 19.2, *a*) the heat is transferred to the soil mainly from the heating layer of sawdust, and participation of the soil in circuit of current is relatively small. Only a small top layer of soil adjacent to the electrodes, included in the circuit is the resistance which generates heat.

The horizontal electrodes are used to warm the soil in a small (up to 0.5–0.7 m) depth, as well as in cases when the vertical electrodes are not applicable due to the low electrical conductivity of the soil or inability of driving in them into the soil, mixed with, for example, with crushed stone.

Warming by vertical electrodes (fig. 19.2, *b*) is more efficient and it is used when the depth of frozen soil is more than 0.7 m, and also at the small contact between the horizontal electrodes and the soil. In hard soils (clay and sand with a moisture more than 15–20%), but then the electrodes are hided to a depth of 20–25 cm, and then are jumped as measure of thaw of the soil. At thaw to a depth of 1.5 m it is recommended to have two sets of electrodes – short and long. On measure of thaw of soil short electrodes are replaced by long. Warming the soil to a depth of 2 m or more should be produced by steps with periodic removal of the thawed layer (at switched off current).



a *b*

Figure 19.2: Electrical curing of the soil: *a* – by horizontal electrodes: 1 – electrodes; 2 – sawdust moistened with a solution of salt; 3 – flow pipe of electric energy; 4 – upper heat insulation (pitch paper, mats, etc); 5 – soil; *b* – by vertical electrodes: 1 – feed of electric energy; 2 – sawdust with a heater; 3 – soil; 4 – electrodes

At vertical electrodes soil is filled with sawdust, which initially serve as a stimulus to the warming up of the top layer of soil. On measure of thaw soil layers are included in the circuit, then sawdust only reduce heat loss of thawed out soil. Instead of sawdust stimulus can be served flutes punched out by chisel between all electrodes at a depth of 6 cm and filled with salt solution. At covering of the surface of warmed soil with a layer of dry sawdust, as practice shows, the device of the flute gives very good results.

In order to save electric energy and maximize the use of the power middle positive temperature of warmed soil should not exceed 2–5°C, at certain points – 15–20°C; warming up should be areas with breaks in feeding their current.

The required power and energy consumption at the soil temperature 15°C on average for each cubic meter make up 3.5 kW at a electricity consumption of 30 kW·h.

In recent years, warm of soil is developed and put into production in northern areas by electric energy by voltage up to 10 kV.

Compared with a voltage 380 V use for electrical curing frozen soil electrodes with a voltage of 10 kV allows you to speed up production of works and reduces their cost. The required number of electrodes is reduced and the distance between them increases. Amount of preparatory work on the immersion of the electrodes into the ground is reduced. The main amount of heat is produced near the electrodes, the rest of the soil warms up to negative temperatures close to 0°C due to the thermal energy accumulated near the electrode. The soil is heated from the bottom up, this reduces the heat loss to the atmosphere. Warming up of the frozen soil to a temperature -1.5...-0.5°C creates very favorable conditions for its development by earthmoving mechanisms, as in the complete thaw the soil freezes to the bucket of excavator or breast of bulldozer. In addition, moistened soil that has been removed in the blade, gets frozen, which causes additional costs when it is loaded in a vehicle or in a reverse its filling.

19.3 Safety engineering at electrical curing

Safety engineering at electrical curing of frozen soil at a voltage up to 10 kV is the complete elimination of the contact people and animals in the zone of dangerous voltages. Multiple measurements the sizes of voltages in soils at an working voltage on the electrodes 10 kV were set; safe step voltage 40 V was observed, typically at a distance of 9–10 m from the electrodes involved in the warming up of the soil. The voltage was measured between the vertical control electrodes, embedded in the ground at 1.5 m and on 5–7 m.

Barrier of dangerous areas of electrical curing provides location at a distance 15 m from the extreme working electrodes multilevel soft rope barrier, mounted on wooden inventory support. The ends of the ropes are attached to the levers of end switches mounted on supports. End switches are triggered at the tension of any of the horizontal rope barriers, which causes disconnection of voltage brought to the installation of electrical curing of the soil.

Key findings

1. Electrical curing of the concrete is an effective method that reduces the time of manufacturing of concrete products and raises their quality.

2. It is convenient to use welding transformers for electrical curing of concrete.

Control questions

1. What methods do electro thermal treatment of concrete use?
2. What electrodes are used in the warming up of concrete?
3. What transformers are used for electrical curing of the concrete?
4. How is the warming up of the concrete by electric furnaces of resistance carried out?
5. What are the peculiarities of the technology of electro steaming of concrete?
6. What are the features of electrical curing by infrared rays?
7. How is electrical curing by horizontal ground electrodes carried out?
8. How is electrical curing by vertical ground electrodes carried out?
9. What are the basic rules of safety engineering?

20 ELECTRIC LIGHTING INSTALLATIONS

Key concepts: luminous intensity, luminous flux, illumination, light sources, incandescent lamp, gas-discharge lamp, luminous tube lamp, mercury lamp, lighting fixture, lamp, method of specific power.

20.1 General information

Proper organization of electric lighting on the construction site is essential for the successful execution of construction assembly works, especially in the autumn-winter period when there is a reduction in daylight hours. Insufficient illumination of the workplace reduces productivity of work, makes worse the quality of work and, in addition, in many cases causes traumatism (accidents).

Adequacy of lighting and its quality are evaluated by the indicators which are determined by light quantities and their units of measure.

In the International system of units (SI) the ***luminous intensity*** (indicated by the letter I_v) is considered to be basic light quantity; the unit of measurement is the *candela* (abbreviated as *cd*).

The second, not less important, light value is ***luminous flux*** (indicated by the letter Φ_v); the unit of measurement is the *lumen* (abbreviated *lm*).

The adequacy of lighting in a particular plane or in a particular point is determined by the amount of ***illuminance*** (indicated by the Latin letter E_v); the unit of measurement of illumination is *lux* (*lx*).

According to the standards the required illumination for performing precise works in mechanical workshops is 100...150 lx and for reading is about 75 lx.

The minimum quantity of illumination required for certain industrial, office and home rooms is set by the construction norms and rules. On their basis standards of electrical lighting of construction and assembly works are developed (tabl. 20.1).

Lighting can be general, local and combined. The general lighting is divided into uniform and localized.

In general uniform lighting the whole room or outdoor area is illuminated, lamps are installed evenly. At general localized lighting there is a big illumination in some areas of the room or external area. In such areas additional lamps are installed or they are placed more frequently. At local lighting the work surfaces are only illuminated. In the combined lighting general and local lighting are used.

In terms of construction both a common (uniform and localized) and combined lighting of work places (the last in repair factories, workshops and other similar areas) are used.

Besides the usual, work lighting, emergency lighting providing a minimum illumination is arranged. A separate power is arranged for emergency lighting.

Table 20.1: Norms of illumination of construction and assembly works

Name of areas of territory and working operations	Illuminance, E_v, lx	The plane in which illumination is standardized	Note
The territory of the building ground in the area of production	2	Horizontal at ground level	Lighting should be multilateral
Highways on construction areas with intensive movement	3	the same	—
Railway lines at the construction territory	0,5	—	—
Earth works, made by earthmoving mechanisms	5...10	Horizontal, vertical	—
Assembly of building structures	25	Horizontal, vertical	—
Concreting	25	On the surface of the concrete	—
Finishing works	50	On the working	—
Crane and rigging works	10	Horizontal	—
Assembly and installation of construction mechanism	50	The same	Additional portable lighting tools are necessary
Installation of equipment	50	On the working surfaces	The same

20.2 Light sources and lighting fixture

20.2.1 Light sources. Incandescent lamps and gas-discharge lamps are used as light sources for construction and industry. They are divided into mercury lamps of low pressure namely luminescent and mercury lamps of high pressure namely ДРЛ lamps.

In incandescent lamps light energy is obtained by heating of thin tungsten filament with passing through it an electric current. The thread is placed in a glass bulb filled with an inert gas; there are also designs of incandescent lamps in which the filament is placed in a vacuum - from a bulb air is evacuated. Ignited (at a temperature of about 3000°C) filament shines brightly. The bulb of lamp is mounted on a metal threaded cap, with which the lamp is screwed in the chuck, served for its connection to the wires of the electrical circuit. Incandescent lamps are produced for voltage 220, 127, 36 and 12 V. As a rule, lamps for 220 V are used in constructions. They are produced by power from 15 to 1500 watts. Incandescent lamps for voltage 36 and 12 V are produced by power from 11 to 100 watts.

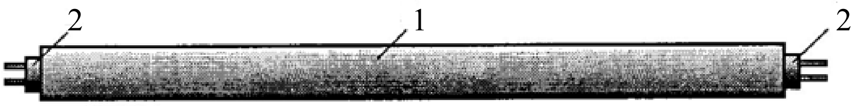
At the reduction of voltage vs nominal luminous flux and luminous efficacy of incandescent lamps are significantly decreased. The voltage rise above 105% rated significantly reduces the lamp life.

The effect of ***gas-discharge lamps*** is based on the electric discharge in rarefied gas environment. In comparison with incandescent lamps they are characterized by lower consumption of electric energy.

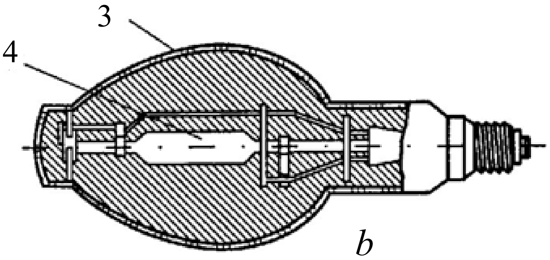
Luminous tube lamp (fig. 20.1, *a*) is a long (about 450 to 1500 mm) glass tube with two caps on the ends, filled with rarefied gas namely argon and a small amount of mercury vapors. A layer of special composition namely phosphor is brought on the inner surface of the tube. Tungsten electrodes are soldered in the cap of the lamp. When the lamp is turned on in the electrical circuit between its electrodes in the mercury vapors in the tube gas discharge and invisible ultraviolet radiation occur under the influence of which the phosphor begins to glow – gives a bright visible light.

Fluorescent lamps are included in the circuit through a special starting and control devices (SCD).

Fluorescent lamps are produced by power 15, 20, 30, 40 and 80 watts, five types of chromaticity (colour) of the emitted light: CDL – day light designed for proper light transmission; DL – day light; CWL – the cold white light; WWL – warm white light and WL – white light.



a



b

Figure 20.1: Gas-discharge lamps: *a* – fluorescent; *b* – mercury; 1 – tube, 2 – cap; 3 – bottle of lamp; 4 – burner made of silica glass

On luminous efficacy of 1 watt of power all fluorescent lamps significantly (2.5...4 times) excel incandescent lamps. White light bulbs (ЛБ) have the highest luminous efficacy. They are recommended for lighting all industrial rooms, except those that are required the correct discrimination of color tones.

Mercury lamp of high pressure of type ДРЛ in appearance is similar to a large incandescent lamp. Its structure is shown in figure 20.1, *b*.

Unlike fluorescent lamps in the ДРЛ lamp electric discharge in mercury vapors occurs not in the entire flask, but in a small tube ("burner") made of quartz glass, transparent to ultraviolet rays (fig. 20.1, *b*). Under the influence of ultraviolet radiation of burner a special phosphor brought on the inner surface of the flask, gives a bright, slightly greenish light (close to white).

ДРЛ Lamps have a threaded cap and are threaded into the same chucks as the incandescent lamps. However, they are included in the circuit as fluorescent lamps, according to a special scheme using a special starting controller (SC) containing the choke, condenser, discharger, etc.

ДРЛ Lamps by power 250, 500, 750 and 1000 watts are produced. They are highly efficient light sources.

20.2.2 Lighting fixture. Well-organized lighting, first of all, should provide sufficient illumination for the human eye could easily without being tired distinguish all the details required for this work. In addition, the lighting should be as uniform as possible, without sharp shadows; the light source should not be directly visible to the eye (in order to avoid glare).

Lighting fixture is served to create the required lighting conditions that meet the specified requirements.

Lighting fixture with placed in it lamp is called ***lamp***. The main types of lamps used in the construction conditions with incandescent lamps, fluorescent and ДРЛ are presented in figures 20.2…20.5.

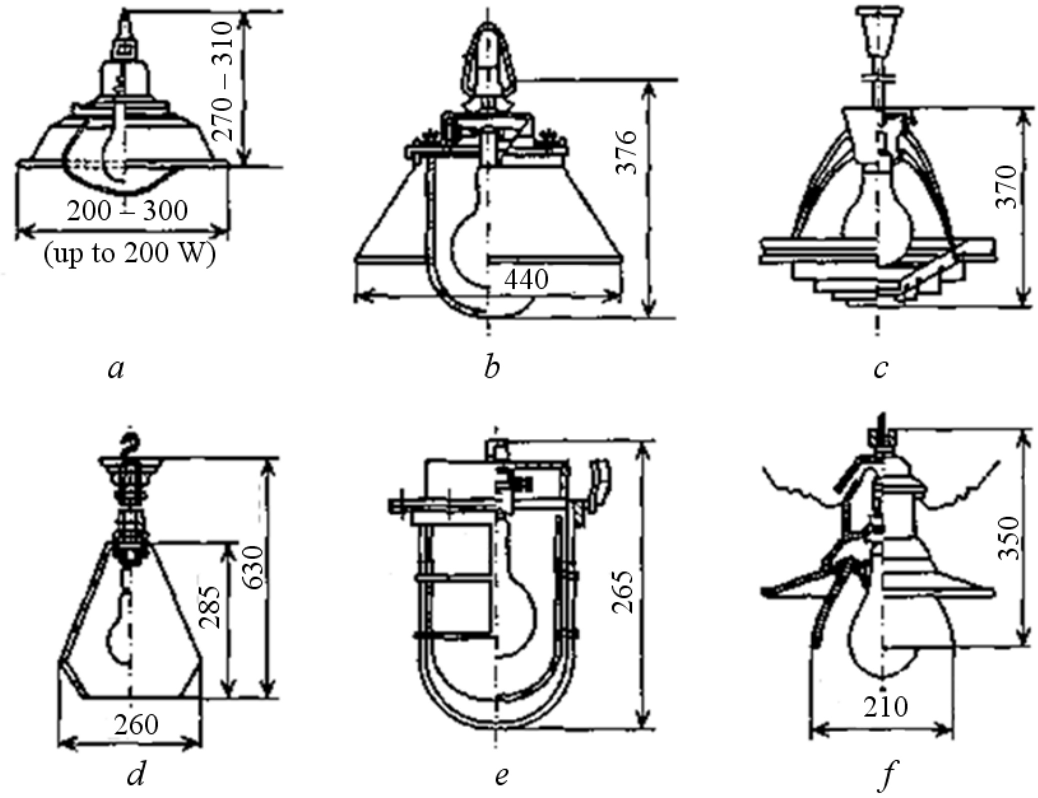


Figure 20.2: Lamps with incandescent lamps: *a* – "Universal"; *b* – industrial compact (IC); *c* – ring type ПМ-1*; d* – “Litsetta”; *e* – mining normal (РН 100); f – outdoor lighting of type СПО

Lamps are served for lighting objects placed at relatively short distances. Different types of projectors are used as lighting fixtures of long action. Floodlight projectors with conventional incandescent lamps with a power from 200 to 1000 W are used for lighting construction areas (see fig. 20.5).

20.3 Installations of electric lighting

Outdoor lighting of territory of construction areas is carried out mostly by projectors of floodlight. Projectors (mainly type ПЗС-35) establish by groups of

3...4 and more on the mast, the height of which depends on the strength of the light and power of projectors: the greater the luminous intensity of projector, the higher it must be installed.

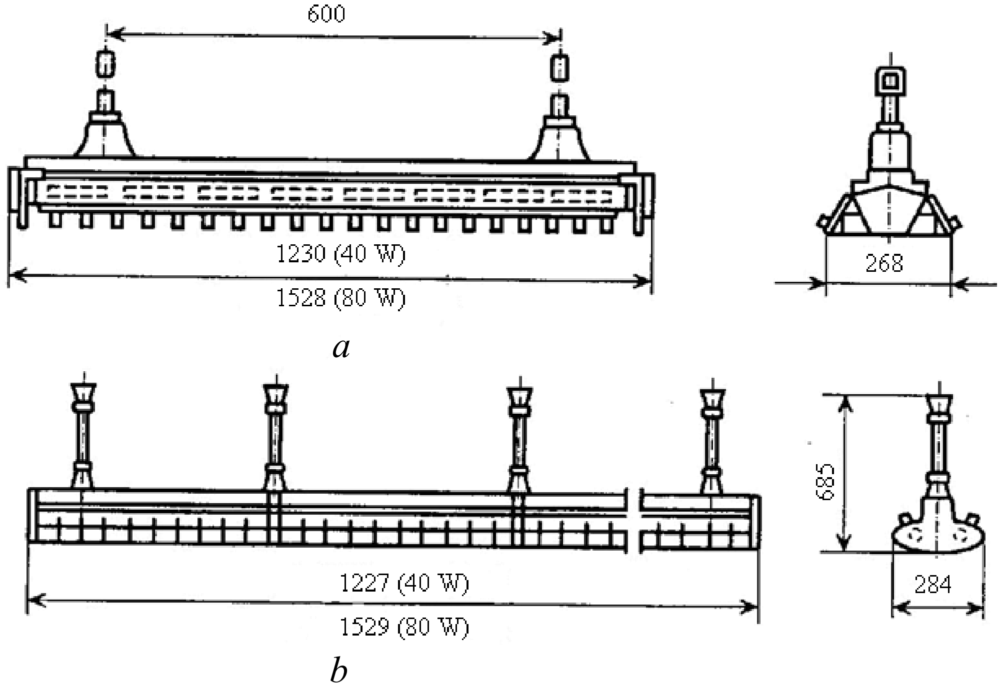


Figure 20.3: Lamps with fluorescent lamps: *a* – type ОДР and ОДОР with two lamps on 40 or 80 watts; *b* – type ШЛД with two lamps on 40 or 80 watts

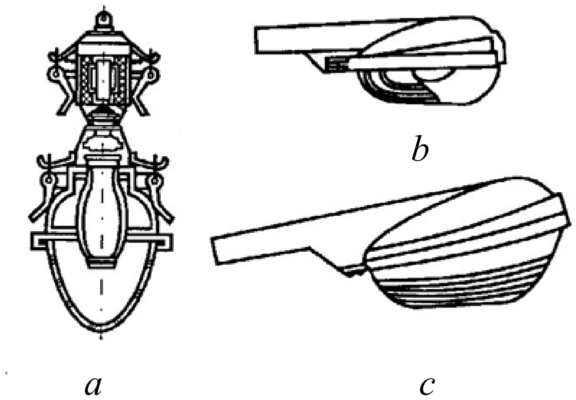


Figure 20.4: Lamps for mercury lamps of type ДРЛ: *a* – suspended type; *b*, *c* – console type

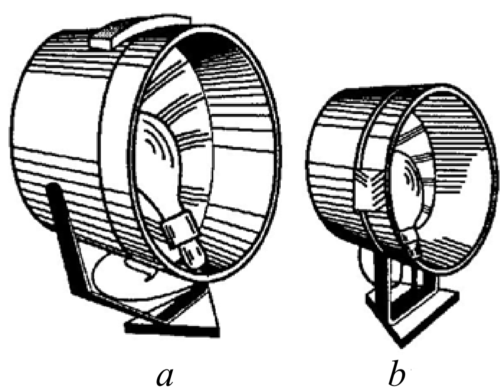


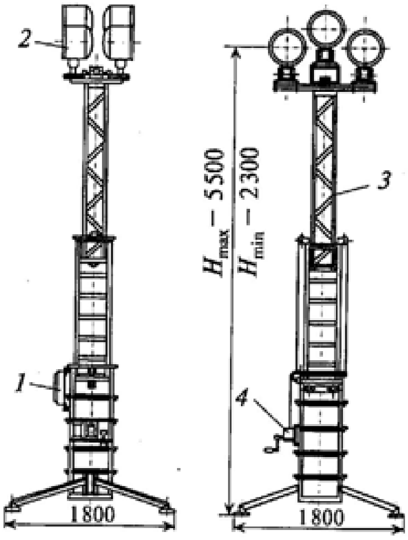
Figure 20.5: Projectors of floodlight: *a* – type ПЗС-45 with lamp 1000 W; *b* – type ПЗС-35 with lamp 500 W

Thus the optical axis of the projector is installed nearly horizontal - at an angle 8...15° down horizontally. Practically the following minimum height of installation of the projectors is accepted above ground level: ПЗС-45 with lamp 1000 W – 21 m, ПЗС-35 with lamp 500 W – 13 m.

It is advisable to use inventory portable searchlight masts. One of such constructions of such masts are presented in figure 20.6.

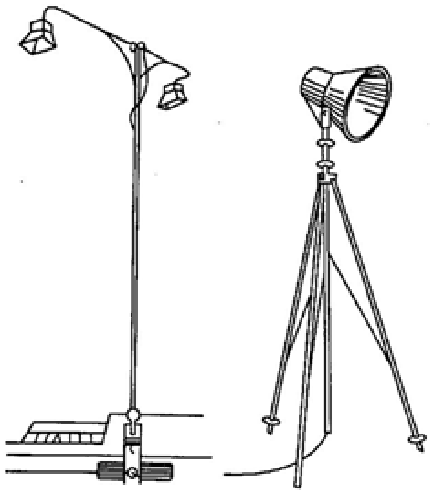
The distance between the searchlight masts is selected usually from 80...100 to 200...250 m (smaller numbers concern to projectors lesser power).

For additional lighting of working areas inventory portable rack with projectors of low power (lamps 200 W) or with lamps. Figure 20.7 presents such inventory rack. In addition, on excavators and other large construction machines also usually mounted projectors of low power, additionally broadcasting the work area.



a *b*

Figure 20.6: Inventory telescopic projector mast of type ТПМ-6: *a* – side view; *b* – front view; 1 – distribution board; 2 – projectors; 3 – metalwork; 4 – hand winch



a *b*

Figure 20.7: Inventory columns with lamps: *a* – column of telescopic type ИПСК-2 with two lamps for lighting of the work of brickwork; *b* – column of ПТС-2 with one lamp for local lighting of working areas

Lighting of roads that do not get into the area, illuminated by projectors, is carried out by lamps with outdoor lighting fixture СПО or "Universal" with lamp by power 200...300 W. Lamps hung up on brackets to the supports (poles) feeding them air line at a height of about 6 m above the ground and at a distance of 25... 35 m from one another. For the same purpose, and also for lighting of specific areas of the territory of construction can be successfully used lamps with more efficient mercury lamps such as HPMV.

General lighting of production enterprises of construction is carried out by either incandescent or fluorescent lamps using lamps (see fig. 20.2, 20.3).

For dry production areas lamps "Universal" are used, ring ПМ-1 with incandescent lamps or lamps of the types of ОД and ОДР with fluorescent lamps. Raw and dusty room are illuminated compacted lamps such as ПУ or РН with incandescent lamps; can also be used lamps "Universal". Lamps usually hang up at a height of 2.5...3.5 m above the work surfaces, the distance between them are taken approximately equal to twice the height of the bracket. Local lighting of work places in the workshops is performed using manufactured for these purposes lamps АМО-60 and the other with incandescent lamps. For temporary lighting of buildings the same above mentioned the lamps with incandescent lamps are used. In addition to the general lighting, as a rule, local lighting of work areas are used using portable inventory racks and suspended devices with lamps. On the safety engineering requirements temporary lighting of buildings is recommended to arrange at a lower voltage (36 V), obtained from step-down transformers. If temporary lighting has a voltage of 220 or 127 V, lamps, according to the rules should be suspended at a height of not less than 2.5 m from the floor or flooring; this should be paid special attention to the state of isolation of the wires of the temporary wiring, the integrity of the insulating shell of chuck etc.

20.4 Standards of illumination and simplified methods of calculation of lighting installations

In terms of construction in some cases (in the absence of the project of electric lighting) can meet the need to determine the number and power of lighting fixture (projectors or lamps) to create a desired norms of illumination in one or another area of the territory of construction or in any room. For these purposes the most convenient simple calculation method, which is called the ***method of specific power***.

Let us consider the order of calculation by this method. Data on illumination standards that should be followed at calculation are given in the table 20.1.

Outer searchlight lighting. The number of projectors needed to illuminate a given area, according to the method of specific power is determined by the formula:

$$N = \omega \, E \, S/P_L \,, \qquad (20.1)$$

where: $\boldsymbol{\omega}$ – specific power of lamps of projectors per 1 m^2 of illuminated area and 1 lx of illumination (it should be taken: for projectors ПЗС-35 is equal to 0.25÷0.4 W/(m^2 lx), and for projectors ПЗС-45 is equal to 0.2÷0.3 W/(m^2 lx));
$\boldsymbol{E}$ – is the illumination, lx (see tab. 19.1);
$\boldsymbol{S}$ – is the area subject to the lighting, m^2;
$\boldsymbol{P_L}$ – power of lamp of projector, watts.

Example 1. It is necessary to illuminate the construction area of size 205 x 100 m.

According to the table. 20.1 the illuminance (E) of area at ground level should take equal to 2 lx.

The type of projectors will accept ПЗС-35 with lamp 500 W.

Find the illuminated area: S = 205 x 100 = 20500 m^2.

Specific power of projectors (ω) is assumed to be 0.30 W/(m^2-lx).

Determine the number of projectors by the formula (19.1): n = 24,6 pieces.

You should install 24 projectors on six masts with a height about 13 m, placing the mast on the contour of the area.

Interior lighting. The calculation of the general lighting within manufacturing, administrative- managerial and other buildings, and in rooms of under construction buildings is performed by a similar method of specific power.

The data necessary for carrying out of simple calculations, are given in table 20.2.

Table 20.2: Specific power of general uniform lighting by lamp "Universal" without gobo with incandescent lamps

Calculated height, m	**Area of room, m^2**	**Specific power ω' (W/m2) at minimum illumination (lx), equal to**					
		20	**30**	**50**	**75**	**100**	**150**
2–3	25–50	6.4	8.6	13.8	19.5	24.5	35
	50–150	5.3	7.2	11.4	16.3	21	29
	150–300	4.7	6.4	10.2	14.3	18.5	26
3–4	30–50	6.4	8.9	14.5	20.5	25	35
	50–120	5.5	7.6	12	17	21.5	29.5
	120–300	4.7	6.6	10.2	14	18	25

Here is the process of the calculation using these tables. Define on table 20.1 the amount of illumination corresponding to the specified conditions. Choose for this room type of lamp, plan (in accordance with the dimensions of the room) the estimated height of suspension of lamps. Then on the table 20.2 for this type of lamp, the estimated height of the suspension, the area of room and the required luminance value of the specific power in W/m^2 is found. The estimated height of the suspension (denoted hp) is called the height of the suspension of lamp over the illuminated work surface (lathe, workbench, table).

Multiplying the found value of the specific power on the area of room, the total power of the lamps of lamps required for this room is calculated:

$$P_{\text{total}} = \omega' \cdot S, \qquad (20.2)$$

where: ***ω'*** – specific power of lamps of lamps, W/m^2;
S – area of room, m^2.

After that, knowing the standard of power of lamps, suitable for this lamp, select the number of lamps and lamps power.

Example 2. It is necessary illuminate workshop of area 190 m^2. The height of the suspension of lamps above the floor should not be less than 3,5 m.

For lighting let's choose incandescent lamps. Choose for workshop lighting the lamp "Universal"; the height of suspension above the floor 3.5 m. Taking the height of the illuminated surfaces (workbenches) above the floor in 0.8 m, we find the calculated height of the suspension of lamp h_p equal to 3.5-0.8 = 2.7 m.

According to the table 20.1 find for workshop minimum illumination of 50 lx (less the norm for the group "accurate work").

According to the table 20.2 find for these conditions the value of the specific power – 10.2 W/m^2. The total power of the lamps for lighting of workshop is determined by the formula (20.2):

$$P_{\text{total}} = 10.2 \cdot 190 = 1938 \approx 2000 \text{ W}.$$

Should be set 10 lamps with lamps 200 W (2 rows of 5 lamps).

Key findings

1. Electric lighting installation are an important item of equipment of construction areas and construction enterprises, providing the necessary standards of illumination of work places.
2. In practice, general, local and combined lighting of work places is used.
3. As light sources incandescent lamps and gas-discharge lamps are used.
4. Gas-discharge lamps compared to incandescent lamps are characterized by a lower consumption of electric energy.

Control questions

1. In what units are luminous intensity, luminous flux and illumination measured?
2. What types of lighting of work places are used in conditions of construction?
3. What light sources are used in the construction and building industry enterprises?
4. What are the features of fluorescent lamps?
5. What types of lamps are used in practice?
6. In what cases are projectors used?

21 ELECTRICAL EQUIPMENT OF ENGINEERING SYSTEMS OF BUILDINGS

Key concepts: system of air conditioning, split system, elevator, electroconvector.

21.1 General information

Modern residential, public and industrial buildings are characterized by saturated engineering systems, providing efficient management of operation of buildings. A significant role in them various electrical installations are performed: conditioning and ventilation of air, heating of buildings, humidifiers and air cleaners, elevator and lifting installations, installations of electric heating, etc.

High-rise buildings and shopping malls become more and more widespread, representing a complex structural system with a large number of engineering communications, with placing on one object variety of systems of technical life support of heightened complexity. Many high buildings are multi-functional and contain either a single core functional element (residential, administrative office, hotel) or two main functional element (office and residential, office and hotel, residential and hotel). In addition, in high-rise buildings supporting functional-structural elements are, designed to serve being in them people, such as car park, technical rooms, swimming pools, trainer rooms, hall for game in bowling, bathhouses -saunas; ambulatory and medical offices, etc. Also in high-rise buildings objects of citywide destination can be: shops, restaurants, cafes, buffets, financial and banking institutions, various offices, etc.

The main components of engineering systems of buildings are: electro, heat and water supply, water treatment, sewerage, ventilation, climatic equipment, air conditioning, electric heating, security and fire systems, refinement of drains, ionizers and air cleaners, lighting and some other.

An important role in the engineering systems of buildings plays ***power electrical equipment***, which includes *electric motors and motor-starting devices of technological, sanitation, fire-prevention equipment, lift-transport installation, harvesting mechanisms*, and also a power collectors of thermal, laboratory, medical equipment and other similar apparatuses and devices of electrical circuit with all sets of conductors, switching centers and wiring products.

In recent years, specialized organizations have shown their effectiveness, which performs the installation, maintenance and repair of technological equipment of engineering systems of buildings (installations of air conditioning, lifts, fire-prevention installations and other), which is often very useful for technical and economic considerations. In addition, in large public buildings engineering services are organized whose employees carry out the operation and maintenance of these electrotechnical installations.

The following ones are some groups of electrical equipment of engineering systems of buildings.

21.2 Systems of ventilation and conditioning

The task of systems of conditioning and ventilation of air- creation, regulation and automatic maintenance of comfort microclimate indoors: temperature, humidity, cleanliness, speed of motion of air etc.

Air conditioning is provided by the complex of hardware, called air conditioning system (ACS). ***In the composition of ACS it is included: technical means of the air intake, the preparation of its parameters (filters, heat exchangers, humidifiers or dehumidifiers), movement (blasts) and its distribution, and means of automation, remote control and monitoring.*** ACS of large public, administrative and industrial buildings are served, as a rule, by complex automated control systems.

Central ACS. Central ACS are used for multistory, functional buildings, rooms in which are united by a common task (hospitals, office buildings, industrial rooms, archives, storehouses, and so on). Such systems are supplied with heat (delivered by hot water, steam or electricity) or cold (delivered by with cold water or a refrigerant) and electricity to ensure operation of ventilators, pumps, etc. Central ACS allow effectively to maintain the desired temperature and relative humidity; provide effective noise and vibro extinguishing, which is important in areas with high requirements on acoustics (radio, TV studios, recording studios, and so on).

Local ACS. Local ACS usually based on autonomous and nonautonomous conditioners that are installed directly in the serviced areas. Among the advantages of such a system – easy installation and mounting, can be installed in already built residential and administrative buildings, in separate rooms of the building, for example, if the mode of consumption of cold/heat in them differs sharply from most other areas. In addition to improve parameters (recirculation) of already having in the room air, there is also a ***split system, providing intake and supply of purified air from the outside***.

In figure 21.1 an example of the air conditioner is shown where asynchronous motor M1 is used for drive the blast giving fresh air into the cooler, and asynchronous motor M2 for drive of the circulation pump of refrigerant.

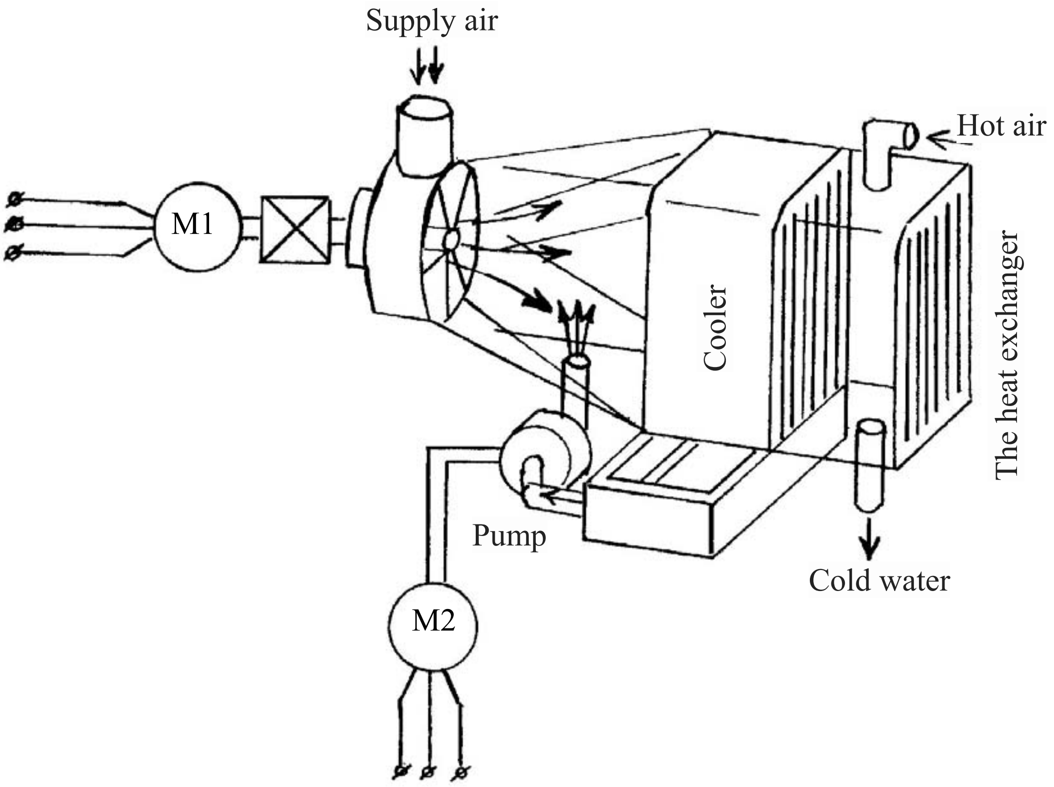


Figure 21.1: The structural elements of air conditioning

21.3 Elevators and escalators

Lift *is a stationary load-lifting machine of periodic action, intended for lifting and lowering of people and loads.* Transportation of passengers is carried out by a separate parties through certain time, meanwhile movement alternates with stops for landing and drop-off of people. The elevator is the most common lift of periodic actions.

To passenger pick-and-place machines funiculars are. These machines are used usually on steep slopes. So, in Yalta funicular is used for lifting the mountain AI-Petri, in Kiev for lifting on Vladimir Hill.

For pick-and-place machines of continuous action escalators are. landing and drop-off of passengers from such machines is done without stopping the latter in the process of work. With significant and intensive passenger traffics, typical for most of public buildings, subways, ports, railway stations and department stores the most widely used escalators are.

However, where the passenger traffic is relatively small and inconsistent (residential buildings, office buildings), elevators are installed. The purpose of the elevators can be a passenger, cargo, cargo-and-passenger, hospital and special. Speed elevators are divided into low-speed – up to 1 m/s, high-speed from 1 to 2.5 m/s and rapid – 2.5 m/c to 9.2 m/s.

Simplified passenger elevator with electric drive is a cabin, suspended by steel ropes in a vertical shaft. Installed in the machine room the winch winds the ropes on the drum. The cabin moves along the guides attached to the walls of the mine. Modern lifts are a fairly complex systems with a load capacity from 50 kg (100, 150 and 250 kg – half cargo) up to 5000 kg. They are equipped with complex systems of automatic devices that prevent an emergencies.

In business centers, banks and other public buildings common practice has become to install lifts, allowing to serve large passenger traffics during arrival and departure of employees. It's not necessary to increase the number of lifts or their load. Today, such problems can be solved also by using special software provided on some models of elevators of leading manufacturers. So, there are special programs to reduce the time of waiting for the arrival of the elevator.

Trade and exhibition complexes are often equipped with panoramic elevators. Panoramic elevator, intended for viewing, a different configuration are possible to have. And to create a maximum of review glass panels from floor to ceiling are often used.

Competition on the lifting market forces companies to constantly improve the quality of their products. This is contributed the modernization of production, the installation of a modern high-tech equipment, the development of new designs elevators. The expansion of nomenclature of enterprises-manufacturer provides to customers with a large selection of lift equipment. So, along with the model and serial projects plants produce lifts for individual orders. For example, in connection with the bringing into action new building norms many manufacturers produce elevators for transportation of fire brigades. In addition, the elevator control systems can be implemented on different hardware components that provide the desired mode of operation of the elevator depending on the type of building in which it was installed.

21.3.1 Electrical equipment of elevators on the composition and functions are similar to the electrical load-lifting machines (chapter 17.3). The voltage from the power source is moved in the machine room of the elevator through the introductory device, which should switch off the power to the drive motor, control circuits, alarm and lighting of the elevator car. For power circuits, signaling and repair of lighting step-down voltage transformers with rated voltage primary windings 220 and 380 V, the secondary winding – 24 and 36 V are used. Powers of step-down transformers 250 – 1500 VA.

To protect electric motors from overload and short circuits automatic switches with thermal and electromagnetic releases are used.

To turn on the power circuits of electric motors and brake electromagnets contactors AC / DC are used.

Figure 21.2 as an example scheme fragment of power circuits of freight elevator is shown. The diagram shows: M – two-speed asynchronous motor, EmB – electromagnetic brake, CD – power contacts of the contactor “down”, CU – power contacts of the contactor "up", CHS – power contacts of the contactor "high speed", CLS – power contacts of the contactor – "low speed".

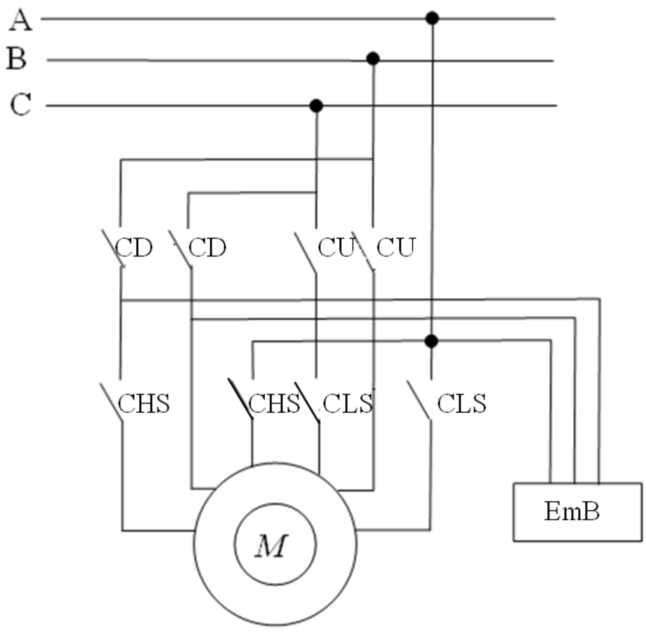


Figure 21.2: The power circuits of the freight elevator

21.4 Elements of water supply systems

Electric motors of service pumps, pump hot water systems and circuits of fireproof water pipeline are related to the electrical equipment of water supply systems. In most cases two pumps (sometimes three or four) with asynchronous motors are installed in water supply networks: the first is a working pump, the second is a backup pump. The mode of work of the electric motor is determined by the purpose of pumps. So in hot water supply systems temperature and pressure of the water are maintained automatically by using various sensors that automatically turn on pumps. And, depending on environmental conditions, the mode of work of the electric motor can be prolonged or repeatedly- short-term. The electric motors of fireproof systems are switched on in emergency situations and work up to elimination of fire (long mode).

Figure 21.3 illustrates the scheme of control by two commercial pumps that are installed in central heating units and serve a group of houses and other buildings in the neighborhood. The pumps are switched on in turn, providing uniform wear of both units. The scheme works as follows.

When the contact of minimum pressure SP1 closes the relay 1K is activated which closes its contacts 1K6 and applies a voltage to the coil of the starter 1KM. The latter puts the pump into operation. Simultaneously, the relay 1K by the contact 1K3 feeds voltage to the coil of the relay 3K, which is self blocked and prepares the circuit of the relay 2K. At repeated closing of contact SP1 the relay 2K is activated which now includes a second pump and de-energizes the relay 3K. Thus, at each closing contact SP1 the relays K1 and K2 are activated in turn, and, as a consequence, the motors of the first M1 and second pumps M2 are alternately turned on. The shutdown of the working pump is carried out by the relay 4K, which is activated at closing contact of maximum pressure SP2.

If a working pump does not raise the pressure in the network, then contact SP1 is not opened, and then with a small time delay provided by the relay 1KT, the relay 1KV or the relay 2KV is activated that switches off the defective pump and switches on the backup one.

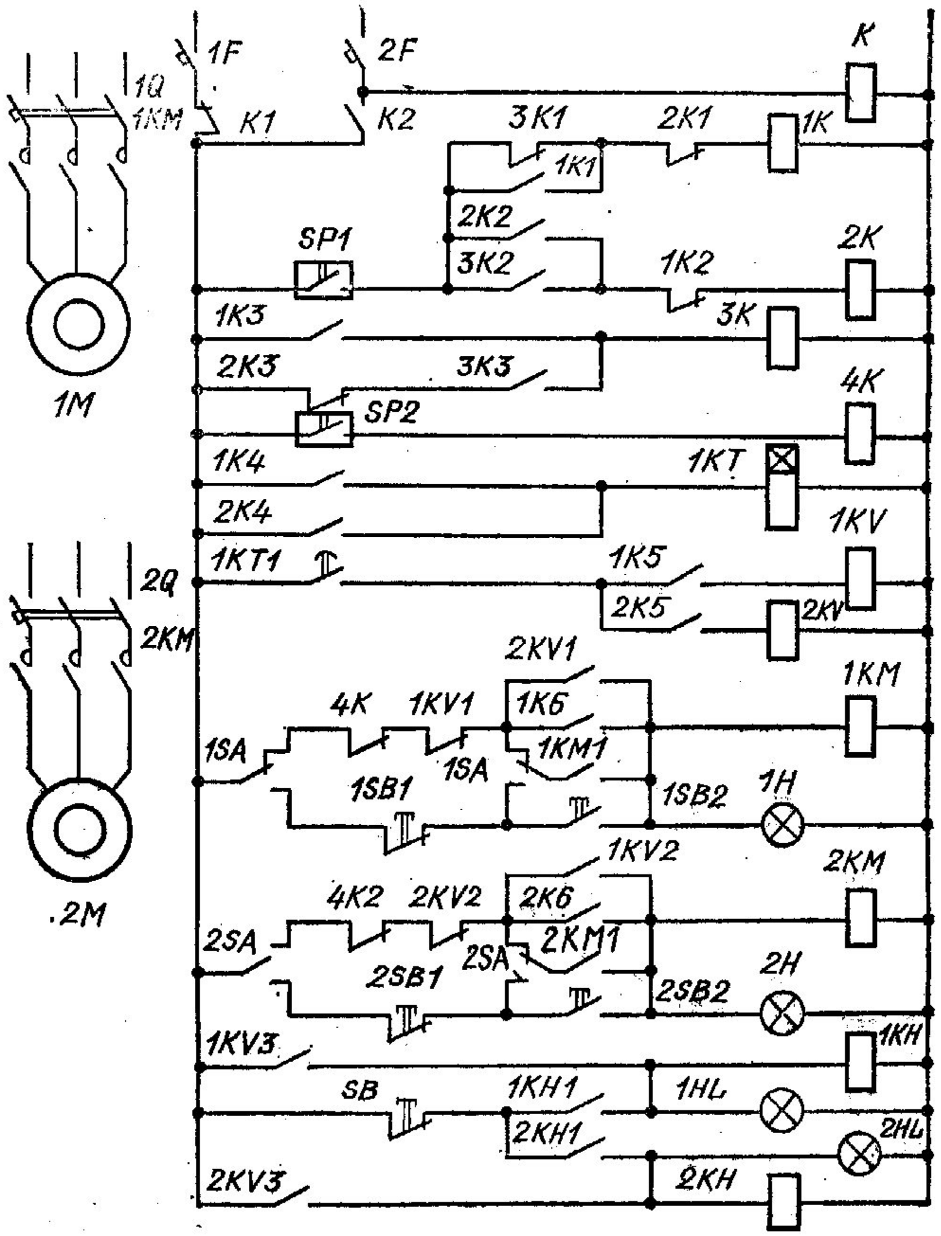


Figure 21.3: The control scheme of pumps of water drink

The work of pumps is indicated by a green light bulbs 1H and 2H, but the accident of pumps is indicated by red light bulbs 1HL and 2HL.

The relay K is used for automatic switching of power of control circuits at disappearance of voltage in the power circuits of one of the electric motors.

For the implementation of fixing and repairing work the pump control is provided by local buttons 1SB1, 2SB2. Translation into local control is accomplished by using switches 1SA and 2SA.

21.5 Electric heating

Quite a long time electric heating was considered to be expensive, flammable and environmentally harmful. Progress in the field of manufacturing of convectors and thermostats, the appearance of new types of electrical heaters, namely cable floor heating and infrared ceiling heaters, radically changed the performance characteristics of modern electric heating. Equipment intended for residential rooms, is environmentally clean (the temperature of the heating elements is insignificant and they do not change the humidity), fireproof, has a corresponding class of protection against hitting of electric current, works silently and does not emit any harmful substances. The electromagnetic field of these devices is at a background level. Modern electrical heating systems of buildings meet the most hard requests on ecology and safety.

Tremendous opportunities for economical and rational use of electric energy appeared in electroheating.

In each heated room a thermostat is set to regulate the temperature in a wide range (5–30^o C). It should be noted that the reduction of temperature at 1C ° reduces electric energy costs on 4–5%. When temperature modes change flexibly the economy can be achieved 30–50% in each separate room. Especially significant savings (up to 80%) are achieved for high (above 4.5 m) objects using infrared heaters or "warm floor". There is a possibility to program any schedule changes in temperature, it’s possible to disable any room, group of rooms or a whole floor. After a temporary disconnection of power heating starts to work without human intervention.

Residential electric heating is very convenient as an addition to an existing central heating system. It is essential for cold weather in the spring-autumn period, when the central heating is turned off. During critical periods in winter at considerable cooling when the central system does not provide the desired level of comfort, the stationary direct electric heating automatically provides the required temperature, meanwhile the operational costs are negligible. It is difficult to assess the value of additional backup electric heating in case of accidents and other emergency situations.

Special problems with heating arise in rooms with high ceilings (5-10 m) and unsatisfactory heat insulation. These are factory buildings, storehouses, motor pools, indoor sports and concert halls, exhibition and shopping pavilions, etc. For such structures the most efficient heating devices are infrared heaters, electric energy economy can be achieved 80%.

Modern heating systems are designed and assembled with the use of ***electric convectors***. The name "convector" reflects the principle of hot air distribution, namely natural convection. Heating of the air occurs when it passes through the heating element. The cold air passes through the heating element, gets warm and goes out through the jalousies, and its place is occupied by the room air. Thus, warm air circulates in the room, providing a fast and comfortable warmth. The convector heats the rooms not using the device for forced air circulation. It makes the convector more reliable and efficient compared to heaters with built-in blast. However unlike fan heaters, convectors, creating efficient movement of warm air, absolutely do not create noise when work.

The temperature of the air is controlled by a built-in thermostat with sensor. The user sets the desired temperature on the thermostat, and the air sensor measures the temperature of the incoming air and upon reaching the given parameters heating is turned off. When the temperature decreases in the room heating up turns on again. Thus, the heater maintains a constant given temperature. For some models of convectors the accuracy of maintenance of room temperature reaches 0.1 degree.

The thermostat of the oil radiators does not control the air temperature. It controls oil temperature. Therefore, the accuracy of maintenance of temperature is much smaller and is approximately 3–5°C.

Modern convectors (APPLIMO, SOLO, EURO PLUS and others) using the latest and unique technologies have the following advantages compared to traditional heaters (oil-filled radiators, fan heaters) and other convectors.

The COP of the heating elements reaches 95–99%.

There is a complete lack of noise during heating and cooling of the heating element. When oil radiator works it produces characteristic clicks ("gurgling") that occur during heating and cooling of the oil inside the device.

Effective heating of the air does not violate its natural moisture and does not burn oxygen.

A great resource of continuous work (some models up to 25 years).

High speed of output at working temperature (1–1.5 min). Oil radiators have up to 15 minutes.

The consumption of electrical energy depends on the heat loss of the heated room. According to average statistical data [62] when outdoor temperature is –20^{O}C and room temperature is +18^{O}C for heating 1 m^3 of space it is necessary 40-45 watts of electrical energy that is 120 watts of installed power on 1 m^2 with a ceiling height 250–270 cm.

Key findings

1. Modern high-rise residential buildings and shopping malls are equipped with complex life-support systems, basic elements of which are electrical equipment for various purposes.
2. Motors are used in the drive of ventilation systems, air-conditioning, elevators and lifts, water supply systems.

Control questions

1. What are the main components of engineering systems of buildings?
2. What elements form the power equipment of engineering systems of buildings?
3. What elements are included into air conditioning system?
4. What are the basic elements of the electrical equipment of elevators?
5. Explain the operation of the control circuit of pumps of water pumps?
6. What are the advantages of modern systems of electric heating?

22 ELECTRICAL SAFETY IN CONSTRUCTION

Key concepts: electrical safety, electric blow, electric burn, electrical sign, metallization of skin, photoelectric ophthalmia, warrant, order, protective equipment, protective earthing, grounding.

22.1 General information

In general, electrical safety is a *system of organizational and technical measures and tools designed to protect the people from the dangerous effects of electric current, electric arc, electromagnetic fields and static electricity.* In this tutorial, we are talking about the protection of personnel working with electric instrument, with electrotechnological installations and equipment in enterprises of construction industry and construction areas. The need to solve this problem is caused by the fact that in the case of passing electrical current through the body, the magnitude depends on the degree of the damage.

The importance of this issue due to the fact that in modern conditions labour productivity in construction, as in other fields of human activity, is largely determined by labor safety. More attention must be paid to electrical safety.

To solve this problem activities have been developed and they can be used individually or in combination in various technical ways and means (protective earthing, grounding, equipotential alignment, isolation of current-carrying parts, warning alarm, locks, safety signs; means of protection and others).

System of requirements and rules is described in [52].

22.2 The effect of electric current on the human organism

Electric current, acting on the human organism, can cause the following types of injuries: electric blow, burn, metallization of skin, electric sign, mechanical damage, photoelectric ophthalmia. Passing through the human organism an electrical current primarily affects the central nervous system. As a result, heart muscles and the respiratory system are broken. The degree of the damage depends on the strength and frequency of the current, as well as from the passage of current through the human organism (see tab. 22.1). Safety rules for absolutely dangerous current strength is accepted 50 mA.

Table 22.1: Degree of influence of electric current on human.

Current strength, mA	AC of frequency 50-60 Hz	DC
0,6–1,5	Start of feeling - weak itch, tingling of skin.	Is not perceptible.
2–3	The sensation of current spread on the wrist, slightly hand got a cramp.	Is not perceptible.
5–7	Pain, cramps in hands.	The itch. Sensation of the heat.
8–10	Hands hardly detached from the electrodes. Severe pain in the hands and cramps.	Aggravation of heat.
20–25	Hands instantly become paralyzed, it is impossible to tear them off from the electrodes. Very severe pain in the hands and chest. Breathing has difficulty.	Else more strengthening of heating, a slight contraction of the muscles of the hands.
50–80	Breath is paralyzed. Start of trembling of heart ventricle.	A strong sensation of heat. Contraction of muscles of the hands. Convulsions. Difficulty of breath.
90–100	Breathing paralysis and fibrillation through 1–3 s.	Breathing paralysis.

Electric blow leads to stimulation of living tissues:

1. electrical injury of I degree – convulsive contraction of the muscles without loss of consciousness;
2. electrical injury of II degree – convulsive contraction of the muscles with loss of consciousness;
3. electrical injury of III – loss of consciousness and breach of functions in cardiac activity or respiratory (quite possibly both);
4. electrical injury of IV degree – clinical death.

The degree of severity of electric lesion depends on many factors: the resistance of the organism, magnitude, duration, type and frequency of the current, path in the body, conditions of the external environment. The outcome of the influence of the electric current depends on the physical condition of the person. If he is sick, tired or is in a state of intoxication, mental distress, the action of the current is especially dangerous. Alternating current to 10 mA and DC – 50 mA. is safe for humans.

Electrical burn is a consequence of short circuits in electrical installations and stay of parts of the body (usually hands) in the field of light (ultraviolet) and thermal (infrared) radiation of the electric arc. It causes burns of III and IV degree with severe consequences – in contact person (directly or through an electric arc) with current-carrying parts with voltage exceeding 1000 V.

The electric sign – is a specific lesion caused by mechanical, chemical, or their combined effects of current. The affected skin is practically painless, there are no inflammatory processes around it. Over time, it hardens, and superficial tissues die off. Electric signs usually quickly cure.

Metallization of skin – is soaking of the skin by tiny vapor or molten metal particles under the influence of mechanical or chemical effects of current. The affected skin becomes hard surface and original color. In most cases, the metallization can be cured, not leaving on the skin traces.

Defeat of eyes by ultraviolet rays, the source of which is the electric arc, it is called ***photoelectric ophthalmia***. As a result of photoelectric ophthalmia the inflammatory process comes in several hours.

In accordance with Ohm's law the current is determined by the voltage and resistance of the circuit. Significant electrical resistance has only a superficial layer of human skin. This resistance depends on many factors (the skin's moisture, degree of expansion of the skin capillaries and others) and varies widely – from 800 to 100 000 Ohms. Resistance is sharply reduced, for example, in the case of alcohol. If you take the resistance of the body of human is equal to 1000 Ohms, then the dangerous one will be current at the voltage $U = I R_{hum} = 0.05 \cdot 1000 = 50$ V, the source should give the power $P = U I = 50 \cdot 0.05 = 2.5$ W.

If the power of source is significantly less than the specified numbers, the high voltage does not lead to the total defeat of the human body, but cause discomfort.

At fault of isolation of electrical installations uninsulated metal constructions can be under voltage. If a person touches a metal construction, that is under voltage, it is called the ***touch voltage*** U_{touch}.

In accordance with [40, 41], it is dangerous to human health the following voltage of touch are: in a dry room $U_{touch} = 65$ V; in damp areas with a relative humidity 75 % and conducting floors $U_{touch} = 36$ V; in particularly dangerous rooms (metal cabin, boilers, rooms with relative humidity 100%) $U_{touch} = 12$ V.

22.3 Classification of conditions of work according to the degree of electrical safety

The work performed in existing electrical installations in respect of security measures is divided into categories: at full taking off voltage; with partial taking off voltage; without taking off voltage stress near and on the current-carrying parts; without taking off voltage away from live parts under voltage.

According to the degree of electrical safety there are the following conditions:

Especially dangerous conditions of defeat people by electric current:

the presence of moisture (rain, snow, frequent sparge and coating of moisture ceiling, walls and objects inside the rooms);

the presence of chemically active environment;

the presence of two or more conditions of high-risk environments.

Conditions with increased risk of lesion of people by electric current:

the presence of humidity (steam or condensing the moisture is released in the form of small drops, and a relative humidity of more than 75%);

the presence of conductive dust (technological or other dust settled on the wires, penetrating inside machines and devices and being deposited on electrical installations, worsen the conditions of cooling isolation, but it will not cause danger of fire or explosion;

the presence of the conducting bases (metal, earth, concrete, brick);

the presence of high temperature regardless of the time of year and various thermal radiation (temperature exceeds 35 °C, short-term 40 °C);

the presence of possibility of simultaneous touch of the person to having a ground connection to the metal structures of buildings, technological equipment with one hand and the metal enclosures of electrical equipment – from other.

Conditions without high-risk lesions of the people by electric current: the absence of conditions that create high or special risk.

22.4 Activities to ensure safe conduct of work with electrical installations

22.4.1 Organizational activities. Work in electrical installations is performed on warrant, the order, in order of current exploitation.

Warrant – writing assignment set forth on a form of prescribed form, specifying the place, time of beginning and termination of the work, conditions of safe working practices, the team and the persons responsible for work security. The works must be performed in accordance with the warrant: with full taking off voltage; with partial taking off voltage; without taking off voltage near and on the current-carrying parts under voltage.

Order – task of work in the electrical equipment, recorded in the operational log. The order has a once-only character, is handed for one work and is valid for one shift or one hour depending on the nature of the work. Such actions must be performed: without taking off voltage away from current-carrying parts under voltage, with a duration of not more than one shift, unplanned short-term and small amount of work (up to 1 Hour), caused by industrial necessity, with full or partial taking off voltage and without taking off voltage near and on the current-carrying parts under tension; some types of work with full or partial taking off voltage in the electrical installations of voltage up to 1000 V with a duration of not more than one shift.

The works, performed in accordance with the order, without taking off the voltage away from current-carrying parts, include: cleaning corridors and business premises of open and closed switchgear; repairing of lighting equipment, lamp replacement (outside of the chambers and cells, at taking off the voltage from the lighting circuit, on which the works are carried out); care for brushes, rings and collectors of electric machines; the renewal of the inscriptions on the covers and others.

The work performed by order in case of production necessity, without taking off voltage near and on the current-carrying parts under voltage are: work on the enclosures of electrical equipment; measuring of current-measuring acaridans; change of the interlocks up to 1000 V; check of the heating of the contact by rod; determination of vibration of the bus bar by rod; phasing; the isolation control by rod. These works are short-term (up to 1 h) and not less than two working man.

Work in electrical installations up to 1000 V with full or partial taking off voltage, executed by order, includes: repair of magnetic starters, start buttons, automatic circuit breakers, cutouts, rheostats, contactors and similar equipment installed outside boards and assemblies; repair of individual power consumers (electric motors, electric heating coil); repair of free-standing magnetic stations and control units; change of the interlocks; repair of lighting wiring. Work must be carried out by two workers.

22.4.2 Technical activities. ***Conduct of works with partial or complete taking off the voltage in installations up to 1000* V**. All power and other transformers are switched off from the highest and lowest voltage. The disconnection may be made by switching device with manual control, the contacts of which are visible from the front side (if the contacts are not visible it is necessary to open the panels, doors of enclosures or take off enclosures themselves); contactors with automatic drive and remote control when taking off interlocks of operating current by detachment of ends of including the coil. In the absence of portable earthing additional measures are accepted, e.g. fuses are removed, insulated lining switch and machines are used, the ends of the supply lines are cut off, and so on.

Verification of absence of voltage shall be done by the voltage pointer. No voltage should be checked on all phases. The check is performed in the dielectric gloves. Verification of absence of voltage in circuits up to 1000 V is produced by a pointer voltage or a portable voltmeter. The use of indicating lamps is admitted at a linear voltage up to 220V.

Measures to ensure safety without taking off voltage. Workplace of an electrician should be positioned so that the current-carrying parts under voltage were in front of him or on one side. It is necessary to use protective equipment. Clothing of working people must be blind and have dropped and buttoned up sleeves. Headdress is mandatory.

22.4.3 Protective equipment. ***Protective equipment*** is appliances, apparatuses, and portable devices used to protect personnel against electric shock. Minimum standards for protective equipment sets for electrical installations with voltage up to 1000 V with their commissioning are as follows: voltage indicator – one; insulated pliers – one; dielectric gloves, overshoes – two pairs; electrician tools with insulated handles – at least two sets; portable ground – at least two; warning signs – at least two sets; insulating mats – two; temporary fencing – at least two sets; safety glasses – one pair; mask – one.

22.5 Protective earthing and grounding

One of the most important activities that significantly improve the safety of working in the construction is the proper device of protective grounding.

Protective grounding is the connection of metal parts of electrical equipment and installations via the earthing conductor with earth electrode having low resistance of connection with ground. It provides a safe voltage contact. Earthing and grounding conductors are called grounding devices.

Protective grounding is used with unearthed neutral.

Grounding is a compound of the metal parts which are usually not under voltage with multiple earthed neutral conductors. Grounding is performed in systems with grounded neutral and ensures reliable disconnection of installation at trunk closing.

Working conditions of electrical installations on construction areas under the open sky like humidity, precipitation, movable mechanisms with electric drive, and temporary electric circuits create an increased danger of lesion of people by electric current.

The reason for lesion of people by electric current can be not only touch to current-carrying live parts. If the isolation is damaged the jars of the electric motors or starting equipment and, most importantly, associated with them metal parts of construction machines and mechanisms get under voltage. Touching them, people in the absence of protective measures is stricken by an electric current. Such cases are especially dangerous because workers serving machines, not expecting danger, are constantly in contact with its metal parts. Protective earthing serves as protection against lesion of current in the transition of the voltage on structural metal part.

The following things are subject to earthing: metal parts of construction machines and mechanisms with electric drive, jars of electric tools, jars of electrical equipment and starting and control devices, constructions, frameworks and enclosures for electrical devices and other metal parts that may become under voltage as a result of damage of the isolation.

Protective grounding is performed differently depending on voltage and power supply system.

Circuits with voltage up to 1000 V (circuit 380/220 V) on construction sites are built on four-wire system known as a "star" with zero. In such networks, according to the rules, it is obligatory that neutral (zero point) of power transformers is earthed (system with dead-earthed neutral).

To do this, each transformer point (TP) is equipped with the earthing circuit which is connected to the zero point of the transformer, and hence the zero wire of the circuit. Resistance of earthing device of TP, according to the rules, should be not more than 4 Ohms (for transformers with power up to 100 kVA this rate increases to 10 Ohms).

Neutral conductor of overhead lines are repeatedly grounded every 250 m, as well as at the ends of lines and branches, including in the area of construction machinery operation like tower cranes, excavators, etc.

In circuits with dead-earthed neutral protective earthing is executed by joining of earthing installation parts to earthed zero wire of electric circuit (fig. 22.1). The effect of earthing (also called grounding) is that in the case of damage of isolation and the appearance of voltage on the jar of the equipment a short circuit in one phase of the transformer through the neutral wire is created, as a result of which in a damaged part of the installation is automatically switched off, as under the influence of the short circuit current immediately fuse of interlock is burnt out or automatic machine is switched off.

Earthing of jars of construction machines is carried out through the earthing strand of hose cable feeding the electric drive of machine. One end of the earthing strand is attached to the earthing bolt on the jar (or metalwares) of the machine and the other is attached to the earthing bolt on the jar of the starting box or connecting point through which power is moved to the machine. The jar of the starting box is attached to the zero wire of the circuit.

The earthing of tower cranes have some features. In addition to the earthing of metal structures and jars of electrical equipment of the crane, which is produced by the fourth strand of hose cable, it is necessary to earth crane tracks.

Wherein all joints between the bridge rail and rail between the two thread are performed by welding. The rails are attached (individual conductors) to re-grounding neutral conductor and the grounding bolt of switching item of a crane.

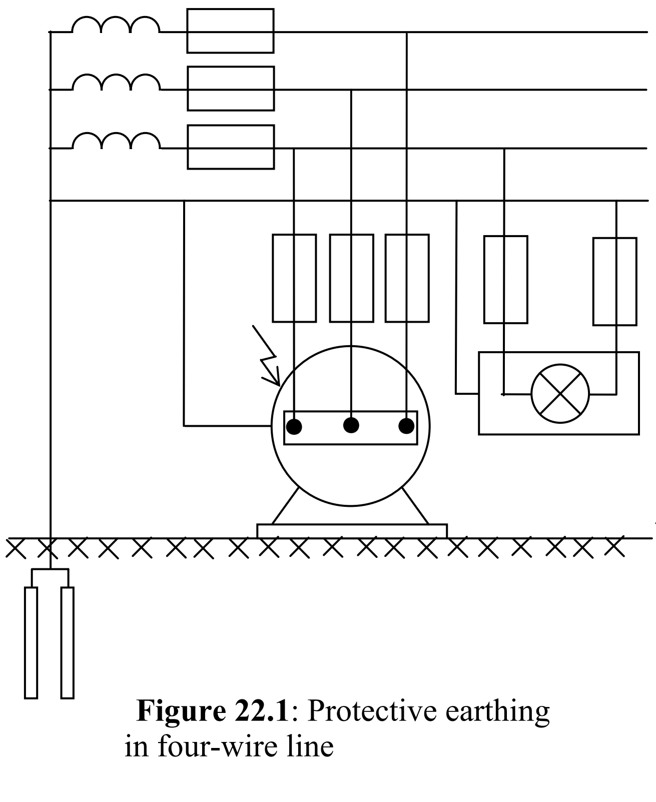


Figure 22.1: Protective earthing in four-wire line

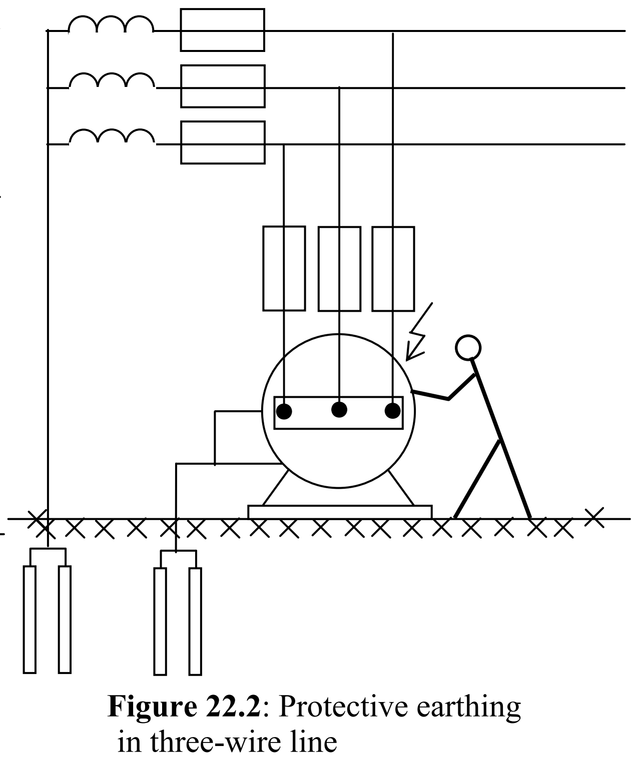


Figure 22.2: Protective earthing in three-wire line

In some cases, in electrical installations of construction companies can be circuits of three-phase current by voltage up to 1000 V (three-wire), working with an isolated (unearthed) neutral point of power transformers. Such circuits are sometimes built on extraction of peat and underground manufactures. In such circuits, as well as in all electrical installations for voltage above 1000 V (for example, at construction machinery with high voltage electric drive) protective earthing is carried out by the construction of local earthing devices (separate earthing contour) with the earthed parts of the equipment joined to it (fig. 22.2). Local earthing device must have low resistance meanwhile. According to the rules of the resistance of such earthing device in installations for voltages up to 1000 V it should not exceed 4 Ohms; in circuits with voltage 6...10 kV this value is determined by calculation, but should not exceed 10 Ohms. Action of earthing in this case is that it reduces to a safe magnitude of the voltage that may appear on the jar of the machine or apparatus with damaged isolation.

As artificial earth electrodes vertically hammered into the ground pieces of angle steel are used by section 50 x 50 mm, length 2...2.5 m or steel rods from round steel by diameter 12...14 mm, length up to 4...5 m (rod earth electrode). Separate earth electrodes are connected between each other in common earthing contour by the steel stripes by section 40 x 4 mm; connection is performed by welding. Earthing conductors must be connected to earth contour (to the steel strip) also by welding, and repair to the jars of apparatus and machinery by bolts. The required number of earth electrodes in the circuit is determined by the calculation. The smaller must be the electrical resistance of the earthing device, the greater earth electrodes are required. Meanwhile great importance is the nature of the soil in which earthing is executed. The most favorable soil is clayey, the least favorable is sandy and rocky. At the arrangement of earthing and during exploitation of electrical equipment a number of measurements are required (verification of correspondence of the earthing device to standards). For this purpose there are special devices for earthing measurement.

Such measurements are made by specialists in accordance with existing guidelines.

Key findings

1. The degree of lesion of a person by an electric current depends on the frequency of the current, the amount of current, the path of current passing through the human body, and the physical condition of the human organism.

2. The value of allowable contact voltage is determined by the degree of electrical safety conditions of staff.

3. To ensure safe conduct of works with the electrical installations organizational and technical measures are carried out and protective equipment is used.

Control questions

1. What current strength is absolutely dangerous to human life?
2. What is electric blow? What is the degree of injury severity from electric shock?
3. What is electrical burn? What is the extent of damage with electrical burns?
4. What is metallization of the skin? What are the degrees of lesion at metallization of the skin?
5. What is photoelectric ophthalmia and its impact on human?
6. What is the touch voltage? What values of touch voltage are considered dangerous to human life?
7. What are the working conditions for the physical safety?
8. What are organizational measures to ensure electrical safety of works?
9. What are technical measures to ensure the electrical safety of works on installations up to 1000 V?
10. What protective equipment is used in electrical installations up to 1000 V?
11. What is protective earthing? What is the principle of its action?
12. What is protective grounding? What is the principle of its action?
13. What are the main reasons of lesion of people by the electric current on construction areas?
14. How to perform earthing of construction machinery jars?
15. What are the characteristics of construction cranes earthing?
16. How does earthing act in electrical installations with isolated neutral?

SECTION VII.
TASKS AND EXAMPLES OF THEIR SOLUTIONS

23 ELECTRICAL DC CIRCUIT

Task 23.1. For the electrical circuits figure 23.1 it is necessary to determine the current I, the voltage at the connection terminals of the user U, the power of the source of power P_1 and the power P_L of the external circuit, the COP of the power supply. It is necessary to build external characteristic $U(I)$ of power source. Initial data for calculation: the EMF of the power source E, the internal resistance of the power supply R_0, the resistance of the load R_L – are given in table 23.1.

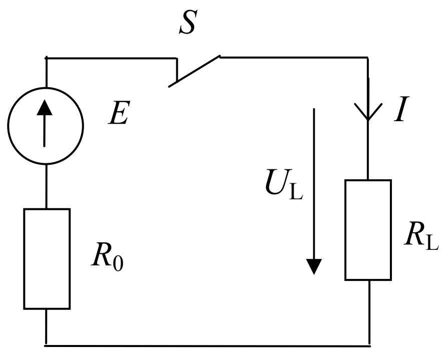


Figure 23.1: Scheme of a circuit to the task 23.1

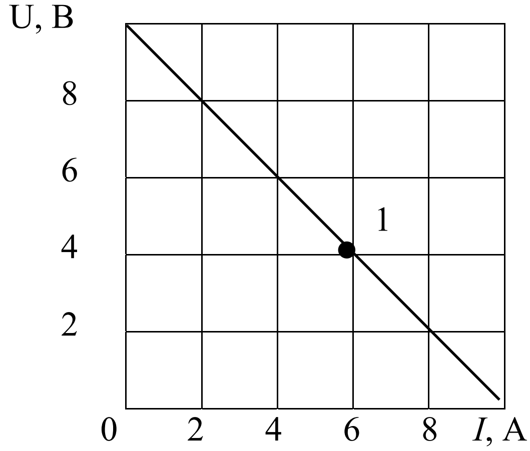


Figure 23.2: External characteristics of the power source

Table 23.1

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
E, V	10	10	10	10	12	12	12	15	15	15	15
R_0, Ω	1	1	0,5	0,5	1	1	0,5	1	1	0,5	0,5
R_H, Ω	4	6	3,5	5,5	5	7	7,5	4	5	5,5	6,5

The ***solution*** of the task for version K.

On the Ohm's law for the entire circuit let's determine the magnitude of the load current

$$I = \frac{E}{R_0 + R_L} = \frac{10}{1+4} = 2 \quad A.$$

The voltage at the connection terminals of the power source and the consumer:

$U = U_L = R_L \cdot I = 4 \cdot 2 = 8$ V.

Power of power supply: $P_1 = E \cdot I = 10 \cdot 2 = 20$ W.

The power of the external circuit (on load): $P_L = U_L \cdot I = 8 \cdot 2 = 16$ W.

The power loss inside source: $P_0 = I^2 \cdot R_0 = 2^2 \cdot 1 = 4$ W.

The COP: $\eta = P_L / P_1 = 16/20 = 0{,}8$ or 80%.

External characteristics of the power source *U(I)* with a constant values *E* and P_{idle}:

At idle (the switch contacts *S* are open) $I = I_{idle} = 0$; $U = U_{idle} = E = 10$ V;

In case of short circuit (the switch *S* is closed and

$R_H = 0$; $I = I_{sc} = =E/R_0 = 10/1 = 10$ A; $U_{SC} = R_{SC} \cdot I_{SC} = 0$.

The dependence of *U(I)* is linear, so the data of modes of idling and short circuit identify the external characteristics of the power source (fig. 23.1). On it and the values of the load current *I* we can determine the corresponding voltage *U* of the source. For example, for point 1, if $I = 6$ A, the voltage $U = 10 - 6 = 4$, as in the second Kirchhoff's law $U=E-R_0 \cdot I$.

The equation of balance of power (the power of power supply is equal to the power allocated in the form of heat in the resistors R_0 and R_0):

$$E \cdot I = R_0 \cdot I^2 + R_L \cdot I^2;\ \ 10 \cdot 2 = 1 \cdot 2^2 + 4 \cdot 2^2;\ 20 = 20 \text{ W}.$$

Task 23.2 For the electric circuit (previous task) to determine on which the load resistance R_H, the power source gives the most power and what is the COP of the installation? Draw a graph showing the change of useful power depending on the load resistance $P_L(R_L)$. To solve the problem in general.

Solution. The power allocated in the load resistance

$$P_L = R_L \cdot I^2 = \frac{E^2}{(R_0 + R_L)^2} R_L.$$

To determine the highest power delivered by the source of electrical energy, is taken as the first derivative of the power in the load resistance and is equal to zero:

$$\frac{dP_L}{dR_L} = \frac{(R_0 + R_L)^2 - 2 \cdot (R_0 + R_L) \cdot R_L}{(R_0 + R_L)^4} \cdot E^2 = 0.$$

After the conversion, get $R_L = R_0$, i.e., the source gives the most power in case of equality of resistances of load and its internal resistance. The maximum power delivered by the power source in the external circuit to the consumer at $R_L = R_0$ is equal to

$$P_{L.\max} = \frac{R_L \cdot E^2}{(R_0 + R_L)^2} = \frac{E^2}{4 \cdot R_0} \text{ W}.$$

The COP of the source

$$\eta = \frac{P_L}{P_1} = \frac{R_L \cdot I^2}{(R_0 + R_L) \cdot I^2} = \frac{R_L}{R_0 + R_L} = \frac{R_0 \cdot I^2}{2 \cdot R_0 \cdot I^2} = 0.5,$$

i.e. when $\eta = 50\%$. It can be shown that when $R_L = 0$ (short circuit) $\eta = 0$; on $R_L = R_0$ COP $\eta = 0{,}5$; on $R_L = \infty$, COP $\eta = 0$.

When change the load resistance R_L useful power is changed in accordance with the equation

$$P_L = \frac{R_L \cdot E^2}{(R_0 + R_L)^2} = \frac{E^2}{R_0}\left[\frac{a}{(a+1)^2}\right] = \frac{E^2}{R_0}K,$$

where $K = a/(a + 1)^2$; $a = R_0/R_L$. On $R_0 = R_L$, $a = 1$, $P_L = 0.25 \cdot E^2/R_0$.

Taking E and R_0 are constant, asking different values of R_0/R_L, it is possible to obtain a graph showing in the relative values the change in useful power $P_L = E^2/R_0$ in function of the load resistance. The obtained dependence is shown in figure 23.3.

Task 23.3 For tasks 23.1 to construct the dependence of the change of the voltage U at the connection terminals of the power source, the power of power supply P_1, power loss P_0 and COP η of installation from the current I in the circuit when the load resistance changes within $R_L = 0 \div \infty$. EMF of power source be considered constant.

The ***solution*** for version K.

The calculation procedure for R_L = 4 ohm it is given in task 1.1. For other values of the load, the calculation results are shown in table 23.2.

Figure 23.4 shows graphs of the change of the corresponding values in function of the load current.

Table 23.2

R_L	**0**	**0.5**	**1**	**2**	**4**	**6**	**7**	**9**	∞
***I*, A**	10	6.7	5	3.3	2	1.4	1.2	1	0
***U*, V**	0	3.3	5	6.7	8	8.6	8.8	9	10
P_1, **W**	100	66.7	50	33.3	20	14.3	12.5	10	0
P_2, **W**	0	22.2	25	22.2	16	12.3	10.9	1	0
P_0, **W**	100	44.4	25	11.1	4	2.1	1.6	1	0
η	0	0.3	0.3	0.7	0.8	0.85	0.88	0,9	1

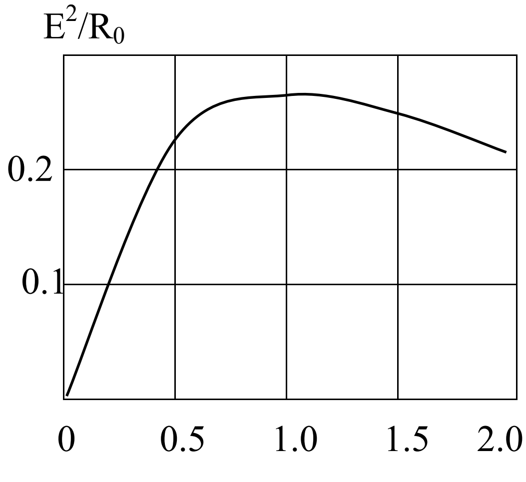


Figure 23.3: Dependence of the useful power from the resistance of load

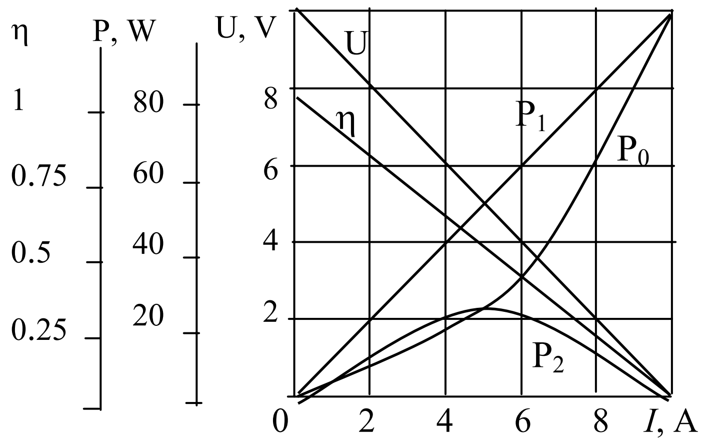


Figure 23.4: Calculation charts

Task 23.4 For electric DC circuit (fig. 23.5) determine the equivalent resistance of the consumer R_{12eq}, load current, the voltage at the connection terminals of the consumer U_{12}, the power of the consumer P_{12} and the power of the power source P_1, the COP of the installation, to make the balance of powers. Source data: the position of the switches S1 – S4, EMF of source E, the internal resistance of the source R_0, the resistance of the resistors R_1 – R_4 for the appropriate versions are listed in the table 23.3.

Table 23.3

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Closed switches	S1	S1	S2	S3	S4		S1	S2	S3	S4	
***E*, V**	12	10	10	10	10	10	8	8	8	8	8
R_0, Ω	0.5	0.6	0.6	0.6	0.6	0.6	0.4	0.4	0.4	0.4	0.4
R_1, Ω	6	5	5	5	7	7	6	5	5	5	5
R_2, Ω	6	4	5	5	5	6	5	6	5	6	6
R_3, Ω	5	6	6	5	6	7	5	5	4	4	5
R_4, Ω	5	5	5	5	7	6	5	5	4	5	4

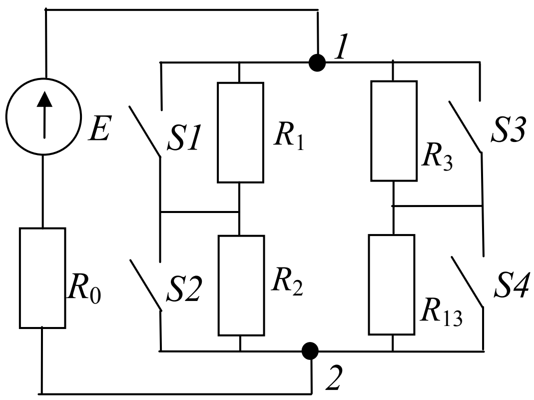


Figure 23.5: Scheme of a circuit for task 3.4

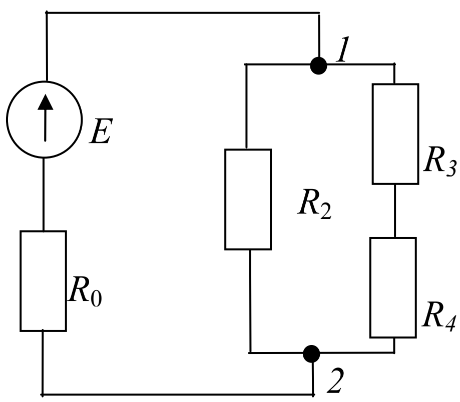


Figure 23.6: Scheme of circuit on version K

The ***solution*** for the version K.

Considering the fact that the switch *S1* for our version is closed, the scheme of circuit is shown in figure 23.6.

The equivalent resistance of the consumer

$$R_{12eq} = \frac{R_2 \cdot (R_3 + R_4)}{R_2 + R_3 + R_4} = \frac{6 \cdot (5+5)}{6+5+5} = 3.75 \ \Omega.$$

Load current

$$I = \frac{E}{R_0 + R_{12eq}} = \frac{12}{0.5 + 3.75} \approx 2.82 \ \text{A}.$$

The voltage at the connection terminals of the consumer

$$U_{12} = I \cdot R_{12eq} = 2.82 \cdot 3.75 \approx 10.59 \ \text{V}.$$

The power of the consumer

$$P_L = I \cdot U_{12} = 2.82 \cdot 10.59 = 29.9 \ \text{W}.$$

The power of power supply

$$P_{\sup} = I \cdot E = 2.82 \cdot 12 = 33.84 \text{ W}.$$

The COP of the installation

$$\eta = \frac{P_L}{P_{\sup}} = \frac{29.9}{33.84} \approx 0.88 \ .$$

The power balance

$$P_{SUP} = \Delta P_{SUP} + P_L = R_0 \cdot I^2 + P_L ,$$

$$33.84 \approx 0.5 \cdot 2.82^2 + 29.9 ,$$

$$33.84 \approx 33.85.$$

Absolute error of calculation

$$\Delta = 33.85 - 33.84 = 0.01 \text{ W}.$$

The relative error of the calculation

$$\delta = \frac{\Delta}{P_{average}} = \frac{0.01}{(33.85 + 33.84)/2} \cdot 100\% = 0.03\% .$$

The obtained accuracy is quite acceptable for engineering calculations.

Task 23.5 For the circuit (fig. 23.7) to determine the equivalent resistance R_{eq} and the total current I in the circuit and the voltage drop on the resistors R_1, R_2, R_8. The internal resistance of the source can be neglected. The initial data are given in table 23.4.

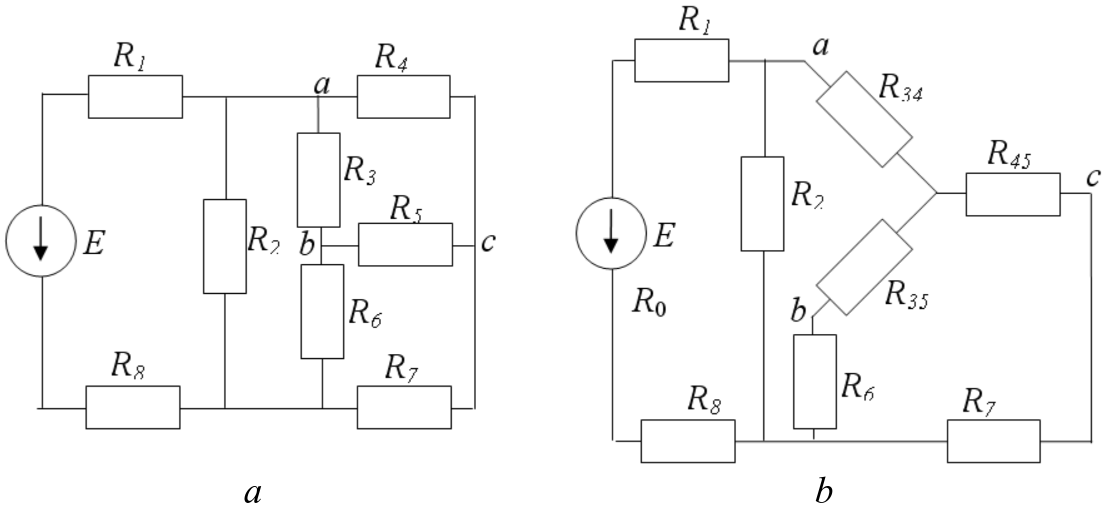


Figure 23.7: The scheme of circuit for the task 23.5: *a* – initial, *b* – converted

Table 23.4

Parameter	Version of task									
	К	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
***E*, V**	50	40	40	50	50	60	60	60	70	70	70
***R*₁, Ω**	5	5	5	5	5	6	6	6	6	6	6
***R*₂, Ω**	4	4	4	4	5	5	5	5	6	6	6
***R*₃, Ω**	20	20	30	30	30	30	30	25	25	25	25
***R*₄, Ω**	30	30	30	35	35	35	35	35	40	40	40
***R*₅, Ω**	50	50	50	50	50	60	60	60	60	60	60
***R*₆, Ω**	100	100	100	90	90	90	90	80	80	80	80
***R*₇, Ω**	5	5	5	5	5	8	8	8	8	8	8
***R*₈, Ω**	1.8	1	1	2	2	3	3	2	2	3	3

The ***solution*** for version K.

Let's convert triangle of resistances R_3, R_4, R_5 in equivalent star, we will receive:

$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4 + R_5} = \frac{20 \cdot 30}{20 + 30 + 50} = 6\ \mathbf{\Omega}, \quad R_{45} = \frac{R_4 \cdot R_5}{R_3 + R_4 + R_5} = \frac{30 \cdot 50}{20 + 30 + 50} = 15\ \mathbf{\Omega},$$

$$R_{35} = \frac{R_3 \cdot R_5}{R_3 + R_4 + R_5} = \frac{20 \cdot 50}{20 + 30 + 50} = 10\ \mathbf{\Omega}.$$

The total (equivalent) resistance of series-connected intercalated resistors R_{45} and R_7: $R_{\text{eq1}} = R_{45} + R_7 = 15 + 5 = 20\ \mathbf{\Omega}$. The total (equivalent) resistance of series-connected resistors R_{35} and R_6: $R_{\text{eq2}} = R_{35} + R_6 = 10 + 10 = 20\ \mathbf{\Omega}$. The total (equivalent) resistance of the branches with a resistor R_{eq1} and R_{34} and R_{eq2}:

$$R_{eq3} = R_{34} + \frac{R_{eq1} \cdot R_{eq2}}{R_{eq1} + R_{eq2}} = 6 + \frac{20 \cdot 20}{20 + 20} = 16\ \mathbf{\Omega}.$$

The total resistance of the whole circuit:

$$R_{tot} = R_1 + R_8 + \frac{R_2 \cdot R_{eq3}}{R_2 + R_{eq3}} = 5 + 1.8 + \frac{4 \cdot 16}{4 + 16} = 10\ \mathbf{\Omega}.$$

The current in the unbranched part of the circuit: $I = E/R_{\text{tot}} = 50/10 = 5$ A.

The voltage drop on the resistors R_1, R_2 and R_8:

$$\Delta U_1 = R_1 \cdot I = 5 \cdot 5 = 25 \text{ B}; \quad \Delta U_8 = R_8 \cdot I = 1.8 \cdot 5 = 9 \text{ V};$$

$$\Delta U_2 = I \frac{R_2 \cdot R_{eq2}}{R_2 + R_{eq2}} = 5 \cdot \frac{20 \cdot 20}{20 + 20} = 16\ \text{V}.$$

Check. On the basis of the second Kirchhoff's law we have:

$$E = U_1 + U_2 + U_3 \text{ or } 50 \text{ V} = 25 + 9 + 16 = 50 \text{ V}.$$

Task 23.6. For the electrical circuit (fig. 23.8) to determine the currents in the branches, the voltage on all the elements of the circuit, the voltage U_{13} between nodes 1–3, power and mode of operation of the source of EMF E_1, the power of the receiver with the resistance R_3. The initial data are given in table 23.5.

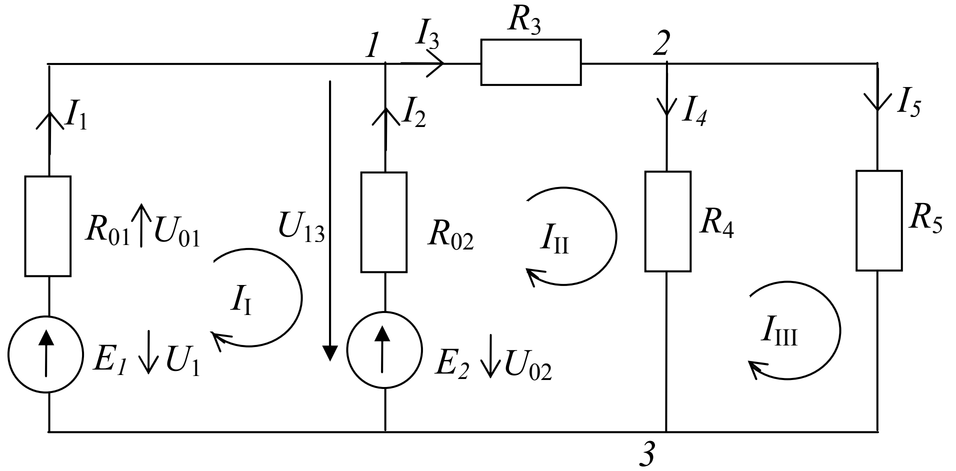


Figure 23.8: The scheme of circuit for the task 23.6.

Table 23.5

Parameter	Version of task										
	К	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
E_1, V	12	10	10	12	12	10	12	13	14	15	10
E_2, V	13.5	12	14	14	12	10	10	11	12	10	15
R_{01}, Ω	0.05	0.1	0.1	0.05	0.05	0.1	0.1	0.05	0.1	0.1	0.1
R_{02}, Ω	0.1	0.1	0.2	0.1	0.05	0.1	0.05	0.05	0.1	0.1	0.1
R_3, Ω	3	4	5	4	4	3	4	4	5	6	4
R_4, Ω	4	4	3	5	4	3	5	5	5	4	6
R_5, Ω	4	3	3	5	4	3	5	5	5	4	6

The ***solution*** of task for version K.

The scheme of the circuit has three nodes and five branches. To find the currents in the branches it is needed to compose five equations: two equations for 1-t Kirchhoff's law and three equations for 2-d Kirchhoff's law.

Let's set the directions of the currents in the branches and bypass of contours (I, II and III) and apply them to the scheme.

Let's write the equation on 1-t Kirchhoff's law for nodes 1 and 2, and the 2nd Kirchhoff's law for the selected contours. The system has the form:

for node 1 $I_1 + I_2 - I_3 = 0$,

for node 2 $I_3 - I_4 - I_5 = 0$,

for contour I $R_{01} \cdot I_3 + R_{02} \cdot I_2 = E_1 - E_2$,

for contour II $R_{02} \cdot I_2 + R_3 \cdot I_3 + R_4 \cdot I_4 = E_2$,

for contour III $- R_4 \cdot I_4 + R_5 \cdot I_5 = 0$.

The solution of the system gives the values of the current in the branches:

$I_1 = -7.93$ A; $I_2 = 11.03$ A; $I_3 = 3.1$ A; $I_4 = I_5 = 1.55$ A

Since the current I_1 has turned out negative, then its actual direction is opposite to the accepted on the scheme (fig. 23.8).

The voltage on the resistors it is determined by Ohm's law:

$U_3 = R_3 \cdot I_3 = 2 \cdot 3.1 = 6.2$ V; $U_4 = R_4 \cdot I_4 = 4 \cdot 1.55 = 6.2$ V;

$U_5 = R_5 \cdot I_5 = 4 \cdot 1.55 = 6.2$ V.

The voltage between nodes 1 and 3 it is found by using the second Kirchhoff's law:

$E_2 = U_{02} + U_{13}$; $U_{13} = E_2 - U_{02} = E_2 - R_{02} \cdot I_2 = 13.5 - 0.1 \cdot 11 = 12.4$ V.

The power of source of EMF E_2

$P_2 = E_2 \cdot I_2 = 13.5 \cdot 11 = 148.5$ W.

The power of the receiver

$P_3 = R_3 \cdot I_3^2 = U_3 \cdot I_3 = 6.2 \cdot 3.1 = 19.2$ W.

The source of EMF E1 current and EMF are directed oppositely. This means that under the given conditions of the problem, it consumes electrical energy.

Because the source of EMF E1 work in the mode of energy consumption, i.e., is a receiver, then the equation of power balance has the form

$E_2 \cdot I_2 = E_1 \cdot I_1 + R_{01} \cdot I_1^2 + R_{02} \cdot I_2^2 + R_3 \cdot I_3^2 + R_4 \cdot I_4^2 + R_5 \cdot I_5^2$.

After substitution the obtained values are

$13.5 \cdot 11 = 12 \cdot 7.9 + 0.05 \cdot 7.9^2 + 0.1 \cdot 11^2 + 2 \cdot 3.1^2 + 4 \cdot 1.55^2$.

148.5 W = 148.5 W.

The obtained equality confirms the correctness of the calculation.

Task 23.7 For a scheme of the electrical circuit and the task of source data 23.6 to determine the currents in the branches of an electric circuit by the method of contour currents.

The ***solution*** of task for version K.

The system of equations of contour currents for a given scheme (fig. 23.8) has the form

$$\begin{cases} R_{11}\, I_{\mathrm{I}} - R_{12}{\cdot}I_{\mathrm{II}} - R_{13}{\cdot}I_{\mathrm{III}} = E_{\mathrm{I}}\ ; \\ -R_{21}{\cdot}I_{\mathrm{I}} + R_{42}{\cdot}I_{\mathrm{II}} - R_{23}{\cdot}I_{\mathrm{III}} = E_{\mathrm{II}}\ ; \\ -R_{31}{\cdot}I_{\mathrm{I}} - R_{32}{\cdot}I_{\mathrm{II}} + R_{33}{\cdot}I_{\mathrm{III}} = E_{\mathrm{III}}\ . \end{cases}$$

where resistance and contour EMF have the following meanings:

$R_{11} = R_{01} + R_{01} = 0.15$ ohm; $R_{22} = R_{02} + R_2 + R_4 = 6.1\ \mathbf{\Omega}$;
$R_{33} = R_4 + R_5 = 8$ ohm; $R_{12} = R_{21} = -R_{02} = -0.1\ \mathbf{\Omega}$;
$R_{13} = R_{31} = 0$, $R_{23} = R_{32} = -R_4 = -4\ \mathbf{\Omega}$,
$E_{\mathrm{I}} = E_1 - E_2 = -1.5$ V; $E_{\mathrm{II}} = E_2 = 13.5$ V; $E_{\mathrm{III}} = 0$.

Solving the received system of equations of contour currents we will get their values $I_{\mathrm{I}} = -7.93$ A, $I_{\mathrm{II}} = 3.1$ A, $I_{\mathrm{III}} = 1.55$ A.

Then let's find the currents in the branches

$I_1 = I_{\mathrm{I}} = -7.93$ A, $I_2 = I_{\mathrm{II}} - I_{\mathrm{I}} = 11.03$ A, $I_3 = I_{\mathrm{II}} = 3.1$ A,
$I_4 = I_{\mathrm{II}} - I_{\mathrm{III}} = 1.55$ A, $I_5 = I_{\mathrm{III}} = 1.55$ A.

As previously noted, the minus sign in the currents I_1 and I_{I} means that their actual direction opposite to the accepted ones in equivalent scheme.

Task 23.8 For a scheme of the electrical circuit and the source data of task 23.6 to determine the currents in the branches of an electric circuit by the method of nodal potentials.

The ***solution*** of task for version K.

Determine the values of the coefficients matrix of conductances of system of nodal potentials (3.8)

$$G_{11} = \frac{1}{R_{01}} + \frac{1}{R_{02}} + \frac{1}{R_3} = \frac{1}{0.05} + \frac{1}{0.1} + \frac{1}{2} = 30.5\ \text{S},$$

$$G_{22} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{0.05} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1\ \text{S},\ G_{12} = G_{21} = -\frac{1}{R_3} = -\frac{1}{2} = -0.5\ \text{S}.$$

The current values of nodes

$$I_{y1} = \frac{E_1}{R_{01}} + \frac{E_2}{R_{02}} = \frac{12}{0.05} + \frac{13.5}{0.1} = 375\ \text{A},\ I_{y2} = 0.$$

The received system has the form

$$\begin{bmatrix} 30.5 & -0.5 \\ -0.5 & 1 \end{bmatrix} \times \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} 375 \\ 0 \end{bmatrix}.$$

Solving the system we get $\varphi_1 = 12.4$ V, $\varphi_2 = 6.2$ V.

Let's determine the currents in the branches

$$I_1 = \frac{E_1 + (\varphi_3 - \varphi_1)}{R_{01}} = \frac{12 + (0 - 12.4)}{0.05} = -7.98 \text{ A},$$

$$I_2 = \frac{E_2 + (\varphi_3 - \varphi_1)}{R_{02}} = \frac{13.5 + (0 - 12.4)}{0.1} = 11.08 \text{ A}, \; I_3 = \frac{(\varphi_1 - \varphi_2)}{R_2} = \frac{12.4 - 6.2}{2} = 3.1 \text{A},$$

$$I_4 = \frac{(\varphi_2 - \varphi_3)}{R_4} = \frac{6.2}{4} = 1.55 \text{ A}, \; I_5 = \frac{(\varphi_2 - \varphi_3)}{R_5} = \frac{6.2}{4} = 1.55 \text{ A}.$$

Task 23.9. For a scheme of the electrical circuit and the task of source data 23.6 using the method of equivalent generator to calculate and to build the dependences of the current in the branch with resistor R3 and the voltage U_{12} between nodes 1 and 2, when the resistance change from zero to 12 ohm.

The ***solution*** of task for version K.

In this case, the internal resistance of the equivalent generator is easier to calculate, but does not determine the short circuit current. At open connection terminals 1 and 2 and shorted the sources of EMF source scheme takes the form shown in figure 23.9, *c*.

Find the equivalent resistance of the branches connected to nodes 1–3 and 2-3 (fig. 23.9, *b*)

$$R_{13} = \frac{R_{01} \cdot R_{02}}{R_{01} + R_{02}} = \frac{0.05 \cdot 0.1}{0.05 + 0.1} = 0.033 \ \mathbf{\Omega}, \quad R_{23} = \frac{R_4 \cdot R_5}{R_4 + R_5} = \frac{4 \cdot 4}{4 + 4} = 2 \ \mathbf{\Omega}.$$

Relative to nodes 1 and 2 resistors R_{13} and R_{23} are connected in series, therefore the internal resistance of the equivalent generator R_{eq}=2.033 **Ω**.

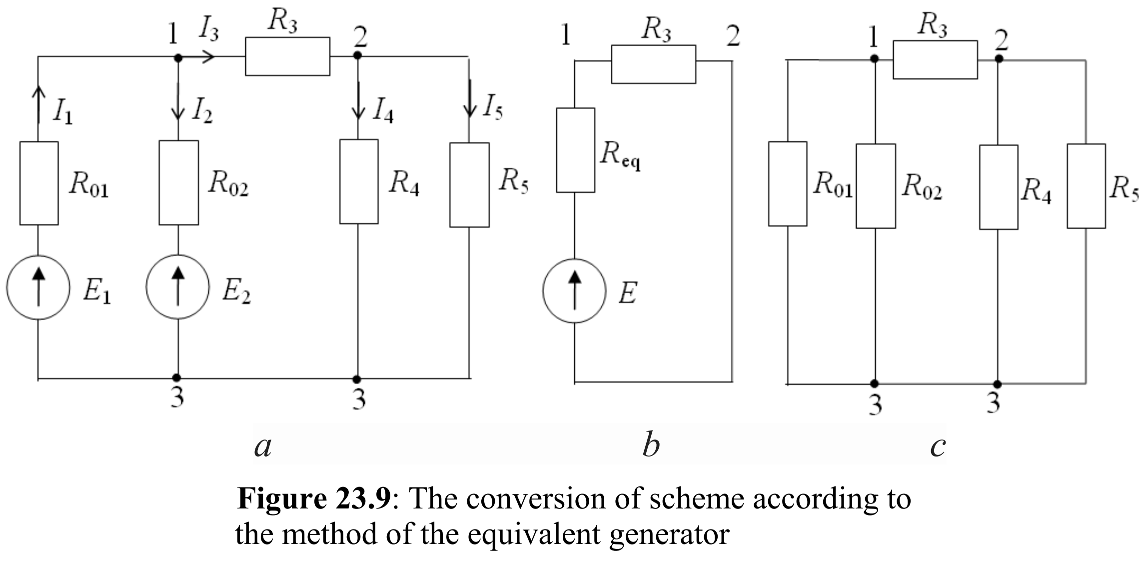


Figure 23.9: The conversion of scheme according to the method of the equivalent generator

To determine the EMF of equivalent generator it is necessary to calculate the potentials of the nodes 1 and 2. In our case they are the easiest to determine by the method of nodal potentials. So, if we take the potential of the node 3 is equal to zero, then open the connection terminals of the currents in the branches with resistors R_4 and R_5 are equal to zero. Therefore, the potential of node 2 is also zero.

The potential of the node 1 let's find on the equation (3.9)

$$\varphi_1 = \frac{I_{y1}}{G_{11}} = \frac{375}{30} = 12.5 \text{ V},$$

where $G_{11} = \frac{1}{R_{01}} + \frac{1}{R_{02}} = \frac{1}{0.05} + \frac{1}{0.1} = 30$ S, $I_{y1} = \frac{E_1}{R_{01}} + \frac{E_2}{R_{02}} = \frac{12}{0.05} + \frac{13.5}{0.1} = 375$ A.

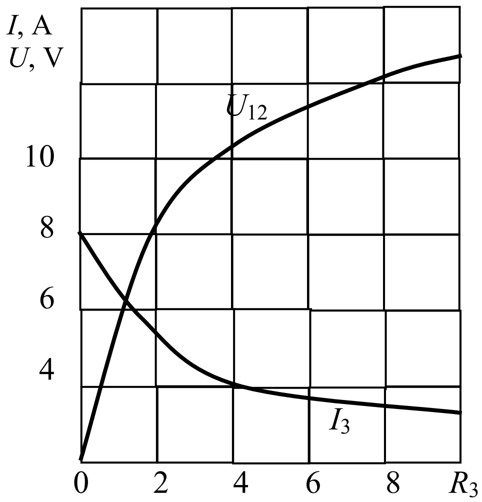


Figure 23.10: Required dependences

EMF of equivalent generator

$E_G = = \varphi_1 - \varphi_2 = \varphi_1 = 12.5$ V.

The current I_3 in the branch with resistor R_3 and the voltage between nodes 1 and 2

$I_3 = E_G/(R_3 + R_{eq})$,
$U_{12} = R_3 \cdot I_3$.

On the obtained ratios let's build a desired graphics (fig. 23.10).

The solution to this problem by other methods requires substantially big costs of time.

24 THE ELECTRIC CIRCUIT OF A SINGLE-PHASE AC

Task 24.1 Current changes according to the law $i = I_m \cdot \sin(\omega t + \psi_i)$. Determine its complex amplitude and complex operating current. The original data are shown in table 24.1.

Table 24.1

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
I_m, **A**	8	6	4	2	3	5	7	9	5	4	3
ψ_i, **rad**	π/18	-π/24	π/30	π/40	π/10	π/16	π/20	π/26	-π/14	π/22	π/28

The ***solution*** for the version K.

For the sinusoidal current with the amplitude $I_m = 8$ A and the initial phase $\psi_i = \pi/18$ the complex current amplitude and complex current are respectively equal to

$$I_m = 8e^{j\frac{\pi}{18}} = 8(\cos\frac{\pi}{18} + j\sin\frac{\pi}{18}) = 7.88 + j1.39,$$

$$I = \frac{8}{\sqrt{2}} e^{j\frac{\pi}{18}} \cong 5.7(\cos\frac{\pi}{18} + j\sin\frac{\pi}{18}) = 5.57 + j0.98.$$

Task 24.2 The complex amplitude of the current is $\dot{I}_m = I_m e^{j\psi_i}$. Write the expression for a sinusoidal current, which varies with the frequency *f*. The initial data are shown in table 24.2.

Table 24.2

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
I_m, **A**	25	23	21	19	17	15	13	9	7	5	3
ψ_i, **rad**	-π/12	-π/18	-π/22	π/20	π/30	π/26	-π/20	π/16	π/24	π/14	π/18
***f*, Hz**	50	50	50	60	60	60	60	50	50	50	50

The ***solution*** for the version K.

The angular frequency of the current is $\omega = 2\pi f = 2\pi\cdot 50 = 314$ 1/s.

To move from the complex amplitude to the instantaneous value of the current it is needed to multiply the complex amplitude $\dot{I}_m = 25e^{-j\frac{\pi}{12}}$ by $e^{j\omega t} = e^{j314t}$ and take the imaginary part of the received complex number

$$i = I_m(25e^{-j\pi/12}\cdot e^{j314t}) = I_m(25e^{j(314t - \pi/12)}) = 25\cdot\sin(314\cdot t - \pi/12).$$

Task 24.3 In the circuit of the alternating current with voltage *U* and frequency *f* an ideal coil with inductance *L* ($R_{coil} = 0$) is turned on. Determine the reactive power *Q* of the coil and the energy W_{Lm} stored in the magnetic field of the coil and write expressions for the instantaneous values of the voltage *u*, current *i*, the EMF of self-induction e_L, the instantaneous power *p* and the average power *P* for the period, if the initial voltage phase is ψ_u. Build temporary and vector diagrams. The initial data are given in table 24.3.

Table 24.3

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
***U*, V**	120	110	100	90	80	70	120	110	100	90	80
***f*, Hz**	50	50	50	50	50	60	60	60	60	50	50
***L*, mH**	9	7	5	6	8	9	7	5	6	8	10
ψ_u, **rad**	π/2	π/2	π/4	π/4	π/4	π/2	π/2	π/2	π/4	π/4	π/4

The ***solution*** for the version **K.**

The inductive resistance of the coil: $X_L = \omega L = 2\pi f = 2\cdot 3.14\cdot 50\cdot 0{,}009 = 3$ **Ω**.

The effective value of the current: $I = U/X_L = 120/3 = 40$ A.

The reactive power of the circuit: $Q = U\cdot I = 120\cdot 40 = 4800$ var.

The maximum energy stored in the magnetic field of the coil:
$W_{Lm} = L\cdot I^2 = 0.009\cdot 40^2 = 14.4$ Joule.

The amplitude value of the voltage $U_m = \sqrt{2}\cdot U = 1.41\cdot 120 = 169.2$ V and current $I_m = \sqrt{2}\cdot I = 1.41\cdot 40 = 56.4$ A.

The instantaneous values of:

the voltage $u = u_L = U_m \cdot \sin(\omega t + \pi/2) = 169.2 \cdot \sin(3{,}14t + \pi/2)$ V;

the current $i = I_m \cdot \sin\omega t = 56.4 \cdot \sin 314$ A;

the EMF of self-induction of the coil $e_L = -u_L = 169.2 \cdot \sin(3.14t - \pi/2)$ V;

the power of the circuit $p = u \cdot i = U_m \cdot \sin(\omega t + \pi/2) \cdot I_m \cdot \sin\omega t = U_m \cdot \cos\omega t \cdot I_m \cdot \sin\omega t = U_m \cdot I_m \cdot \sin 2\omega t/2$, on the strength $\sin(\omega t + \pi/2) = \cos\omega t$, and $\sin 2\omega t = 2 \cdot \sin\omega t \cdot \cos\omega t$.

For the effective values of the voltage and current:

$p = u \cdot i = U \cdot I \cdot \sin 2\omega t = 120 \cdot 40 \cdot \sin 2 \cdot 314 \cdot t = 4800 \cdot \sin 628t$ VA.

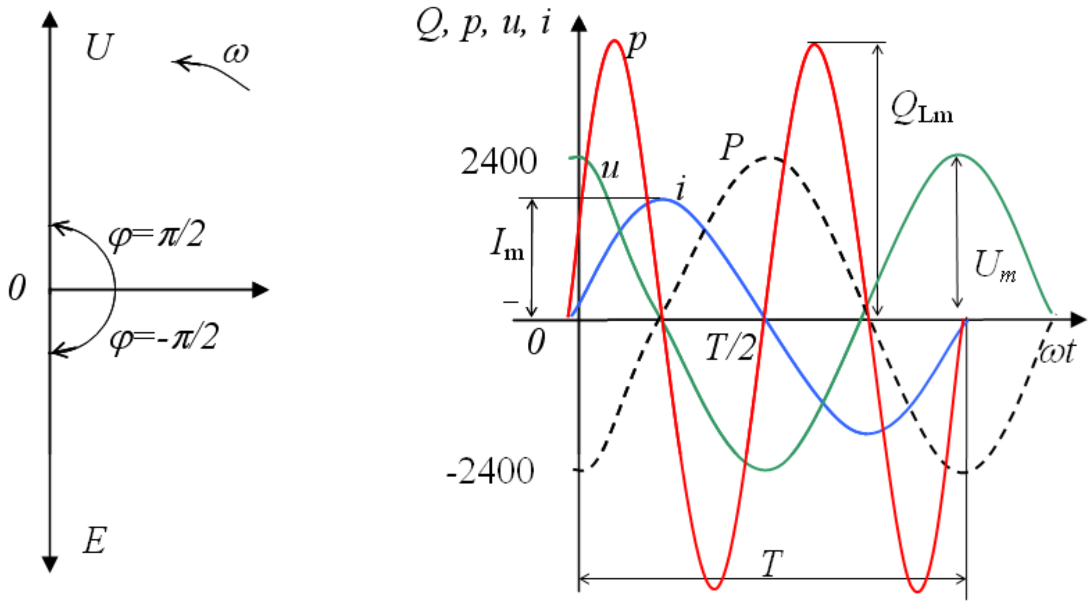
The average power during the period:

$$P = \frac{1}{T}\int_0^T p \cdot dt = \frac{1}{T}\int_0^T U \cdot I \cdot \sin 2\omega t \cdot dt = 0 \ .$$

The vector diagram for the effective values of the voltage and current is shown in figure 24.1, *a*.

A graph of change of the instantaneous power is a sine wave with double frequency and amplitude Q_{Lm} (fig. 24.1, *b*). Meanwhile the reactive power is

$$Q_L = \frac{U_m \cdot I_m}{2} = \frac{\sqrt{2}U \cdot \sqrt{2}I}{2} = U \cdot I = 120 \cdot 40 = 4800 \text{ var}.$$



a *b*

Figure 24.1: Vector diagram (*a*) and graphs of instantaneous values (*b*) to the task 24.3

Task 24.4 A capacitor with capacitance C (R_C =0) is included in the circuit of the alternating current with the voltage U and frequency f. Determine the reactance of the capacitor X_C, current i, reactive power Q_C, maximum energy W_{Cm} stored in the electric field of the capacitor. Write the expressions for the instantaneous values of the current i and the instantaneous power p. Build temporary and vector diagrams. The initial data are shown in table 24.4.

Table 24.4

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
***U*, V**	220	220	220	210	210	210	127	127	127	120	120
***f*, Hz**	50	50	50	60	60	60	60	50	50	50	50
***C*, μF**	20	18	14	10	20	18	14	10	16	20	30
ψ_u, rad	0	0	0	30	30	30	60	60	60	157	157

The ***solution*** for the version K.
The reactance of the capacitor:

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2 \cdot 3.14 \cdot 50 \cdot 20 \cdot 10^{-6}} = 160 \ \Omega.$$

The current in the circuit of the capacitor: $I = U / X_C = 220/160 = 1.37$ A.

The reactive power of the circuit: $Q_C = U \cdot I = 220 \cdot 1.37 = 302$ var.

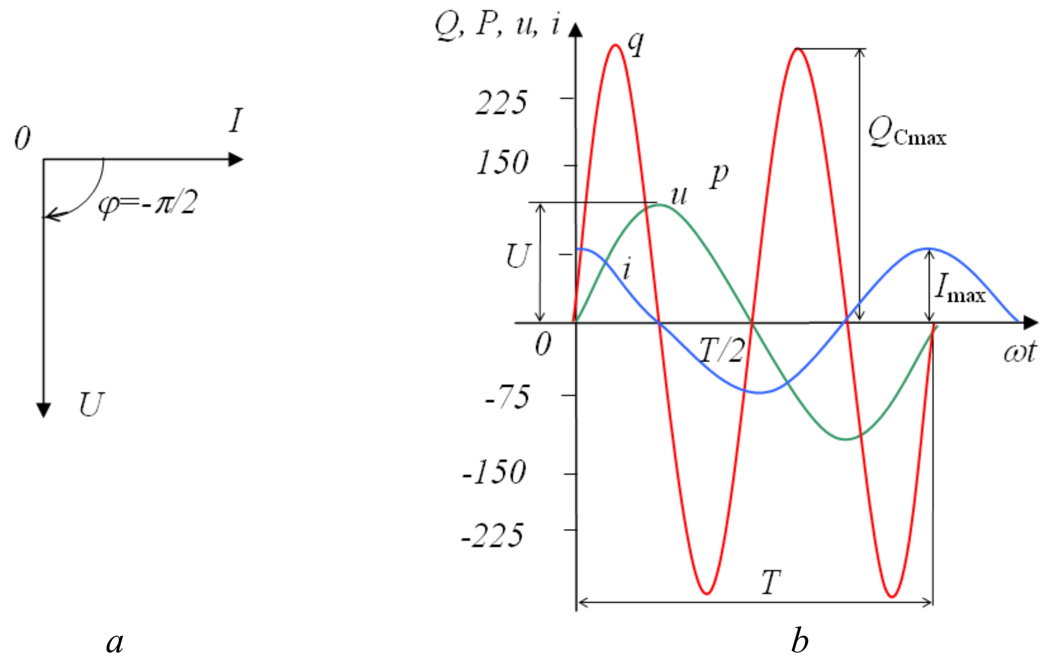


Figure 24.2: Vector diagram (*a*) and graphs of instantaneous values (*b*) to the task 24.4

The maximum energy stored in the electric field of the capacitor:

$W_{Cm} = C \cdot U^2 = 20 \cdot 10^{-6} \cdot 220^2 = 968 \cdot 10^{-3}$ J.

The instantaneous value of the current in the circuit:

$$i = C\frac{du}{dt} = C\frac{d(U_m \sin\omega t)}{dt} = C \cdot U_m \cdot \omega \cdot \sin(\omega t + \pi/2) = I_m \cdot \cos\omega t .$$

The instantaneous power of the circuit:

$$p = u \cdot i = U_m \sin\omega t \cdot I_m \cos\omega t = U_m \cdot I_m \frac{\sin 2\omega t}{2}$$

or for the effective values of the current and voltage:

$$p = U \cdot I \cdot \sin 2\omega t = 220 \cdot \frac{220}{160} \cdot \sin(2 \cdot 314t) = 302 \sin 628t\,.$$

The vector diagram of the current and voltage is shown in figure 24.2, *a*.

The time graphs of the voltage, current and power are shown in figure 24.2, *b*. The graph of change of the instantaneous power over time is a sine wave with double frequency and amplitude equal to the reactive power:

$$Q_{Cm} = \frac{U_m \cdot I_m}{2} = \frac{\sqrt{2}U \cdot \sqrt{2}I}{2} = U \cdot I = 220 \cdot 1.37 = 302 \text{ var.}$$

Task 24.5 In the electric circuit of the alternating current the voltage *U* and current *I* are changed in accordance with the expressions:

$u = U_m \cdot \sin(\omega t + \psi_u)$, V; $i = I_m \cdot \sin(\omega t + \psi_i)$, A.

Determine the active *P*, reactive *Q* and total *S* of the power circuit. The initial data are shown in table 24.5.

Table 24.5

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
U_m, **V**	28.2	32	30	26	24	25	27	29	31	33	35
ω, **1/s**	628	628	628	314	314	314	314	314	628	628	628
ψ_u, **rad**	4π/9	4π/9	4π/9	2π/9	2π/9	π/2	π/2	π/4	π/4	π/2	π/4
I_m, **A**	2.82	2.6	2.4	2.2	2.3	2.5	2.7	2.9	3.1	3.0	2.8
ψ_i, **rad**	5π/18	5π/18	π/30	π/40	π/2	π/4	π/8	2π/8	π/10	π/6	π/8

The solution for the version K.

The instantaneous power value of the circuit:

$p = u \cdot i = 28.2\sin(628t + 4\pi/9) \cdot 2.28\sin(628t + 5\pi/18) =$

$$= 28.2 \cdot 2.28\{\frac{1}{2}[\cos(628t + 4\pi/9 - 628t - 5\pi/18) - \cos(628t + 4\pi/9 + 628t + 5\pi/18)]\} =$$

$$= 79.5\{\frac{1}{2}[\cos\pi/6 - \cos(1225t + 13\pi/18)]\},$$

or for the effective values of the voltage and current:

$$p = \frac{28.2 \cdot 2.28}{\sqrt{2} \cdot \sqrt{2}} \cdot \frac{1}{2}[\cos\pi/6 - \cos(1225t + 13\pi/18)] =$$

$$= 19.8\cos\pi/6 - 19.8\cos(1225t + 13\pi/18).$$

The power circuit: active $P = 19.8\cos\pi/6 = 19.8\frac{\sqrt{3}}{2} = 17.1$ W;

reactive $Q = 19.8\sin\pi/6 = 19.8\frac{1}{2} = 9.9$ var; full $S = U \cdot I = 19.8$ VA.

Task 24.6 The effective value of the voltage applied to the electric circuit (fig. 24.8) U, the frequency of the voltage f, the resistance of the resistor R, the inductance of the coil L, the capacitance of the capacitor C. Using a complex method find the current values of the currents in the circuit branches and the voltages on its elements, full S, active P and reactive Q of the power circuit. The initial data are given in table 24.6.

Table 24.6

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
***U*, V**	100	110	120	130	100	110	120	130	90	90	80
***f*, Hz**	50	50	50	50	60	60	60	60	60	50	50
***R*, Ω**	10	14	16	18	22	20	17	15	13	12	11
***L*, mH**	31.8	30	28	26	27	29	32	34	30	29	27
***C*, μF**	318.5	310	300	280	260	250	270	290	300	310	305

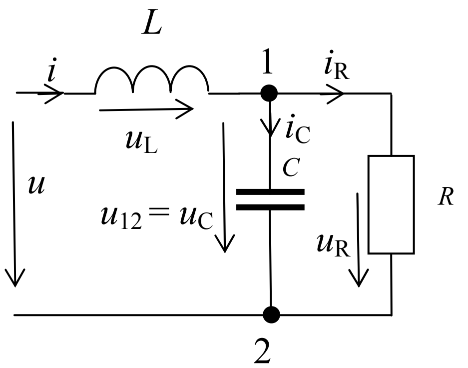


Figure 24.3: Scheme of a circuit to the task 24.6

The solution for version K.

Taking the initial phase of voltage equal to zero, for complex of voltage we can write

$$\dot{U} = 100e^{j0} = 100.$$

The complex resistance of the inductive coil and condenser:

$$\underline{Z}_L = jX_L = j\omega L = j314 \cdot 31{,}8 \cdot 10^{-3} = j10 = 10e^{j\frac{\pi}{2}},$$

$$\underline{Z}_C = -jX_C = -j\frac{1}{\omega C} = -j\frac{1}{314 \cdot 318.5 \cdot 10^{-6}} =$$

$$= -j10 = 10e^{-j\frac{\pi}{2}},$$

where the angular frequency: $\omega = 2\pi f = 314$ s^{-1} .

To determine the complex currents we can use any known calculation method of electric circuits, for example, the method of nodal potentials. Considering the complex potential of the node 2 is equal to zero $\varphi_2 = 0$, we have $\underline{Y}_{11} \cdot \dot{\varphi}_1 = \dot{I}_{11}$, from where

$$\dot{\varphi}_1 = \frac{\dot{I}_{11}}{\underline{Y}_{11}} = \frac{-j10}{0.1} = -j100 = 100e^{-j\frac{\pi}{2}},$$

where complex nodal conductivity and calculated complex current in the node are respectively equal to:

$$\underline{Y}_{11} = \frac{1}{\underline{Z}_{1L}} + \frac{1}{\underline{Z}_C} + \frac{1}{R} = \frac{1}{j10} + \frac{1}{(-j10)} + \frac{1}{10} = -j0.1 + j0.1 + 0.1 = 0.1,$$

$$\dot{I}_{11} = \frac{\dot{U}}{\underline{Z}_{11}} = \frac{100}{j10} = -j10 \ .$$

The complex currents in the branches:

$$\dot{I} = \frac{\dot{U} - \dot{\varphi}1}{\underline{Z}} = \frac{100 + j100}{j10} = \frac{100\sqrt{2}e^{j\frac{\pi}{4}}}{10e^{j\frac{\pi}{2}}} = 10\sqrt{2}e^{-j\frac{\pi}{4}},$$

$$\dot{I}_C = \frac{\dot{\varphi}_1}{\underline{Z}_C} = \frac{-j100}{-j10} = 10, \quad \dot{I}_R = \frac{\dot{\varphi}_1}{R} = \frac{-j100}{10} = -j10 = 10e^{-j\frac{\pi}{2}}.$$

The effective values of the currents: $I = 14.1$ A, $I_R = I_C = 10$ A.

The complex voltages on the inductive coil, the capacitor and the resistor:

$$\dot{U}_L = \underline{Z}_L \cdot \dot{I} = 10e^{j\frac{\pi}{2}} \cdot 10\sqrt{2}e^{-j\frac{\pi}{4}} = 141.2e^{j\frac{\pi}{4}};$$

$$\dot{U}_R = R \cdot \dot{I}_R = 10 \cdot 10e^{-j\frac{\pi}{2}} = 100e^{-j\frac{\pi}{2}};$$

$$\dot{U}_C = \underline{Z}_C \cdot \dot{I}_C = 10e^{-j\frac{\pi}{2}} \cdot 10 = 100e^{-j\frac{\pi}{2}}.$$

The effective values of the voltages: $U_L = 141.2$ V, $U_C = U_R = 100$ V.

The complex power:

$$\underline{S} = \dot{U} \cdot \dot{I} = 100 \cdot 10\sqrt{2}e^{-j\frac{\pi}{4}} = 1410(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}) = 1000 + j1000.$$

Consequently, the total, active and reactive power:

$$S = |\underline{S}| = 1410 \text{ VA}, \quad P = \mathrm{Re}(\underline{S}) = 1000 \text{ Watt}, \quad Q = \mathrm{Im}(\underline{S}) = 1000 \text{ var}.$$

Task 24.7 For the electric circuit of AC (fig. 24.9, *a*) determine the readings of the ammeter A, A_1, A_2, angles of phase shift φ, φ1 and φ2 between the respective currents $\dot{I}$, $\dot{I}_1$ and $\dot{I}_2$ and voltages $\dot{U}$; construct the vector diagram of the currents and voltages. The initial data (the feeding voltage *U*, the active and reactive resistance of circuit) are shown in table 24.7.

Table 24.7

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
***U*, V**	120	110	100	90	80	86	94	104	114	124	136
R_1, Ω	2	1.6	1.4	1.4	1.8	2	2.1	2.2	2.6	2.9	3.2
R_2, Ω	1	0.8	0.6	0.7	0.9	1.3	1.5	1.6	1.4	1.1	2.0
X_L, Ω	6	5	4	5	6	7	6	5	4	7	8
X_C, Ω	9.95	9	8	7	10	12	8	9	7	10	9

The solution for the version K.

The impedance of the branches of the circuit:

$$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{2^2 + 6^2} = 6.23 \ \Omega; \ Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{1^2 + 9.95^2} = 10 \ \Omega.$$

The angles of phase shift between the currents and voltages of the respective parallel branches:

$$\cos\varphi_1 = \frac{R_1}{Z_1} = \frac{2}{6.32} = 0.316;\ \ \varphi_1 = 71^o35';$$

$$\cos\varphi_2 = \frac{R_2}{Z_2} = \frac{1}{10} = 0{,}1;\quad \varphi_2 = -84^o15'.$$

The readings of the ammeters A_1 and A_2 in the parallel branches:

$$I_1 = U/Z_1 = 120/6.32 = 19\ \text{A};\ I_2 = U/Z_2 = 120/10 = 12\ \text{A}.$$

The active components of the currents in the parallel branches:

$$I_{a1} = I_1\cdot\cos\varphi_1 = 19\cdot 0.316 = 6{,}01\ \text{A},\ I_{a2} = I_2\cdot\cos\varphi_2 = 12\cdot 0.1 = 1.2\ \text{A}.$$

The reactive components of the currents in the parallel branches:

$$I_{p1} = I_1\cdot\sin\varphi_1 = I_1\cdot X_L/Z_1 = 19\cdot 6/6.32 = 18.01\ \text{A};$$

$$I_{p2} = I_2\cdot\sin\varphi_2 = I_1\cdot X_C/Z_2 = 12\cdot 9.95/10 = 11.93\ \text{A}.$$

The active and reactive components of the total current:

$$I_a = I_{a1}\cdot + I_{a2} = 6.01 + 1.2 = 7.21\ \text{A};$$

$$I_p = I_{p1}\cdot + I_{p2} = 18.01 - 11.93 = 6.08\ \text{A}.$$

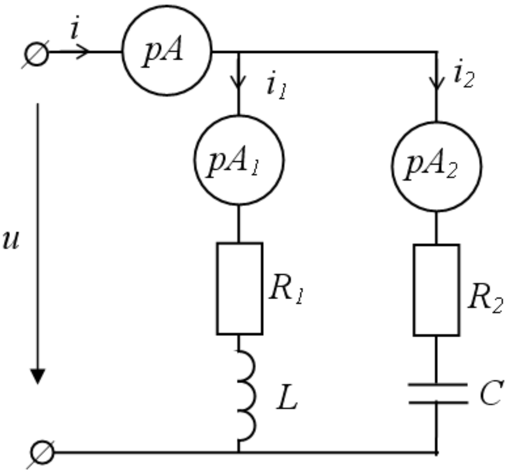
The total current in the circuit:

$$I = \sqrt{I_a^2 + I_p^2} = \sqrt{7.21^2 + 6.08^2} = 9.43\ \text{A}.$$

The phase shift angle between the current I and the applied voltage U:

$$\cos\varphi = I_a/I = 7.21/9.43 = 0.756;\ \ \varphi = 40^o10'.$$

The vector diagram of the currents and voltages for this version of the calculation is shown in figure 24.4, *b*.



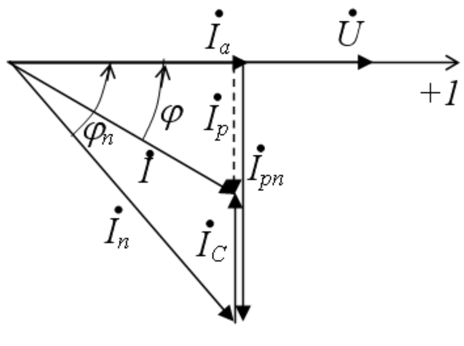


Figure 24.4: To task 24.7: *a* – the scheme of circuit;
b – vector diagrams for version K

Task 24.8 Using the conductivity method solve the task 24.7

The solution for the version K.

Let's determine the values of the impedances Z_1, Z_2, currents I_1, I_2 and power coefficients $\cos\varphi_1$, $\cos\varphi_2$ using the method stated in the task 24.7.

The active and reactive conductivity of the parallel branches:

$$G = G_1 + G_2 = \frac{R_1}{Z_1^2} + \frac{R_2}{Z_2^2} = \frac{2}{6.32^2} + \frac{1}{10^2} = 0.06 \text{ S};$$

$$B = B_1 + B_2 = \frac{X_L}{Z_1^2} - \frac{X_C}{Z_2^2} = \frac{6}{6.32^2} - \frac{9.95}{10^2} = 0.05 \text{ S}.$$

The full conductivity of the whole chain:

$$Y = \sqrt{G^2 + B^2} = \sqrt{0.06^2 + 0.05^2} = 0.0784 \text{ S}.$$

The total current in the circuit: $I = U \cdot Y = 120 \cdot 0.0784 = 9.4$ A.

The phase shift angle between the current I and the applied voltage U:

$$\cos\varphi = G/Y = 0.06/0.0784 = 0.765;\ \varphi = 40^{\circ}10' .$$

Task 24.9 Determine the real P, reactive Q and apparent power S of the electrical circuit (fig. 24.5). The values of the currents I_1, I_2, I_3, active R_1, R_2, R_3 and reactive X_L and X_C resistances are given in table 24.8.

Table 24.8

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
I_1, A	5	5	5	4	4	4	3	3	3	5	5
I_2, A	3	3	3	2	2	2	1	1	1	2	2
I_3, A	4	4	4	5	5	5	4	4	4	3	3
R_1, Ω	10	10	10	9	9	9	8	8	8	7	7
R_2, Ω	6	6	6	5	5	5	4	4	4	3	3
R_3, Ω	5	5	5	6	6	6	7	7	7	5	5
X_L, Ω	8	8	8	7	7	7	6	6	6	8	8
X_C, Ω	5,6	5	5	6	6	6	7	7	7	5	5

The solution for the version K.

The real power of the circuit:

$$P = P_1 + P_2 + P_3 = I_1^2 \cdot R + I_2^2 \cdot R + I_2^3 \cdot R = 5^2 \cdot 10 + 3^2 \cdot 6 + 4^2 \cdot 5 = 384 \text{ W}.$$

The reactive power of the circuit:

$$Q = Q_L - Q_C = I_2^2 \cdot X_L - I_3^2 \cdot X_C = 3^2 \cdot 8 - 4^2 \cdot 5.6 = -17.6 \text{ var}.$$

The apparent power of the circuit: $S = \sqrt{P^2 + Q^2} = \sqrt{384^2 + (-17.6)^2} = 385$ VA.

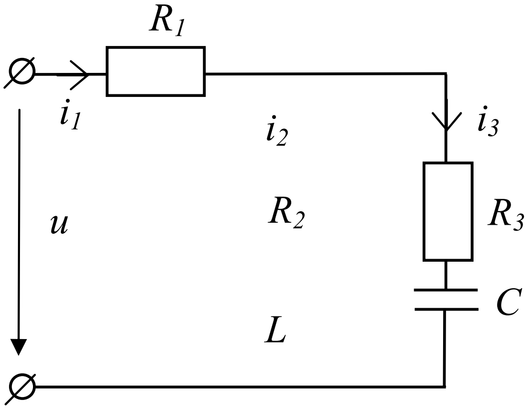


Figure 24.5: Scheme of the electric circuit to the task 24.9

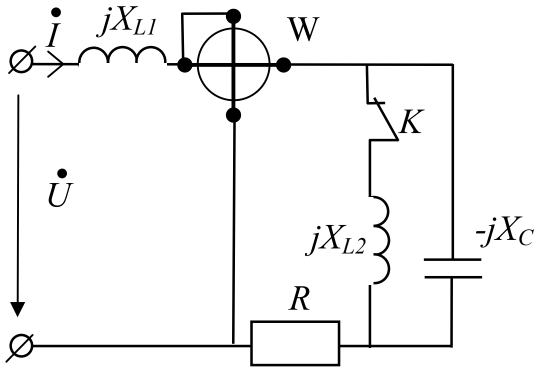


Figure 24.6: Scheme of the electric circuit to the task 24.10

Task 24.10 Determine the readings of the wattmeter W in the electric circuit (fig.24.6) in the closed and open switch K. The power supply voltage U, the real R and reactive X_{L1}, X_{L2} and X_C resistance specified in table 24.9.

Table 24.9

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
U, V	100	100	110	110	120	120	130	130	90	90	90
R, Ω	5	4	3	6	5	4	7	6	5	4	3
X_{L1}, Ω	5	5	4	4	6	6	5	5	4	4	6
X_{L2}, Ω	5	4	3	6	7	5	7	8	5	5	4
X_C, Ω	5	5	4	4	6	6	5	6	5	4	3

The solution for the version K

In this case, there is a resonance voltage in the electric circuit. The impedance of the circuit when the switch is open:

$$Z = \sqrt{R^2 + (X_{L1} - X_C)^2} = \sqrt{5^2 + (5-5)^2} = 5\ \Omega.$$

The current in the circuit when the switch is open: $I = U/Z = 100/5 = 20$ A. The readings of the wattmeter in this case: $P = R{\cdot}I^2 = 5{\cdot}20^2 = 2000$ W = 2 kW.

The readings of the wattmeter when the switch is closed: $P{=}R{\cdot}I^2{=}5{\cdot}0 = 0$ W, as there is the resonance of currents on the parallel plot and the current in the circuit of the resistor R does not flow.

25 THREE-PHASE CIRCUIT OF THE ELECTRIC CURRENT

Task 25.1 Three-phase generator is connected to symmetric receiver of electrical energy (fig. 25.1). Determine the phase voltage of the generator, currents, phase and line voltages of the receiver, the voltage drop in the line wires, a power of receiver. Construct the vector diagram of voltages and currents. Table 25.1 shows the initial data for solving the problem: line voltage of generator U_L, resistance wire $\underline{Z}_w = R_w + jX_w$, the resistance of the receiver $\underline{Z} = R + jX$.

Table 25.1

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
***U*L, V**	220	220	220	220	220	220	380	380	380	380	380
***R*W, ohm**	0.5	0.5	0.6	0.6	0.5	0.5	0.8	0.8	1	1.2	1.2
***X*W, ohm**	1	1.2	1.4	1.2	1.4	1.2	1.2	1.4	1.2	1.0	1.4
***R*, ohm**	10	12	12	12	10	14	18	20	22	22	24
***X*, ohm**	6	6	8	6	8	8	10	12	14	16	16

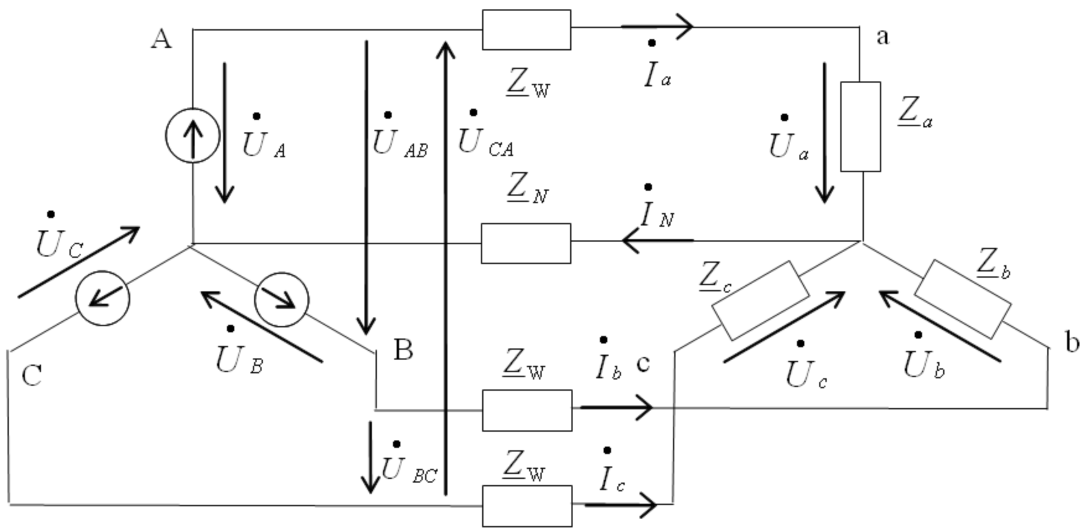


Figure 25.1: Scheme of a circuit to the task 25.1

The ***solution*** for version K.

The task is solved by the method of complex numbers.

The mode of work of three-phase circuit is symmetrical, so the voltage between the neutral points $U_N = 0$.

In a symmetric system of linear voltages of generator phase voltage is equal to

$$U_{ph} = U_L / \sqrt{3} = 220 / \sqrt{3} = 127 \text{ V}.$$

System of complex phase voltages of the generator, if we take the voltage $\dot{U}_A$ totally real

$$\dot{U}_A = 127,\ \dot{U}_B = 127e^{-j120^0},\ \dot{U}_C = 127e^{j120^0}.$$

Complex phase resistances and conductivities

$$\underline{Z}_{ph} = \underline{Z} + \underline{Z}_w = (10 + j6) + (0.5 + j1) = 10.5 + j7 = 12.6e^{j34^0},$$

$$\underline{Y}_{ph} = \frac{1}{\underline{Z}_{ph}} = \frac{1}{12.6e^{j34^0}} = 7.9 \cdot 10^{-2} \cdot e^{-j34^0} = (6.6 - j4.4) \cdot 10^{-2}.$$

Due to the fact that the systems of phase and line voltages of the generator and receiver are symmetrical, three-phase systems of currents, phase and line voltages of the receiver are also symmetric.

Phase currents of receiver:

$$\dot{I}_a = \dot{U}_a \cdot \underline{Y}_a = 127 \cdot 7.9 \cdot 10^{-2} \cdot e^{-j34^0} = 10 \cdot e^{-j34^0},$$

$$\dot{I}_b = \dot{I}_a \cdot e^{-j120^0} = 10 \cdot e^{-j154^0}, \quad \dot{I}_c = \dot{I}_a \cdot e^{j120^0} = 10 \cdot e^{j86^0}.$$

Effective values of phase currents

$I_a = I_b = I_c = 10$ A.

System of phase voltages of receiver:

$$\dot{U}_a = \dot{I}_a \cdot \underline{Z}_a = 10 \cdot e^{-j34^0}(10 + j6) = 10 \cdot e^{-j34^0} \cdot 11{,}7 \cdot e^{j31^0} = 117 \cdot e^{-j3^0};$$

$$\dot{U}_b = \dot{U}_a \cdot e^{-j120^0} = 117 \cdot e^{-j123^0}; \quad \dot{U}_c = \dot{U}_a \cdot e^{j120^0} = 117 \cdot e^{j117^0}.$$

Effective values of phase voltages of the receiver

$U_a = U_b = U_c = 117$ V.

Line voltages on the receiver:

$$\dot{U}_{ab} = \dot{U}_a - \dot{U}_b = 117 \cdot e^{-j3^0} - 117 \cdot e^{-j123^0} = 116.8 - j6.1) - (64.8 - j98.1) =$$
$$= 180 + j92 = 202 \cdot e^{j27^0};$$

$$\dot{U}_{bc} = \dot{U}_{ab} \cdot e^{-j120^0} = 202 \cdot e^{j27^0} \cdot e^{-j120^0} = 202 \cdot e^{-j93^0};$$

$$\dot{U}_{ca} = \dot{U}_{ab} \cdot e^{j120^0} = 202 \cdot e^{j27^0} \cdot e^{j120^0} = 202 \cdot e^{j147^0}.$$

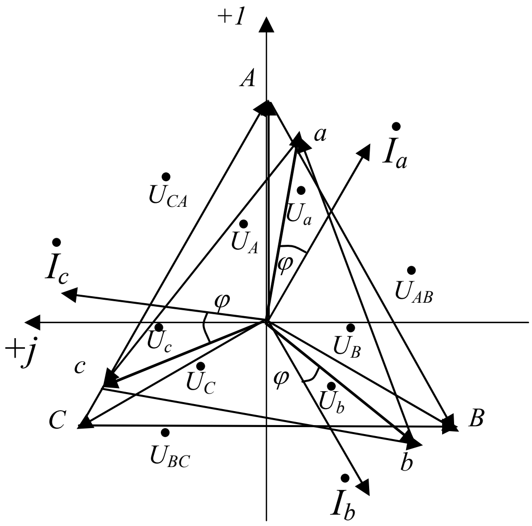


Figure 25.2: Vector diagram of currents and voltages to the task 25.1

Voltages drops in the line wires:

$$\Delta\dot{U}_a = \dot{I}_a \cdot \underline{Z}_{np} = 10 \cdot e^{-j34^0} \cdot (0.5 + j1) =$$
$$= 10 \cdot e^{-j34^0} \cdot 1.12 \cdot e^{j63^0} = 11.2 \cdot e^{j29^0};$$

$$\Delta\dot{U}_b = \dot{I}_b \cdot \underline{Z}_{np} = 10 \cdot e^{-j154^0} \cdot (0.5 + j1) =$$
$$= 10 \cdot e^{-j154^0} \cdot 1.12 \cdot e^{j63^0} = 11.2 \cdot e^{-j91^0};$$

$$\Delta\dot{U}_c = \dot{I}_c \cdot \underline{Z}_{np} = 10 \cdot e^{j86^0} \cdot (0.5 + j1) =$$
$$= 10 \cdot e^{j86^0} \cdot 1.12 \cdot e^{j63^0} = 11.2 \cdot e^{j149^0}.$$

Phase apparent, real and reactive powers of receiver:

$$\underline{S}_{ph} = \underline{S}_a = \underline{S}_b = \underline{S}_c = \dot{U}_a \cdot \overset{*}{I}_a = 117 \cdot e^{-j3^0} \cdot 10 \cdot e^{j34^0} = 1170 \cdot e^{j31^0} = 1003 + j603;$$

$S_{ph} = 1170$ VA; $P_{ph} = 1003$ W; $Q_{ph} = 603$ var.

Apparent, real and reactive powers of receiver:

$$\underline{S} = \sum \underline{S}_{ph} = 3\sum \underline{S}_{ph} = 3510 \cdot e^{j51} = 3010 + j1810.$$

$S = 3510$ VA, $P = 3010$ W, $Q = 1810$ var.

A vector diagram of voltages and currents is shown in figure 25.2.

Task 25.2 Three-phase electricity consumer with active and reactive resistances R_{ab}, R_{bc}, R_{ca}, X_{ab}, X_{bc}, X_{ca} is connected by a triangle (fig. 25.3) and are included in the three-phase circuit with a linear voltage U_L on symmetric power. Determine the phase I_{ph} and line currents I_L, real P, reactive Q and apparent S power of each phase and the entire electrical circuit. Construct the vector diagram of currents and voltages. The initial data are shown in table 25.2.

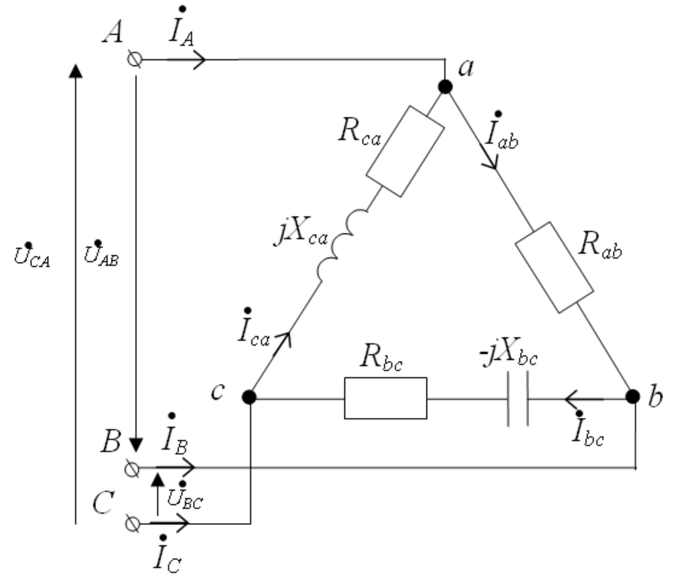


Figure 25.3: Scheme of a circuit to the task 25.2

Table 25.2

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
***U*L, V**	220	220	220	220	127	127	127	127	100	100	100
***R*ab, Ω**	10	12	14	10	10	14	14	10	8	10	10
***X*ab, , Ω**	0	0	0	0	0	0	0	0	0	0	0
***R*bc, , Ω**	5	8	10	8	5	10	8	12	12	8	10
***X*bc, , Ω**	5	8	10	8	5	10	8	12	12	8	10
***R*ca, , Ω**	5	8	10	10	5	10	10	12	12	8	10
***X*ca, , Ω**	5	8	10	10	5	10	10	12	12	8	10

The solution for version K.

Phase currents of the consumer:

$$I_{ab} = \frac{U_{ab}}{Z_{ab}} = \frac{U_{ab}}{R_{ab}} = \frac{220}{10} = 22 \text{ A};$$

$$I_{bc} = \frac{U_{bc}}{Z_{bc}} = \frac{U_{bc}}{\sqrt{R^2{}_{bc} + X^2{}_{bc}}} = \frac{220}{\sqrt{5^2 + 5^2}} = 31.11 \text{ A};$$

$$I_{ca} = \frac{U_{ca}}{Z_{ca}} = \frac{U_{ca}}{\sqrt{R^2{}_{ca} + X^2{}_{ca}}} = \frac{220}{\sqrt{5^2 + 5^2}} = 31.11 \text{ A}.$$

Vector diagram of currents and voltages with respect to the character of the load is shown in figure 25.4.

The active components of the phase currents:

$$I'_{ab} = I_{ab} \cos\varphi_{ab} = I_{ab}\frac{R_{ab}}{Z_{ab}} = I_{ab}\frac{R_{ab}}{\sqrt{R^2_{ab} + X^2_{ab}}} = 22\frac{10}{\sqrt{10^2 + 0}} = 22 \text{ A};$$

$$I'_{bc} = I_{bc} \cdot \cos\varphi_{bc} = I_{bc}\frac{R_{bc}}{Z_{bc}} = I_{bc}\frac{R_{bc}}{\sqrt{R^2_{db} + X^2_{bc}}} = 31.11\frac{5}{\sqrt{5^2 + 5^2}} = 22 \text{ A};$$

$$I'_{ca} = I_{ca} \cdot \cos\varphi_{ca} = I_{ca}\frac{R_{ca}}{Z_{ca}} = 31.11\frac{5}{\sqrt{5^2 + 5^2}} = 22 \text{ A}.$$

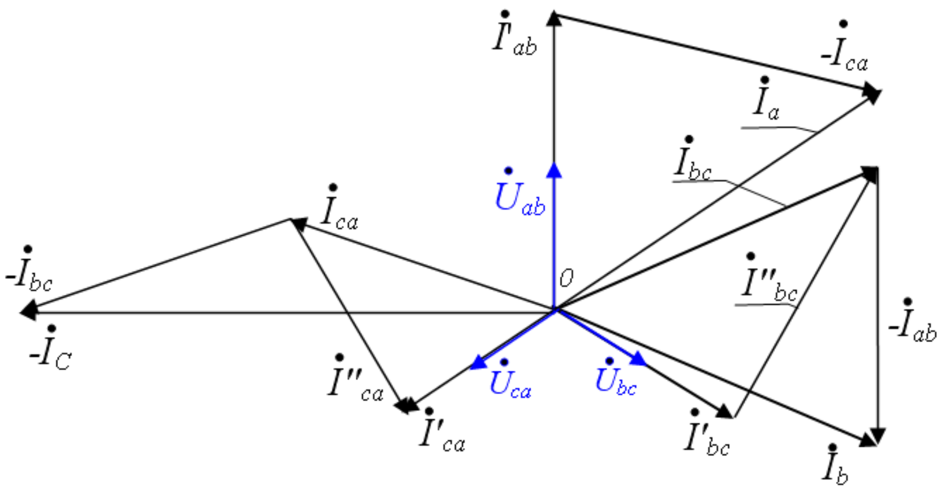


Figure 25.4: Vector diagram of currents and voltages to the task 25.2

Reactive components of phase currents:

$$I''_{ab} = I_{ab} \cdot \sin\varphi_{ab} = I_{ab}\frac{X_{ab}}{Z_{ab}} = I_{ab}\frac{X_{ab}}{\sqrt{R^2_{ab} + X^2_{ab}}} = 22\frac{0}{\sqrt{5^2 + 0}} = 0;$$

$$I''_{bc} = I_{bc} \cdot \sin\varphi_{bc} = I_{bc}\frac{X_{bc}}{Z_{bc}} = I_{bc}\frac{X_{bc}}{\sqrt{R^2_{bc} + X^2_{bc}}} = 31.11\frac{5}{\sqrt{5^2 + 5^2}} = 22 \text{ A};$$

$$I''_{ca} = I_{ca} \cdot \sin\varphi_{ac} = I_{ca}\frac{X_{ca}}{Z_{ca}} = I_{ca}\frac{X_{ca}}{\sqrt{R^2_{ca} + X^2_{ca}}} = 31.11\frac{5}{\sqrt{5^2 + 5^2}} = 22 \text{ A}.$$

Linear currents of consumers is defined by a vector diagram (fig. 25.4).
Real (active) powers of phases of consumer:

$$P_{ab} = U_{ab} \cdot I'_{ab} = 220 \cdot 22 = 4840 \text{ W};$$

$$P_{bc} = U_{bc} \cdot I'_{bc} = 220 \cdot 22 = 4840 \text{ W};$$

$$P_{ca} = U_{ca} \cdot I'_{ca} = 220 \cdot 22 = 4840 \text{ W}.$$

Reactive powers of phase of user:

$Q_{ab} = U_{ab} \cdot I''_{ab} = 220 \cdot 0 = 0$;

$Q_{bc} = U_{bc} \cdot I''_{bc} = 220 \cdot 22 = -4840$ var;

$Q_{ca} = U_{ca} \cdot I''_{ca} = 220 \cdot 22 = 4840$ var.

Apparent powers of phases of consumer:

$S_{ab} = P_{ab} = U_{ab} \cdot I'_{ab} = 220 \cdot 22 = 4840$ VA;

$S_{bc} = U_{bc} \cdot I_{bc} = 220 \cdot 31.11 = 6844.2$ VA;

$S_{ca} = U_{ca} \cdot I_{ca} = 220 \cdot 31.11 = 6844.2$ VA.

Powers of the whole circuit:

real $P = P_{ab} + P_{bc} + P_{ca} = 4840 + 4840 + 4840 = 14520$ W;

reactive $Q = Q_{ab} + Q_{bc} + Q_{ca} = 0 - 4840 + 4840 = 0$ var;

apparent $S = \sqrt{P^2 + Q^2} = \sqrt{14520^2 + 0^2} = 14520$ VA = 14.52 kVA.

26 ELECTRICAL MEASUREMENTS

Task 26.1 According to table 26.1 determine the largest absolute and relative error of the results of measuring of voltage with a voltmeter accuracy class Kт with the upper limit of measurement $U_{\max}$, if the readings U_x. Determine the smallest value of the voltage which can be measured by this voltmeter at the highest allowable measurement error ±10%.

Table 26.1

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Kт	2.5	1.5	1.0	0.5	2.5	1.5	1.0	0.5	0.2	0.2	1.0
$U_{\max}$, **V**	150	300	150	150	300	150	150	300	100	150	300
U_x, **V**	90	210	110	80	250	60	120	240	60	70	220

The solution for the version K.

Because the accuracy class of the voltmeter, defined as the number of 2.5, it is defined by the most reduced error, maximum absolute error is defined by the ratio (6.9)

$$\Delta X = \frac{\gamma_m \cdot X_{\max}}{100} = \pm\frac{2,5 \cdot 150}{100} = \pm 3.75 \text{ V}.$$

The largest relative error of measurement is defined by the ratio (8.2)

$$\delta = \frac{\Delta X}{X} \cdot 100\% = \frac{\pm 3.75 \cdot 100\%}{90} = \pm 4.17\%.$$

The lowest value of voltage which can be measured by this voltmeter with maximum error ±10%, will be found using the ratio (8.5)

$$X = \frac{\gamma_m \cdot X_{\max}}{\delta} = \frac{2.5 \cdot 150}{10} = 37.5 \text{ V}.$$

Task 26.2 By digital voltmeter of accuracy class c/d voltage U_x was measured at the limit of measurement U_{max}. Determine the largest absolute and relative measurement error. Write the result with the received error. The initial data are shown in table 26.2.

Table 26.2

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
c/d	0.2/0.1	0.2/0.1	0.2/0.1	0.2/0.1	0.5/02	0.5/0.2	0.5/0.2	1.5/1.0	1.5/1.0	1.5/1.0	1.5/1.0
U_{max}, **V**	99,99	100	100	150	150	150	150	300	300	300	300
U_x, **V**	50.2	64.8	78.4	110	69	135	48.3	179	246	261	285

The solution for option K.

The largest relative error in the measurement by instrument, accuracy class of which is defined as the ratio *c/d*, and it is determined by the ratio (8.6)

$$\delta = \pm\left[c + d\left(\frac{U_{Lar}}{U}\right) - 1\right]\% = \pm\left[0.2 + 0.1\left(\frac{99.99}{50.2} - 1\right)\right]\% = \pm 0.299\%.$$

The largest absolute error of measurement is determined from the relation (8.2)

$$\Delta U = \frac{\delta \cdot U}{100} = \pm\frac{0.299 \cdot 50.20}{100} = \pm 0.1501 \text{ V}.$$

The measurement result with the specified error will be

$$U_0 = U - \Delta U = 50.20 \pm 0.15 \text{ V}.$$

Task 26.3 The limit of measure of current by ammeter with shunt I_{rat}. It is made on the basis of the magnetoelectric milliammeter with resistance R_{dev}, the limit of the measurement $I_{dev.max}$ and scale α_n divisions. Determine the resistance of the shunt R_{sh} and the circuit current I, which includes an ammeter, if its arrow deviated to α_X divisions (see fig. 7.10). The initial data are shown in table 26.3.

Table 26.3

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
I_{rat}, **A**	2.5	2.5	2.5	5	5	5	5	10	10	10	10
R_{dev}, **Ω**	0.4	0.6	0.4	1	1	1	1	2	2	2	2
$I_{dev.max}$, **mA**	500	250	500	250	250	500	500	250	250	500	500
α_n, **divisions**	50	50	100	50	50	100	100	75	75	150	150
α_X, **divisions**	40	30	60	40	45	60	80	50	60	80	120

The solution for version K.

The division ratio of current (7.13) $p = \frac{I_H}{I_{np.\max}} = \frac{2{,}5}{500 \cdot 10^{-3}} = 5$.

The resistance of the shunt (7.14) $R_{sh} = \frac{R_{dev}}{p-1} = \frac{0.4}{5-1} = 0.1$ Ω.

Constant of ammeter with shunt $C_i = \frac{I_{rat}}{\alpha_H} = \frac{2.5}{50} = 0{,}05$ A/division.

The current in the circuit $I \ = C_i \cdot \alpha_X = 0.05 \cdot 40 = 2$ A.

Task 26.4 Double-limit milliammeter (fig. 26.1) is made on the basis of the magnetoelectric microammeter with current of full deflection $I_{\text{dev.max}}$ and resistance R_{dev}. To determine the resistances of resistors R_1 and R_2 double-limit shunt, if the limits of the milliammeter $I_{1.\max}$ and $I_{2.\max}$.

The initial data are shown in table 26.4.

Table 26.4

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$I_{\text{dev.max}}$, **microampe re**	100	100	100	100	150	150	150	150	300	300	300
R_{dev}, **Ω**	2000	2400	2400	2000	2000	2400	2400	2000	2000	2400	2400
$I_{1.\max}$, **mA**	10	50	75	50	75	50	75	10	50	75	75
$I_{2.\max}$, **mA**	75	100	150	100	150	100	150	75	150	250	300

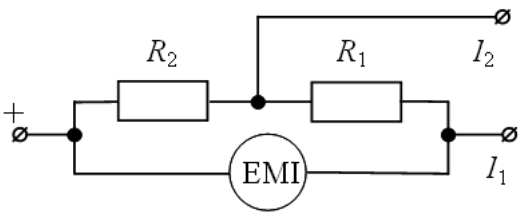


Figure 26.1: The scheme to the task 26.4

The solution for version K.

The bridging coefficients (7.13):

$$p_1 = \frac{I_{1.\max}}{I_{dev.\max}} = \frac{10 \cdot 10^{-3}}{100 \cdot 10^{-6}} = 100;$$

$$p_2 = \frac{I_{2.\max}}{I_{dev.\max}} = \frac{75 \cdot 10^{-3}}{100 \cdot 10^{-6}} = 750.$$

Using (7.14) let's write the system of equations:

$$\begin{cases} R_1 + R_2 = \dfrac{R_{dev}}{p_1 - 1}; \\ R_2 = \dfrac{R_1 + R_{dev}}{p_2 - 1}; \end{cases}$$

from which we find $R_2 = \frac{R_{dev} \cdot p_1}{(p_1 - 1) \cdot p_2} = \frac{2000 \cdot 100}{(100-1) \cdot 750} = 2.69$ Ω.

Then $R_1 = \frac{R_{dev}}{p_1 - 1} - R_2 = \frac{2000}{100-1} - 2{,}69 = 17.51$ Ω.

27 TRANSFORMERS

Task 27.1 Determine the transformation coefficient n of the transformer, the number of turns of the primary winding w_1 when the number of turns of the secondary winding w_2, and rated currents I_{1rat} and I_{2rat} in the windings of single-phase transformer with a rated power S_{1rat} connected to the supply main with voltage U_{1rat} = 127, the voltage at the connection terminals of the secondary winding at idle U_{20}. Initial data for calculation are given in table 27.1

Table 27.1

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
w_2, turns	40	50	60	70	80	80	70	60	60	40	50
S_{1rat}, кVA	3	3,6	4	6	10	5	6,6	6	4,4	3,8	4,2
U_{1rat}, V	127	127	220	220	220	127	220	220	127	127	127
U_{20}, V	60	100	40	60	80	40	40	60	50	40	40

The solution for version K.

The transformation coefficient of the transformer

$$n = \frac{w_1}{w_2} = \frac{E_1}{E_2} = \frac{U_1}{U_{20}} = \frac{127}{60} = 2.11 \ .$$

As $U_{20} = E_2$, at idle the transformer the voltage drop on the primary winding is negligible. The number of turns of the primary winding:

$$w_1 = n \cdot w_2 = 2.11 \cdot 40 = 84.4 \,.$$

The rated current of the primary winding (including total power of windings $S_1 \cong S_2$):

$$I_{1rat} = \frac{S_{1rat}}{U_{1rat}} = \frac{3 \cdot 1000}{127} = 23.6 \ \text{A}.$$

The rated current of the secondary winding of the transformer (including $U_{2НОМ} = U_{20}$)

$$I_{2rat} = \frac{S_{1rat}}{U_{20}} = \frac{3000}{60} = 50 \ \text{A}.$$

Task 27.2 Determine the transformation coefficient n of three-phase transformer and the rated active values of the primary and secondary phase $U_{1ph.rat}$, $U_{2ph.rat}$ and linear $U_{2L.rat}$ voltages, at the connection of the windings on schemes "star" and "star - triangle". The primary winding has a number of turns per phase w_1, secondary – w_2. Nominal line voltage of primary winding $U_{1L.rat}$. Initial data for calculation are listed in table 27.2.

Table 27.2

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
w_1, turns	2002	1980	1200	1600	1400	2300	2000	1900	1800	1700	1980
w_2, turns	134	126	100	106	94	140	130	127	120	112	126
$U_{1L.rat}$, V	6000	6000	3300	3300	3300	10000	10000	6000	6000	6000	6000

The solution for version K.

The transformation coefficient of phase voltage

$$n = \frac{w_1}{w_2} = \frac{2002}{134} = 15 .$$

Rated primary phase voltage of transformer

$$U_{1ph.rat} = \frac{U_{1L.rat}}{\sqrt{3}} = \frac{6000}{1.73} = 3470 \text{ V}.$$

Rated secondary voltage at the connection of the windings of the transformer according to the scheme "star-star":

linear $U_{2L.rat} = U_{1L.rat} / n = 6000/15 = 400$ V;

phase $U_{2ph.rat} = U_{2L.rat} / \sqrt{3} = 400/1.73 = 230$ V.

The transformation coefficient of the transformer on connection of the windings in a schema “star – star”:

linear $n_{Y.л} = U_{1L.rat} / U_{2L.rat} = 6000/400 = 15$;

phase $n_{Y.ph} = U_{1ph.rat} / U_{2ph.rat} = 3479/230 = 15$.

The transformation coefficient of the transformer on connection of the windings on a "star - triangle":

linear $n_{\Delta.л} = U_{1L.rat} / U_{2L.rat} = 6000/230 = 26$;

phase $n_{\Delta.ph} = U_{1ph.rat} / U_{2ph.rat} = 3479/230 = 15$.

Task 27.3 Three-phase transformer has a capacity of S_{rat} , the nominal voltage of the primary and secondary windings $U_{1.rat}$, $U_{2.rat}$ when the circuit frequency f = 50 Hz. Idle losses at rated voltage P_{idle} = 180 W, short circuit losses P_{shc} = 1000 W. Determine the COP of the transformer at specified $\cos\varphi_2$ and the load coefficient β, changing in the range from 0.4 to 1. Build the dependence of COP from load coefficient. Initial data for calculation are given in table 27.3.

Table 27.3

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
S_{rat}, kVA	40	63	25	100	100	160	160	250	250	400	400
$U_{1L.rat}$, kV	10	10	10	10	10	10	10	10	10	10	10
$U_{2L.rat}$, kV	0.4	0,4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
P_{idle}, kW	0.18	0.24	0.14	0.49	0.49	0.73	0.73	1.05	1.05	1.45	1.45
P_{shc}, kW	1.0	1.4	0.8	1.97	1.97	2.65	2.65	3.7	3.7	5.5	5.5
$\cos\varphi_2$	0.9	0.8	0.84	0.74	0.86	0.76	0.88	0.73	0.87	0.68	0.84

The solution for version K.

To determine the COP of the transformer we will use the relation (9.16). COP on the load coefficient $\beta = 1{,}0$

$$\eta = \frac{\beta \cdot S_{rat} \cdot \cos\varphi_2}{\beta \cdot S_{rat} \cdot \cos\varphi_2 + \Delta P_S + \Delta P_{MAG} \cdot \beta^2} = \frac{\beta \cdot S_{rat} \cdot \cos\varphi_2}{\beta \cdot S_{rat} \cdot \cos\varphi_2 + P_{idle} + P_{shc} \cdot \beta^2} =$$

$$= \frac{1{,}0 \cdot 40 \cdot 0{,}9}{1{,}0 \cdot 40 \cdot 0{,}9 + 0{,}18 + 1 \cdot 1^2} = 0{,}968$$

The calculation results for other values of the load coefficient are shown in table 27.4. Figure 27.1 shows the resulting dependence of the COP from the load coefficient of the transformer.

Table 27.4

β	0.1	0.2	0.3	0.5	0.7	0.8	0.9	1.0
η	0.949	0.970	0.975	0.976	0.974	0.972	0.970	0.968

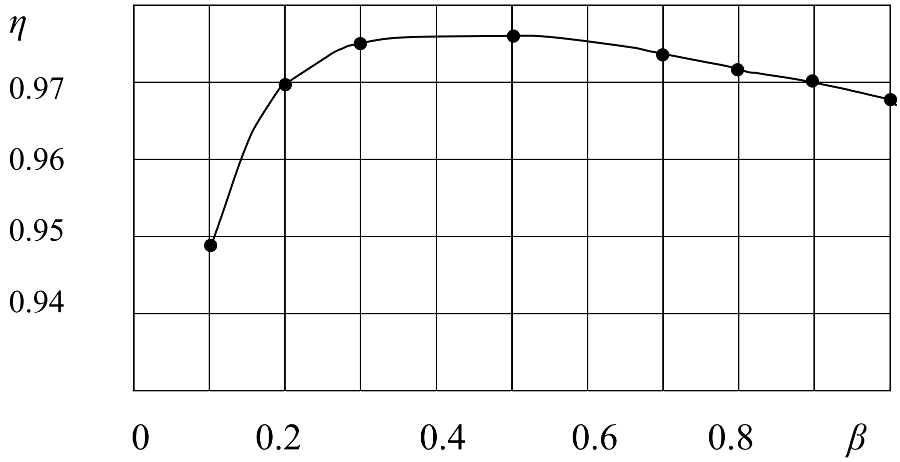


Figure 27.1: Calculated dependence of the COP from β

Task 27.4 Determine the parameters of a simplified Γ-shaped circuit of the transformer 9 (fig. 9.4, *b*) with a rated power S_{1rat}. Transformer windings are connected in star schema, the nominal line voltage of primary and secondary windings $U_{1L.rat}$, $U_{2L.rat}$, the idle current I_{idle},=%· I_{rat} power of idle P_{idle}, short circuit voltage U_{shc}, short-circuit power P_{shc}. Initial data for calculation are shown in table 27.5.

Table 27.5

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
S_{1rat}, **kVA**	50	63	25	100	100	160	160	250	250	400	400
$U_{1L.rat}$, **kV**	6	10	10	10	10	10	10	10	10	10	10
$U_{2L.rat}$, **kV**	0.53	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
I_{idle}, **%**	7.0	6.3	8.0	2.6	2.6	2.4	2.4	2.3	2.3	2.1	2.1
P_{idle}, **kW**	0.35	0.24	0.14	0.49	0.49	0.73	0.73	1.05	1.05	1.45	1.45
U_{shc}, **%**	1.0	1.4	0.8	1.97	1.97	2.65	2.65	3.7	3.7	5.5	5.5
P_{shc}, **kW**	0.325	0.8	0.84	0.74	0.86	0.76	0.88	0.73	0.87	0.68	0.84

The solution for version K.

Rated phase (line) currents of transformer (counting $S_{1rat} \approx S_{2rat}$):

$$I_{1rat} = \frac{S_{1rat}}{\sqrt{3} \cdot U_{1rat}} = \frac{50 \cdot 10^3}{1.73 \cdot 6000} = 4.82 \text{ A},$$

$$I_{2rat} = \frac{S_{1rat}}{\sqrt{3} \cdot U_{2rat}} = \frac{50 \cdot 10^3}{1.73 \cdot 525} = 55 \text{ A}.$$

Rated phase voltage of transformer

$$U_{1ph.rat} = U_{1L.rat} / \sqrt{3} = 6000/1.73 = 3460 \text{ V},$$

$$U_{2ph.rat} = U_{2L.rat} / \sqrt{3} = 525/1.73 = 303 \text{ V}.$$

The idle current of the transformer

$$I_{idle} = \frac{7\%}{100} \cdot I_{1rat} = 0{,}07 \cdot 4.82 = 0.338 \text{ A},$$

The active resistance of the magnetizing circuit of the Γ-shaped equivalent schemes

$$R_0 = \frac{P_{idle}}{3 \cdot I_{idle}^2} = \frac{350}{3 \cdot 0.338^2} = 1040 \ \Omega.$$

Resistance of magnetizing circuit:

impedance $Z_0 = U_{1ph.rat} / I_{idle} = 3460/0.338 = 10250 \ \Omega$;

inductive $X_0 = \sqrt{Z_0^2 - R_0^2} = \sqrt{10250^2 - 1040^2} = 9800 \ \Omega$.

Resistance of short circuit transformer:

impedance $Z_\kappa = U_\kappa \dfrac{U_{1.rat}^2}{S_{rat}} = 0.055 \dfrac{6000^2}{50000} = 39.6 \ \Omega$;

active $R_\kappa = R_1 + R_2' = \dfrac{P_{idle}}{3 \cdot I_{1rat}^2} = \dfrac{325}{3 \cdot 4.82} = 4 \ \Omega$;

reactive $X_\kappa = X_1 + X_2' = \sqrt{Z_\kappa^2 - R_\kappa^2} = \sqrt{39.6^2 - 4^2} = 39.5 \ \Omega$.

28 DC MACHINES

Task 28.1 Determine the MDC of the generator with parallel actuation and the resistance of the actuation circuit (fig. 10.12) for the given values of voltage on the generator connection terminals *U*, the load current *I*, the resistance of the armature winding R_a and actuation current I_{ac}. The initial data are shown in table 28.1.

Table 28.1

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
***U*, V**	115	120	110	116	118	120	112	114	115	116	118
***I*, 32 A**	32	34	30	31	33	34	30	31	30	28	28
R_a, Ω	0.18	0.2	0.2	0.19	0.21	0.21	0.18	0.16	0.16	0.17	0.19
I_{ac}, A	1.35	1.4	1.42	1.37	1.4	1.42	1.3	1.3	1.3	1.4	1.5

The solution for version K.

The resistance of the actuation circuit

$R_{ac} = \frac{U}{I_{ac}} = \frac{115}{1,35} = 85,2\ \Omega.$

Current of armature $I_A = I - I_{ac} = 32 - 1,35 = 30,65$ A.

EMF of generator (from 10.11)

$E = I_A \cdot R_A + I_{ac} \cdot R_{ac} = 30.65 \cdot 0.18 + 1.35 \cdot 85.2 = 121$ V.

Task 28.2 Four-pole DC generator with parallel actuation develops rated power P_{rat} kW at a voltage V and the speed of rotation of the armature n turn /minute. Winding of armature consists of N conductors connected in four parallel branches (α = 2). The total resistance of the actuation circuit R_{ac} Ω, the resistance of the armature winding R_A Ω. Determine:

- the magnetic flux of one pair of poles;
- the resistance of one branch of the armature winding.

The initial data are shown in table 28.2.

Table 28.2

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
P_{rat}, кW	25	24	22	26	25	23	22	20	23	21	25
U_{rat}, V	115	118	116	118	112	120	116	110	114	118	112
***n*, turns/minute**	1300	1200	1100	1400	1300	1260	1360	1200	1250	1170	1120
***N*, coils**	300	290	310	280	280	290	300	280	270	260	280
R_{ac}, Ω	12.5	13.8	13	14	12	13	14	12	14	12	13
R_A, Ω	0.024	0.02	0.03	0.02	0.3	0.26	0.2	0.3	0.2	0.3	0.32

The solution for version K.

Determine the armature current of the generator

$$I_A = I_{rat} + I_{ac} = \frac{P_{rat}}{U_{rat}} + \frac{U_{rat}}{r_{AC}} = \frac{25000}{115} + \frac{115}{12.5} = 226\ \text{A}.$$

Let us calculate EMF of generator

$$E = U_{rat} + I_A \cdot R_A = 115 + 226 \cdot 0.024 = 120.5\ \text{V}.$$

Find the magnetic flux of the pair of poles from the equation of EMF of generator (10.1)

$$E = \frac{p \cdot N}{60 \cdot \alpha} n \cdot \Phi,$$

$$\Phi = \frac{60 \cdot \alpha \cdot E}{p \cdot N \cdot n} = \frac{60 \cdot 2 \cdot 120.5}{2 \cdot 300 \cdot 1300} \cdot 1.85 \cdot 10^{-2}, \text{ Wb}.$$

The resistance of one of the parallel branches of the armature is four times larger than the total resistance of the armature winding, i.e.

$$R_{1A} = 4 \cdot R_A = 4 \cdot 0.024 = 0.096\ \Omega.$$

Task 28.3 The EMF of the armature of a DC generator with mixed actuation E V (see fig. 10.15). Determine the voltage at the generator connection terminals U and the resistance of the parallel winding $R_{ac.sh}$, at given resistance of the armature R_A, the serial resistance of the winding $R_{a.ser}$, the actuation current $I_{ac.sh}$ and the load current I. The initial data are shown in table 28.3.

Table 28.3

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
E, V	240	230	220	210	200	190	180	160	150	140	130
R_A, Ω	0.125	0.11	0.1	0.12	0.11	0.12	0.13	0.13	0.1	0.2	0.25
$R_{a.ser}$, Ω	0.025	0.03	0.04	0.035	0.05	0.02	0.03	0.025	0.04	0.05	0.03
$I_{ac.sh}$, A	2	2.2	2.4	2.6	2.8	1.9	2	2.2	2.3	2.4	2.5
I, A	64	62	60	58	56	54	52	50	48	46	44

The solution for version K.

Determine the armature current of the generator

$$I_A = I + I_{ac \cdot sh} = 64 + 2 = 66 \text{ A}.$$

Let us calculate voltage at the connection terminals of the load

$$U = E - I_A (R_A + R_{a.ser}) = 240 - 66 \cdot 0.15 = 230.1 \text{ V}.$$

The voltage at the connection terminals of the parallel circuit of actuation (on connection terminals of armature)

$$U_A = E - I_A \cdot R_A = 240 - 66 \cdot 0.125 = 231.7 \text{ V}.$$

The resistance of the parallel circuit of actuation

$$R_{ac.sh} = \frac{U_A}{I_{Ac.sh}} = \frac{231.7}{2} = 115.9 \ \Omega.$$

Task 28.4 Generator with parallel actuation (fig. 10.12) has the following rated data: P_{rat}; U_{rat}; n_{rat}. The resistance of the armature winding and the additional poles R_A; the resistance of the actuation circuit R_{ac}; losses in steel and mechanical losses $\Delta P_{st} + \Delta P_{mech}$ = % of P_{rat}. Determine the armature current, EMF and the COP of generator at rated load. The initial data are shown in table 28.4.

Table 28.4

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
P_{rat}, kW	25	24	22	21	20	26	28	30	32	34	36
U_{rat}, V	230	220	230	220	230	220	230	220	230	220	230
n_{rat}, turns/min	2850	2800	2600	2700	2500	2900	3000	2800	2700	2600	2800
R_A, Ω	0.15	0.14	0.13	0.15	0.16	0.18	0.15	0.14	0.13	0.16	0.17
R_{ac}, Ω	140	136	134	142	144	140	138	136	142	144	142
$\Delta P_{st}+\Delta P_{mech}$, %	3.2	3.0	2.8	3.0	3.2	3.4	3.6	3.4	3.2	3.0	3.2

The solution of task for version K.

The armature current of the generator at rated load

$$I_{A.rat} = I_{rat} + I_{ac.rat} = P_{rat}/U_{rat} + U_{rat}/R_{ac} = 100 \text{ A}.$$

EMF of generator

$$E = U_{rat} + R_A \cdot I_{Arat} = 230 + 0.15 \cdot 110 = 246.5 \text{ V}.$$

The power loss in the armature circuit

$$\Delta P_A = R_A \cdot I^2_{Arat} = 0.15 \cdot 110^2 = 1820 \text{ W}.$$

Loss in a parallel circuit of actuation

$$\Delta P_{ac} = U^2_{rat} / r_{ac} = 230^2/140 = 380 \text{ W}.$$

Loss in steel and mechanical ones

$$\Delta P_{st} + \Delta P_{mech} = 3.2 \cdot 25000 / 100 = 800 \text{ W}.$$

The mechanical power delivered to the shaft of the generator,

$$P_1 = \text{Prat} + \Sigma\Delta P = 25 + (1.82 + 0.38 + 0.8) = 28 \text{ kW}.$$

The generator COP at rated load

$$\eta = P_{rat}/P_1 = 25/28 = 0.89.$$

Task 28.5 DC machine in a mode of motor has the following rated data: P_{rat}; U_{rat}; I_{rat}; R_a; I_{ac}; n_{rat}.

Determine the required speed of rotation of the armature of DCM working in mode of generator with voltage U_G. Calculate the rated power of the generator, if to take the same saturation of the steel and heat as in motor. The initial data are shown in table 28.5.

Table 28.5

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
P_{rat}, **kW**	11	13	15	18	16	14	12	10	12	14	16
U_{rat} , **V**	220	230	240	230	220	230	240	230	220	230	240
n_{rat}, **turn/min**	1000	960	900	840	800	1100	1180	960	900	860	800
I_{rat} , **A**	62	64	66	68	66	64	62	60	62	64	66
R_A, **Ω**	0.09	0.1	0.14	0.16	0.08	0.1	0.12	0.14	0.16	0.13	0.15
I_{ac} , **A**	3	3.4	3.6	3.8	4	3,8	3.6	3	3.2	3.4	3.6
U_G , **V**	230	240	230	240	230	240	230	240	230	240	230

The solution of task for version K.

Determine the armature current of the motor and generator

$$I_{A.m} = I_{rat} - I_{ac} = 62 - 3 = 59 \text{ A} = I_{rat.g} .$$

Let us calculate EMF of the motor and generator without taking into account the voltage drop in the brush contact:

$$E_m = U_{rat} - I_{am} \cdot R_a = 220 - 59 \cdot 0.09 = 214.7 \text{ V};$$

$$E_G = U_G + I_{ag} \cdot R_я = 230 + 59 \cdot 0.09 = 235.3 \text{ V}.$$

The speed of rotation of the armature of the generator is determined from the relation

$$\frac{E_{Г}}{E_{ДВ}} = \frac{c \cdot n_{Г} \cdot \Phi_{Г}}{c \cdot n_{ДВ} \cdot \Phi_{ДВ}} = \frac{n_{Г}}{n_{ДВ}},$$

as on condition $\Phi_G = \Phi_m$,

$$n_G = \frac{E_G}{E_M} n_M = \frac{235.3}{214.7} 1000 = 1096 \text{ turns/min.}$$

If to consider the voltage drop in the brush contact, for example when $\Delta U_{щ} = 2$ V, then

$$E'_m = U_{rat} - I_{am} \cdot R_A - \Delta U_B = 214{,}7 - 2 = 212.7 \text{ V},$$
$$E'_G = U_G + I_{AG} \cdot R_A + \Delta U_B = 235{,}3 + 2 = 237.3 \text{ V},$$

and the speed of rotation should be

$$n_G = \frac{E'_G}{E'_m} n_m = \frac{237.3}{212.7} 1000 = 1116 \text{ turns/min.}$$

Lets us find the rated current and rated power:

$$I_{rat.G} = I_{A.G} - I_{ac} = 59 - 3 = 56 \text{ A},$$
$$P_{ratG} = U_G \cdot I_{rat\ G} = 230 \cdot 56 = 12880 \text{ W} = 12.9 \text{ kW}.$$

Task 28.6 Determine the speed of rotation of the armature of the generator with P_{rat} = 16.5 kW, U_{rat} = 230 V, $n_{rat.G}$ = 1460 turns/min, R_A = 0.18 Ω, R_{ac} = 82 Ω when work by the motor with U_m = 220 V, if to take the same saturation of the steel and the heat of the motor as in generator. Calculate the electromagnetic power of the motor.

The initial data are shown in table 28.6.

Table 28.6

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
$P_{rat.G}$, kW	16.5	18	20	22	21	19	17	15	16	18	20
$U_{rat.G}$, V	230	240	230	240	230	240	230	240	230	240	230
$n_{rat.G}$, turns/min	1460	1400	1540	1500	1460	1400	1540	1600	1500	1460	1400
R_A, Ω	0.18	0.2	0.22	0.21	0.2	0.19	0.18	0.17	0.16	0.18	0.2
R_{ac}, Ω	82	84	86	85	83	81	80	82	84	85	83
U_m, V	220	220	220	230	210	220	210	220	210	220	220

The solution of task for version K.

Find the armature current of the generator and motor:

$$I_{A.G} = I_{rat} + I_{ac} = P_{rat.G}/U_{rat.G} + U_{rat}/R_{ac} =$$
$$= 16500/230 + 230/82 = 72 + 2.8 = 74.8 \text{ A}.$$
$$I_{a.m} = I_{a.g} \textit{ (on condition)}.$$

The current consumed by the motor from the network, equal

$$I_m = I_{a.m} + I_{ac} = 74.8 + 2.8 = 77.6 \text{ A}.$$

Let us calculate EMF of the generator and motor:

$$E_G = U_{rat.G} + I_{a.G} \cdot R_a = 230 + 74.8 \cdot 0.18 = 243.5 \text{ V};$$
$$E_m = U_m - I_{am} \cdot R_a = 220 - 74.8 \cdot 0.18 = 206.5 \text{ V}.$$

The speed of rotation of the motor armature is found from the ratio

$$n_M = \frac{E_M}{E_G} n_{rat.G} = \frac{206.5}{243.5} 1460 = 1238 \text{ turns/min.}$$

Let us define electromagnetic power of engine

$$P_{em} = E_m \cdot I_{a.m} = 206.5 \cdot 74.8 = 15446 \text{ W} = 15.5 \text{ kW.}$$

Rated power of the motor will be slightly less.

29 ASYNCHRONOUS ELECTRIC MACHINES

Task 29.1 A six-pole motor at rated load works with slip s = 4%. Circuit frequency f_1 = 50 Hz. Determine the rotation speed of the motor. The initial data are given in table 29.1.

Table 29.1

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Number of poles	6	6	6	6	6	6	4	4	4	4	4
***s*, %**	4	3.6	3.2	2.8	4.4	4.8	3.3	3.5	3.8	4.2	4.5
f_1 = 50 Hz.	50	60	50	60	50	60	50	60	50	60	50

The solution of task for the version K.

The number of pairs of poles p = 6/2 = 3 ; synchronous speed

$n_1 = f_1 \cdot 60/\text{p} = 50 \cdot 60/3 = 1000$ turns/min.

The speed of rotation of the rotor

$n_2 = n_1 \cdot (1 - s) = 1000 \cdot (1 - 0{,}04) = 960$ turns/min.

Task 29.2 The rotation speed of the asynchronous motor in rated load is n_2 turns/minute, at idle - n_{idle} turns/minute. Determine the slip under load and idle; f_1 = 50 Hz. The frequency scale of rotation: 3000, 2200, 1600, 1500, 1000, 850, 800, 750, 650, 600 turns/minute.

The initial data are shown in table 29.2.

Table 29.2

Parameter	**Version of task**										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
N_{idle}, turns/min	2940	2160	1550	820	630	580	580	740	730	780	840
n_2, turns/min	2850	2040	1320	740	560	510	490	680	660	690	760
f_1 = 50 Hz	50	50	50	50	60	60	50	60	50	60	50

The solution of task for version K.

The synchronous speed for a given motor n_1 = 3000 turns/min (the nearest big). Slip under load

$$s = \frac{n_1 - n_2}{n_1}100 = \frac{3000 - 2850}{3000}100 = 5\% \ ;$$

at idle

$$s = \frac{3000 - 2940}{3000}100 = 2\% .$$

Task 29.3 Motor with contact rings is included in the circuit with a voltage U_R. Is measured on open rings of the rotor voltage U_2. The number of turns of the phase windings w_1 = 60, winding coefficient k_1 = 0.94, the windings of the rotor – w_2 = 36, k_2 = 0.96. The windings are connected in star. Circuit frequency f_1 Hz.

Determine the flow passing through the poles of the motor, and the stator EMF E_1. The initial data are shown in table 29.3.

Table 29.3

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
U_R, V	380	380	220	220	400	400	400	390	390	380	380
U_2, V	228	224	129	127	300	292	268	240	236	210	216
w_1, turns	60	60	54	54	66	68	70	64	62	62	60
k_1	0.94	0.92	0.94	0.92	0.95	0.94	0.95	0.95	0.94	0.94	0.93
w_2, turns	36	35	32	30	40	38	42	40	38	38	36
k_2	0.96	0.94	0.96	0.92	0.93	0.96	0.96	0.96	0.95	0.96	0.94
f_1, Hz	50	50	50	50	60	60	60	60	50	50	50

The solution of task for version **K.**

Phase EMF of stationary rotor

$$E_{2S} = \frac{U_2}{\sqrt{3}} = \frac{228}{1.73} = 132 \text{ V.}$$

Flux $\Phi = \dfrac{E_{2S}}{4.44 f_1 \cdot w_2 \cdot k_2} = \dfrac{132}{4.44 \cdot 50 \cdot 36 \cdot 0.96} = 0.0173$ Wb.

The EMF of the stator

$$E_1 = 4.44 \cdot f_1 \cdot w_1 \cdot k_1 \cdot \Phi = 4.44 \cdot 50 \cdot 60 \cdot 0.94 \cdot 0.0173 = 216 \text{ V.}$$

Task 29.4 Determine the magnitude and phase of the rated current of the motor rotor with contact rings, at given E_{2rat}, R_2, X_{2rat} and s_{rat}.

The initial data are shown in table 29.4.

Table 29.4

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
E_{2rat}, V	132	130	126	122	136	140	138	135	133	137	131
R_2, Ω	0.24	0.22	0.2	0.2	0.25	0.27	0.26	0.25	0.23	0.27	0.22
X_{2rat}, Ω	1.2	1.1	1.0	1.2	1.3	1.4	1.35	1.3	1.1	1.4	1.1
s_{rat}	0.035	0.03	0.04	0.02	0.04	0.06	0.05	0.02	0.03	0.04	0.035

The solution of task for version K.

$$I_{2rat} = \frac{E_{2rat} \cdot s}{\sqrt{R_2^2 + X_{2rat}^2 \cdot s_{rat}^2}} = \frac{132 \cdot 0{,}035}{\sqrt{0.24^2 + 1.2^2 \cdot 0.035^2}} = 19 \text{ A};$$

$$tg\psi_2 = \frac{X_{2rat} \cdot s_{rat}}{R_2} = \frac{1.2 \cdot 0.035}{0.24} = 0.175; \qquad \psi_2 = 9055' .$$

If at determination of the rated current of the rotor to neglect the inductive resistance $X_2 = X_{2rat} \cdot s$, then

$$I_{2rat} = \frac{E_{2rat} \cdot s}{R_2} = \frac{132 \cdot 0.035}{0.24} = 19.2 \text{ A}.$$

Task 29.5 The rated power of the motor of P_{rat} kilowatt, voltage U_{rat} V, COP η_{rat}, power factor $\cos\varphi_{rat}$, loss in steel ΔPs from P_{rat}. The power loss in the windings of the stator in the rated mode ΔPe.st from P_{rat}. Determine the consumed current and the electromagnetic power of the motor in rated mode.

The initial data are shown in table 29.5.

Table 29.5

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
P_{rat}, kW	10	11	7.5	7.5	5.5	5.5	4	4	3	2.2	2.2
U_{rat}, V	380	380	380	380	380	380	380	220	220	220	220
η_{rat}	0.88	0,9	0.78	0.84	0.86	0.88	0.86	0.85	0.84	0.84	0.82
$\cos\varphi_{rat}$	0.87	0.84	0.87	0.86	0.85	0.84	0.8	0.84	0.82	0.8	0.76
ΔP_s	0.05	0.04	0.04	0.05	0.04	0.06	0.05	0.04	0.06	0.06	0.05
$\Delta P_{e.st}$	0.03	0.05	0.03	0.03	0.02	0.05	0.04	0.03	0.04	0.03	0.03

The solution of task for version K.

Consumed power

$$P_1 = \frac{P_{rat}}{\eta_{rat}} = \sqrt{3} U_{rat} \cdot I_{rat} \cdot \cos\varphi_{rat} .$$

Rated current

$$I_{rat} = \frac{P_{rat}}{\sqrt{3} U_{rat} \cdot \cos\varphi_{rat} \cdot \eta_{rat}} = \frac{10 \cdot 10^3}{1.73 \cdot 380 \cdot 0.87 \cdot 0.88} = 20 \text{ A}.$$

Electromagnetic power in rated mode

$$P_{em} = P_1 - P_s - P_{e.st} = P_{rat}/\eta_{rat} - 0{,}05 \cdot P_{rat} - 0{,}03 \cdot P_{rat} =$$
$$= 10/0.88 - 0.05 \cdot 10 - 0.03 \cdot 10 = 10.55 \text{ kW}.$$

Task 29.6 Asynchronous short-circuit motor has the following data: nominal power P_{rat}, nominal voltage U_{rat}, COP η, $\cos\varphi = 0.89$, order of starting current $k_c = I_s/I_{rat} = 7$, the order of starting moment $k_s = M_s/M_{rat} = 1.3$ the rotor rotation speed n_2.

Determine the starting moment and starting current of the motor.

The initial data are given in table 29.6.

Table 29.6

Parameter	Version of task										
	K	0	1	2	3	4	5	6	7	8	9
P_{rat}, kW	20	22	18.5	15	11	7.5	30	37	45	55	75
U_{rat}, V	380	380	380	380	380	380	380	380	380	380	380
η_{rat}, %	87.5	89	86.4	84	84	83	88	88	89	90	91
$\cos\varphi$	0.89	0.88	0.87	0.86	0.85	0.84	0.89	0.86	0.9	0.9	0.9
k_c	7	6.6	6.5	6.5	6.4	6.4	7	7	7	7	7
k_s	1.3	1.4	1.4	1.3	1.3	1.5	1.5	1.4	1.6	1.6	1.6
n_2, turns/min	2930	2910	2940	2950	1440	1450	960	970	980	980	980

The solution of task for version K.

Rated torque

$$M_{rat} = 975 \cdot P_{rat}/n_2 = 975 \cdot 20/2930 = 6{,}65 \text{ kg·m}.$$

Starting torque

$$M_s = M_{rat} \cdot k_s = 6.65 \cdot 1.3 = 8{,}64 \text{ kg·m}.$$

Rated current

$$I_{rat} = \frac{P_{rat}}{\sqrt{3} \cdot U_{rat} \cdot \cos\varphi \cdot \eta} = \frac{20 \cdot 10^3}{1.73 \cdot 380 \cdot 0.89 \cdot 0.875} = 39.6 \text{ A}.$$

Starting current

$$I_s = I_н \cdot k_c = 39.6 \cdot 7 = 277 \text{ A}.$$

Task 29.7 How do the starting torque and the motor current considered in task 29.6 change if for the start time the voltage is decreased by 20%.

The solution of task for version K.

The starting moment is proportional to U^2, so

$$M_{S1} = 0.8^2 \cdot M_S = 0.64 \cdot 8.64 = 5.5 \text{ kg·m},$$

(reduction on 34%).

Starting current can be considered proportional to the voltage U. Thus, it will be reduced on 20% and will be equal to

$$I_{S1} = I_S \cdot 0{,}8 = 277 \cdot 0.8 = 221.6 \text{ A}.$$

30 THE ELECTRIC DRIVE

Task 30.1 The motor is loaded with a constant that does not depend on the speed, the drag torque is M_{dt}. Total reduced moment of inertia is J.

Determine the time of acceleration of the motor to rated speed n_{rat} from the state of rest, if the average rotating moment of motor during acceleration is M. The initial data are shown in table 30.1.

Table 30.1

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
M_{dt}, **kg·m**	7	8	9	10	9	8	7	6	5	7	8
J, **kg·m·s²**	0.3	0.4	0.5	0.25	0.34	0.44	0.52	0.48	0.4	0.3	0.4
n_{rat}, **turns/min**	960	1200	1400	1600	1400	1300	1200	1100	1000	980	920
M, **kg·m**	15	16	17	18	17	16	14	13	15	17	16

The solution for option K.

To determine the acceleration time we use the equation of motion of the electric drive (14.13), from which it follows

$$dt = \frac{J}{M - M_c} d\omega,$$

how to express the time of acceleration

$$t = \int_{\omega=0}^{\omega_н} \frac{J}{M - M_c} d\omega = \frac{J}{M - M_c} \omega_н = \frac{J}{M_д} \omega_н .$$

The average of dynamic moment on the motor shaft during acceleration can be found by the formula

$$M_d = M - M_{dt} = 15 - 7 = 8 \text{ kg m}.$$

The nominal value of the angular speed is determined as follows:

$$\omega_{rat} = \frac{2\pi \cdot n_{rat}}{60} = \frac{2 \cdot 3{,}14 \cdot 960}{60} = 100{,}5 \text{ rad/s}.$$

Substituting values, we get the time of acceleration

$$t = \frac{0{,}3}{8} \cdot 100{,}5 = 3{,}8 \text{ s}.$$

Task 30.2 Determine reduced to the motor shaft M the static moment of resistance and moment of inertia of the mechanism of the lifting winch with load (fig. 30.1). Known: the weight of load G, the speed of the lifting of load V, the speed of rotation of motor n, the moment of inertia of the motor J_m, the moment of inertia of clutch and winch mechanism J_{mech}, the COP of the winch η_{winch}. The initial data are shown in table 30.2.

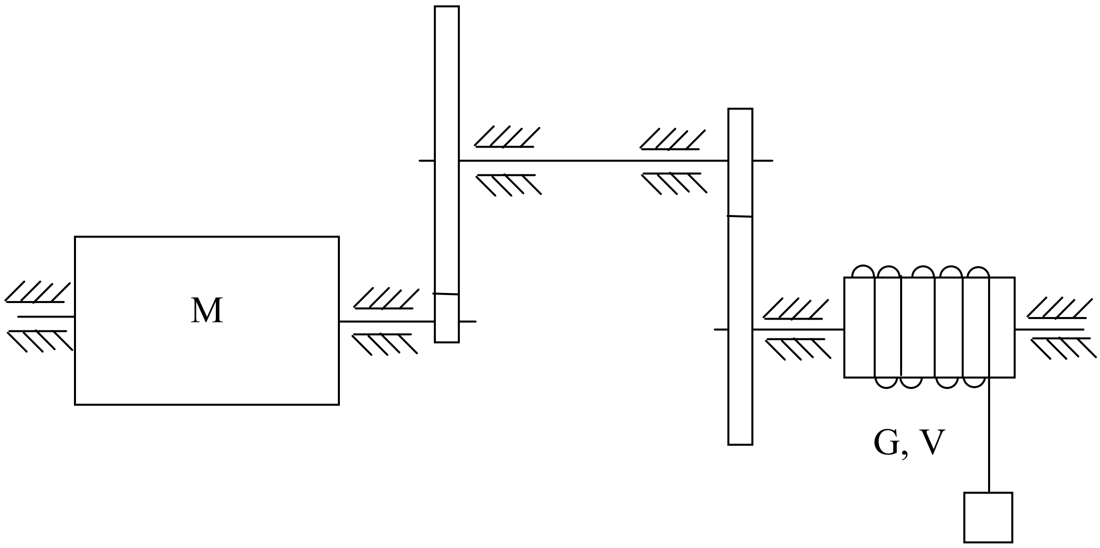


Figure 30.1: Diagram of a lifting mechanism

Table 30.2

Parameter	Version of task										
	K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
G, kg	1000	1100	1200	960	980	920	900	960	1000	1200	1000
***V*, m/min**	25	26	27	24	23	22	21	23	26	25	27
***n*, turns/min**	730	760	800	840	800	760	730	700	750	770	800
J_m, **kg m·s^2**	0.08	0.07	0.06	0.07	0.08	0.09	0.1	0.09	0.08	0.07	0.06
J_{mech}, **kg m·s^2**	0.02	0.03	0.04	0.05	0.05	0.04	0.03	0.02	0.03	0.04	0.05
η_{winch}	0.8	0.7	0.6	0.6	0.7	0.8	0.9	0.9	0.8	0.7	0.6

The solution for version K.

Given moment of inertia of steadily moving elements is determined from the equality

$$\frac{mv^2}{2} = J\frac{\omega^2}{2},$$

from where

$$J = m\left(\frac{v}{\omega}\right)^2.$$

Determine the angular speed of the motor and weight of the load

$$\omega = \frac{2\pi n}{60} = \frac{2 \cdot 3.14 \cdot 730}{60} = 76.4 \text{ rad/s};$$

$$m = \frac{G}{g} = \frac{1000}{9.81} = 101.94 \text{ kg}.$$

Then the moment of inertia of the steadily moving load

$$J_s = 101.94 \cdot \left(\frac{25}{60 \cdot 76.4}\right)^2 = 0.0032 \text{ kg m·s}^2.$$

Total reduced moment of inertia

$$J_\Sigma = J_m + J_{mech} + J_s = 0.08 + 0.02 + 0.0032 = 0.1032 \text{ kg m·s}^2.$$

Given static drag torque on the motor shaft in accordance with (14.9) is equal

$$M_s = G \cdot \rho / \eta.$$

ρ is defined by the formula

$$\rho = \frac{v}{\omega} = \frac{25}{76.4 \cdot 60} = 0.0055 \text{ m},$$

then

$$M_s = \frac{1000 \cdot 0.0055}{0.8} = 6.9 \text{ kg m}.$$

Task 30.3 Using the catalog choose asynchronous short-circuit motor for the lifting mechanism, which operates in a repeatedly short-term mode on given schedule of load. The rotation speed is n, the duration of the step of the load is t and the corresponding values of load moment *M* are given in table 30.3.

Table 30.3

Parameter		**Version of task**										
		K	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
***n*, turns/min**		900	800	800	900	900	1000	1000	950	950	1000	800
1 step	***t*, s**	40	10	15	20	30	35	20	40	35	80	90
	***M*, kg m**	5	8	12	60	45	15	80	120	95	70	25
2 step	***t*, s**	90	90	80	60	120	70	150	180	20	30	50
	***M*, kg m**	2.1	4	6	45	88	12	50	100	35	40	20
3 step	***t*, s**	430	30	45	120	80	60	30	20	120	180	30
	***M*, kg m**	0	0	0	0	0	0	0	0	0	0	0

The solution for version K.

Load diagram is shown in figure 14.8.

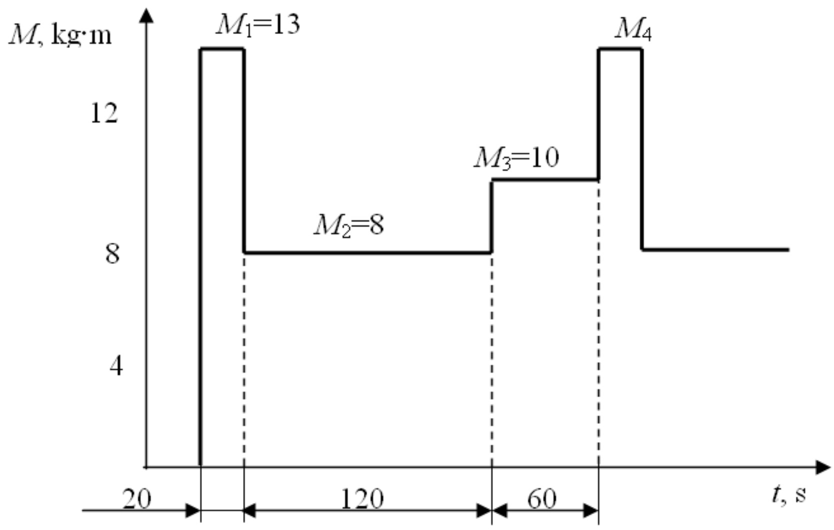


Figure 30.2: Graph of load moment

Solution. Determine the equivalent load moment

$$M_{eq} = \sqrt{\frac{M_1^2 \cdot t_1 + M_2^2 \cdot t_2 + ... + M_n^2 \cdot t_n}{t_1 + t_2 + ... + t_n}} = \sqrt{\frac{13^2 \cdot 20 + 8^2 \cdot 120 + 10^2 \cdot 60}{20 + 120 + 60}} = 9.4 \text{ kg m.}$$

The equivalent power of the motor is determined by the formula

$$P_{eq} = M_{eq} \cdot \omega = M_{eq} \frac{\pi \cdot n}{30} = 9.4 \cdot 9.81 \cdot \frac{3.14 \cdot 970}{30} = 9300 \text{ W} = 9.3 \text{ kW.}$$

Changes in motor speed when the load changes are neglected.

On catalog we accept motor AO63-6, P_{rat}=10 kW; n = 980 turns/min, η= 87%, $M_к/M_{rat}$ = 2.2, M_s/M_{rat} = 1.4.

Let us do a check on overload capacity and starting moment:

$$M_{rat} = 975 \frac{P_{rat}}{n} = 975 \frac{10}{980} \approx 10 \text{ kg·m;}$$

$$M_к = 10 \cdot 2.2 = 22 \text{ kg·m;}$$

$$M_s = 10 \cdot 1.4 = 14 \text{ kg·m.}$$

On transmission and starting properties motor come up.

Task 30.4. Using the catalog, select asynchronous short-circuit motor for lifting mechanism working in the repeatedly short-term mode with the schedule of load shown in figure 30.3. The speed of rotation is 900 turns/min.

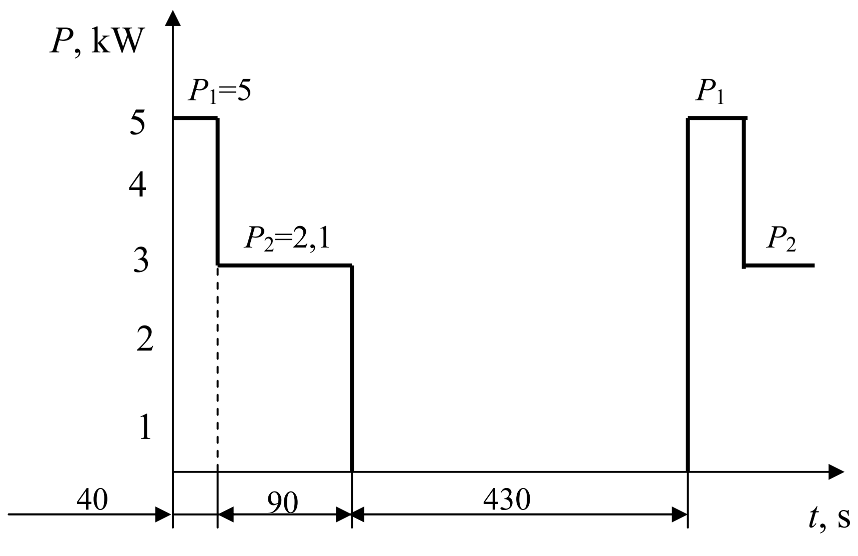


Figure 30.3: Load diagram

Solution. Determine the equivalent power for the working period

$$P_e = \sqrt{\frac{P_1^2 \cdot t_1 + P_2^2}{t_1 + t_2}} = \sqrt{\frac{5^2 \cdot 40 + 2.7^2 \cdot 90}{40 + 90}} = 3{,}6 \text{ kW.}$$

The actual duration of inclusion

$$\Pi B_1 = \frac{t_p}{t_p + t_0} \cdot 100 = \frac{t_1 + t_2}{t_1 + t_2 + t_0} \cdot 100 = \frac{40 + 90}{40 + 90 + 430} \cdot 100 = 23\% .$$

Let us recalculate the equivalent power on standard value ΠB_2=25%

$$P_e' = P_e \sqrt{\frac{\Pi B_1}{\Pi B_2}} = 3.6 \sqrt{\frac{23}{25}} = 3.46 \text{ kW}.$$

On value P_e'=3,46 kilowatt from the catalogue let us select the asynchronous short-circuit motor of crane series of type MD-6, for which at ID = 25%, P_{rat} = 3.5 kilowatt, n = 883 turns/min:

$$\frac{M_\kappa}{M_{rat}} = 2.6 ; \qquad \frac{M_s}{M_{rat}} = 2.6 .$$

The selected motor is also suitable for transmission and starting properties.

LIST OF ACCEPTED ABBREVIATIONS

AC – alternating current
AM – asynchronous machine
AMT – asynchronous motor
AFC – amplitude-frequency characteristic
ACS – air conditioning system

COP – coefficient of performance
CNandR – construction norms and rules

DC – direct current
DCM – DC motor
DCM – DC machine

EMI – electrical measuring instrument
EMF – electromotive force
ED – electric drive
EMM – electric manual machine

LLM – load-lifting machine

RAEI – the rules of arrangement of electrical installations
RTE – rules of technical exploitation
RSE (ПТБ) – rules of safety engineering

s.c. – short-circuit
SG – synchronous generator
SM – synchronous motor
SM – synchronous machine
SRD – starting-up and regulating device

TP – transformer point

USSD – unified system for structural documentation
VAC – volt-ampere characteristic

SUBJECT INDEX

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APPENDIXES

Appendix 1

The Greek alphabet

Sequence number	Letter	Sequence number	Sequence number
1	Α, α	13	Ν, ν
2	Β, β	14	Ξ, ξ
3	Γ, γ	15	Ο, ο
4	Δ, δ	16	Π, π
5	Ε, ε	17	Ρ, ρ
6	Ζ, ζ	18	Σ, σ
7	Η, η	19	Τ, τ
8	Θ, θ	20	Υ, υ
9	Ι, ι	21	Φ, φ
10	Κ, κ	22	Χ, χ
11	Λ, λ	23	Ψ, ψ
12	Μ, μ	24	Ω, ω

The Latin alphabet

Sequence number	Letter	Sequence number	Letter
1	A, α	14	N, n
2	B, b	15	O, o
3	C, c	16	P, p
4	D, d	17	Q, q
5	E, e	18	R, r
6	F, f	19	S, s
7	G, g	20	T, t
8	H, h	21	U, u
9	I, i	22	V, v
10	J, j	23	W, w
11	K, k	24	X, x
12	L, l	25	Y, y
13	M, m	26	Z, z

Appendix 2

Letter designations of the basic electrical and magnetic quantities

The letters of the Latin and Greek alphabets are used for the designations.
2.1 Designations by the Latin alphabet letters:
A – the magnetic vector potential.
B – magnetic induction.
B, b – reactive conductivity.
C – capacity.
E – intensity of electric field.
E, e – electromotive force.
F – magnetomotive force.
f – the oscillation frequency.
G, g – active conductivity.
H – intensity of magnetic field.
I, i – current.
j – current density.
L – self-inductance.
M – mutual inductance.
m – number of phases in multiphase systems.
N – number of turns.
n – transformation coefficient.
P – active power.
p – electric moment, number of pairs of poles.
Q – reactive power.
R, r – electrical resistance, active resistance.
S – apparent power.
T – period of oscillation.
U, u – voltage.
W – electromagnetic energy.
w – number of turns.
X, x – reactance.
Y, y – complex admittance.
Z, z – impedance.

2.2 Designations by the Greek alphabet letters:
γ – specific conductance.
δ – damping coefficient, the loss angle.
ε – dielectric capacitivity (ε_0 – electric constant).
λ – the length of the electromagnetic wave, power factor.
μ – magnetic permeability (μ_0 – magnetic constant).
ρ - resistivity constant.
Φ – magnetic flux.
φ – electric potential, the phase shift between current and voltage.
ψ – flux linkage.
Ω, ω – circular frequency of oscillations.

Constant electrical value and the current value of the variable are denoted by a capital letter; the instantaneous variable is denoted by a small letter. Amplitude value of the sine value is designated as the current value with the index m. For example:

I, U, E – constant current, voltage, EMF, active values of AC, voltage, EMF;

i, u, e – instantaneous value of a sinusoidal current, voltage, EMF;

I_m, U_m, E_m – the amplitude of the sinusoidal current, voltage, EMF.

2.3 Complex values are denoted as follows:

$$\underline{A} = A' + \mathrm{j}A'';$$

$$\underline{A} = \mathrm{Re}\,\underline{A} + \mathrm{jIm}\,\underline{A};$$

$$\underline{A} = A\,e^{j\alpha};$$

$$\underline{A} = |\underline{A}|e^{j\varepsilon};$$

$$\underline{A} = \cos\alpha + \mathrm{j}\sin\alpha,$$

where: $\underline{A}$ is a complex value;

$A' = \mathrm{Re}\,\underline{A}$ is the real part of the complex value;

$A'' = \mathrm{Im}\,\underline{A}$ is the imaginary part of the complex value;

$A = |\underline{A}|$ is the module of the complex value;

α is the argument of the complex value.

In the designation of electrical complex quantities the letter designations adopted for these quantities are used, for example:

$$\underline{S} = P + jQ$$

where: $\underline{S}$ is a apparent power;

P is an active power;

Q is a reactive power.

To denote complex quantities, which are sinusoidal functions of time, their main literal designation with a dot over it is used. For example:

$$\dot{I} = Ie^{j\alpha};$$

$$\dot{U} = U\cos\alpha + jU\sin\alpha;$$

$$\dot{\Phi} = \Phi' + j\Phi''.$$

Reduced complex value of the above marked with an asterisk, i.e:

$$\overset{*}{\underline{A}} = A' - \mathrm{j}A'';$$

$$\overset{*}{\underline{A}} = \mathrm{Re}\,\underline{A} - \mathrm{jIm}\,\underline{A};$$

$$\overset{*}{\underline{A}} = A\,e^{-j\alpha};$$

$$\overset{*}{\underline{A}} = |\underline{A}|e^{-j\varepsilon};$$

$$\overset{*}{\underline{A}} = \cos\alpha - \mathrm{j}\sin\alpha.$$

To denote vectors of current, voltage, EMF and other values their letter symbols are used with dashes above the magnitude of the vector: $\bar{I}$, $\bar{U}$, $\bar{E}$ etc.

Appendix 3

Units of the international system (SI)

Name of value	Units		
	Name	Dimension	International designation
Main units			
Length	metre	m	m
Weight	kilogram	kg	kg
Time	second	s	s
Electric current strength	ampere	A	A
Temperature	degree of Kelvin	K	K
Intensity of light	candela	cd	col
Derived units of electrical quantities			
Density of electric current	ampere per square metre	A/m^2	A/m^2
Quantity of electricity	coulomb	$A \cdot s$	C
Electric voltage, EMF	volt	$\frac{kg \cdot m^2}{A \cdot s}$	V
Electric capacity	farad	$\frac{A^2 \cdot s^4}{kg \cdot m^2} = \frac{s}{ohm}$	F
Electrical resistance	ohm	$\frac{kg \cdot m^2}{A^2 \cdot s^3} = \frac{V}{A}$	Ω
Specific electrical resistance	ohm on metre	$\frac{kg \cdot m^3}{A^2 \cdot s^3} = ohm \cdot m$	$\Omega \cdot m$
Electrical conductivity	siemens		S
Derived units of magnetic quantities			
Magnetic flux	weber	$\frac{kg \cdot m^2}{A \cdot s^2}$	Wb
Magnetic induction	tesla	$\frac{kg}{A \cdot s^2}$	
Magnetic field strength	ampere per metre	$\frac{A}{m}$	$\frac{A}{m}$
Inductance	Henry	$\frac{kg \cdot m^2}{A^2 \cdot s^2} = ohm \cdot s$	H

Continuation of appendix 3.

Name of value	Unit		
	Name	Dimension	International designation
Magnetic permeability	Henry per meter	$\frac{H}{m}$	$\frac{H}{m}$
Magnetomotive force	ampere	A	A
Electromagnetic energy	Joule		J
Active power	watt	$V\,A\cdot$	W
Reactive power	var	$V\,A$	var
Apparent power	volt-ampere	$V\,A$	VA

Appendix 4
Multipliers and prefixes for the formation of multiple and partial units

Multiplier	Name of prefix	International designation	Name of multiplier
10^{18}	Exa	E	quintillion
10^{15}	Peta	P	quadrillion
10^{12}	Tera	T	trillion
10^{9}	Giga	G	milliard
10^{6}	Mega	M	million
10^{3}	Kilo	k	thousand
10^{2}	Hecto	h	hundred
10^{1}	Deca	da	ten
10^{-1}	Deci	d	one-tenth
10^{-2}	Santi	c	one-hundredth
10^{-3}	Milli	m	one thousandth
10^{-6}	Micro	µ	one millionth
10^{-9}	Nano	n	one billionth
10^{-12}	Pico	p	one trillionth
10^{-15}	Femto	f	one quadrillionth
10^{-18}	Ato	a	one quintillionth

Appendix 5

The current load on the wires at welding

The conductor cut, mm^2	Permissible load of the welding wire, A			
	single-conductor		twin-core	
	long	repeatedly - short-term	long	repeatedly - short-term
The air temperature -5 ^0C				
10	119	165	198	275
16	158	220	251	248
25	211	293	330	460
35	251	349	396	550
50	310	431	489	679
70	383	533	620	863
95	467	650	-	-
120	546	760	-	-
The air temperature +25 ^0C				
10	90	125	150	208
16	120	167	190	264
25	160	222	250	348
35	190	264	300	416
50	235	327	370	514
70	290	404	470	654
95	354	492	-	-
120	414	575	-	-
The air temperature +30 ^0C				
10	71	99	118	164
16	95	132	150	203
25	126	175	197	275
35	160	208	237	320
50	186	258	292	405
70	229	319	371	515
95	280	382	-	-
120	327	458	-	-

Note. With repeatedly – short-term load the main welding time is not more than 4 minutes, total cycle time is 10 minutes.

Appendix 6

DC motors of continuous regime type ПН, 220 V, protected with a speed regulation of turn up to 1:2

Type	P_{nom}, kW	N_{nom}, turns/min	$I_{a.nom}$, A	The number of active wires of the armature, N	The number of parallel branches of armature	The number of pairs of poles, 2p	Resistance			The turns on the pole		Nominal current of parallel winding, A	Nominal flux of pole Ф, mWb	Moment of inertia of the armature J, kg m^2	Motor weight, kg
							Armature+additional poles, $R_a+R_{a.p}$, Ω	Stabilizing winding, $R_{s.w}$, Ω	Parallel winding, $R_{par.w}$, Ω	Stabilizing winding, $w_{s.w}$	Parallel winding, $w_{par.w}$				
ПН-45	2.5	1000-1950	14.1	1218	2	4	1.37	0.11	372	12	2400	0.48	4.8	0.07	107
	4.4	1500-2100	23.5	812	2	4	0.6	0.043	298	9	2300	0.6	5.0	0.07	107
	6.6	2200- 2400	35.0	522	2	4	0.26	0.043	298	9	2300	0.6	5.4	0.07	107
ПН-68	3.7	1000- 1750	21.0	1116	2	4	0.89	0.05	250	11	2200	0.71	5.2	0.125	135
	6.5	1550- 2000	35.0	744	2	4	0.364	0.012	250	6	2200	0.71	5.2	0.125	135
	10.0	2250- 2400	52.2	496	2	4	0.17	0.012	188	6	1900	0.95	5.6	0.125	135
ПН-85	5.6	1000-1750	30.0	744	2	4	0.48	0.04	228	8	1750	0.78	8.1	0.16	175
	9.0	1500- 2000	48.0	496	2	4	0.22	0.02	228	6	1750	0.78	8.3	0.16	175
ПН-100	5.8	780- 1500	34.0	1112	2	4	0.62	0.013	136	6	1800	1.3	7.0	0.4	290
	10.0	1090- 1900	55.0	834	2	4	0.33	0.009	136	6	1800	1.3	6.6	0.4	290
	15.0	1560- 2000	81.5	556	2	4	0.143	0.004	96	4	1500	1.8	7.1	0.4	175
ПН-145	8.5	780- 1500	46.5	834	2	4	0.36	0.014	166	8	1700	1.1	9.3	0.5	330
	13.5	1050- 1500	73.0	556	2	4	0.12	0.006	150	4	1700	1.2	10.5	0.5	330
	21.0	1500- 1900	110	420	2	4	0.09	0.004	150	4	1700	1.2	9.7	0.5	330
ПН-205	14.0	750- 1500	76	700	2	4	0.22	0.01	92	8	1500	1.9	11.5	1.0	480
	20.5	970- 1700	110	556	2	4	0.08	0.006	92	6	1500	1.9	11.4	1.0	480
	33.5	1580- 1900	174	350	2	4	0.054	0.002	92	3	1500	1.9	11.2	1.0	480
ПН-290	19.0	750- 1300	102	524	2	4	0.122	0.007	86	6	1400	2.1	15.6	1.2	530
	29.0	1000- 1600	151	396	2	4	0.070	0.002	57	3	1250	3.1	15.5	1.2	530
	46.5	1500- 1800	238	278	2	4	0.035	0.002	59	3	1250	3.0	15.0	1.2	530

Appendix 7

Crane asynchronous motors with phase rotor of types MT and MTB, 380 V, 50 Hz, duration of inclusion = 25% (isolation of class E, B)

Type	P_{nom}, kW	$n_н$, turns/min	M_{max}/M_{nom}	Stator						Rotor					Moment of inertia of rotor J, kg m^2	Motor weight, kg
				cosφ		*I*st.nom, A	*I*st.idl, A	*R*st, ohm	*X*st, ohm	*E*r, V	*I*r.nom, A	*R*r, Ω	*X*r, Ω	Transformation coefficient of voltage k_e ($k_R = k_e^2$)		
				rated	idle											
MT011-6	1.4	885	2.3	0.65	0.15	5.3	3.9	5.98	3.93	112	9.3	0.695	0.57	3.14	0.0212	51
MT012-6	2.2	895	2.3	0.67	0.13	7.5	5.4	3.6	2.58	144	11.0	0.67	0.58	2.5	0.0288	58
MT111-6	3.5	915	2.3	0.70	0.11	10.5	6.6	2.16	2.03	181	13.7	0.525	0.755	1.96	0.0488	76
MT112-6	5.0	925	2.5	0.69	0.12	14.8	9.5	1.32	1.39	206	16.6	0.50	0.43	1.72	0.0675	88
MT211-6	7.5	935	2.5	0.70	0.09	20.8	11.8	0.68	1.07	255	19.8	0.44	0.88	1.38	0.115	120
MTB311-6	11.0	945	2.8	0.73	0.09	28.6	16.7	0.54	0.57	172	42.5	0.11	0.225	2.1	0.225	170
MTB311-8	7.5	695	2.5	0.71	0.09	21.0	14.0	0.88	0.965	251	20.5	0.47	0.72	1.41	0.275	210
MTB312-6	16	955	2.8	0.77	0.08	37.6	20.6	0.33	0.41	208	49.5	0.0099	0.25	1.75	0.313	210
MTB312-8	11	710	2.8	0.66	0.10	33.0	22.1	0.53	0.56	182	41.0	0.13	0.23	1.96	0.387	210
MTB411-6	22	965	2.8	0.71	0.07	55.0	33.2	0.19	0.31	225	61.0	0.066	0.23	1.6	0.5	280
MTB411-8	16	715	2.8	0.65	0.08	45.7	30.2	0.285	0.43	207	49.5	0.103	0.25	1.73	0.538	280
MTB412-6	30	970	2.8	0.73	0.06	70.5	42.0	0.125	0.23	259	72.0	0.055	0.225	1.4	0.675	345
MTB412-8	22	720	2.8	0.69	0.07	58.0	37.1	0.207	0.32	234	59.0	0.09	0.24	1.53	0.75	345
MTB511-6	30	720	2.8	0.68	0.06	77.0	46.0	0.123	0.245	280	67.5	0.082	0.28	1.28	1.025	410
MTB512-8	40	730	2.8	0.69	0.06	101	60	0.08	0.17	322	76.5	0.072	0.24	1.12	1.4	500

Appendix 8

Crane asynchronous motors with short-circuit rotor of types MTK and MTKB, 380 V, 50 Hz, duration of inclusion = 25% (isolation of class E, B)

Type	P_{nom}, kW	$n_н$, turns/min	M_{max}/M_{nom}	M_{start}/ M_{nom}	Stator							
					$I_{st.start}$/ $I_{st.nom}$	cosφ			$I_{st.nom}$, A	$I_{st.idl}$, A	R_{st}, Ω	Xst, Ω
						starting	rated	idle				
MTK011-6	1.4	870	2.8	2.8	3.0	0.86	0.69	0.15	4.8	3.2	5.98	3.93
MTK012-6	2.2	875	2.8	2.8	3.1	0.85	0.7	0.13	7.2	4.6	3.6	2.58
MTK111-6	3.5	870	2.8	2.8	3.5	0.85	0.74	0.11	10.1	6.1	2.16	2.03
MTK112-6	5.0	890	3.0	3.0	3.9	0.80	0.75	0.12	13.5	8.4	1.32	1.39
MTK211-6	7.5	905	2.9	2.8	4.3	0.72	0.79	0.09	18.4	11.0	0.68	1.07
MTKB311-6	11.0	910	3.1	2.8	4.9	0.72	0.8	0.09	26	15.7	0.54	0.575
MTKB311-8	7.5	680	3.1	2.9	4.4	0.76	0.74	0.09	20	13.2	0.88	0.965
MTKB312-6	16	905	3.1	2.8	4.9	0.70	0.79	0.08	37.8	19.8	0.33	0.41
MTKB312-8	11	690	3.1	3.1	4.6	0.77	0.71	0.10	30.4	21.3	0.53	0.56
MTKB411-6	22	935	3.0	2.8	5.2	0.61	0.78	0.07	50	29.7	0.19	0.31
MTKB411-8	16	695	3.3	3.0	4.8	0.66	0.73	0.08	41	28.2	0.285	0.43
MTKB412-6	28	945	3.3	2.8	5.6	0.60	0.81	0.06	62	35.1	0.125	0.23
MTKB412-8	22	695	3.3	3.0	5.0	0.65	0.76	0.07	53.2	35.8	0.207	0.32
MTKB511-6	28	700	3.4	3.1	5.4	0.61	0.75	0.06	68	40	0.123	0.245
MTKB512-8	37	705	3.6	3.3	5.8	0.61	0.72	0.06	91	55	0.08	0.17

Appendix 9

Technical parameters of synchronous motors

Type	$U_н$, kV	$S_н$, kVA	$P_н$, kW	$n_с$, turns/min	cos$\varphi_н$, passing ahead	$I_{stator.nom}$, A	Iр.н at cos$\varphi_н$, A	Iр.н.idl, A	$M_м/M_н$
ДС3-2121-16	10	17000	14070	375	0.85	983	561	350	2.1
МС325-12/12	10	7000	5400	500	0.8	404	367	212	2.4
МС325-12/12	6	8000	6150	500	0.8	770	408	211	1.85
МС321-7/6	6	900	675	1000	0.8	86.5	278	139	2.0
МС323-14/8	3	3850	3300	750	0.9	740	466	295	2.25
МС321-6/6	3	850	640	1000	0.8	164	164	190	2.25

Continuation of appendix 9

Type	Asynchronous start				Moment of inertia of rotor J, tonne m^2	Motor weight, tonne	Number of turns		Winding coefficient of the stator, $k_{об}$	Pole division τ, mm
	$I_{с.п}/I_{с.н}$	$M_п/M_н$	$M_{вх}/M_н$	cos$\varphi_н$			phases of stator, $w_с$	one pole of rotor		
ДС3-2121-16	6.1	0.545	1.88	0.1	50	110	72	44.5	0.945	536
МС325-12/12	6.5	0.65	1.1	0.149	12.7	54	126	54.5	0.89	607
МС325-12/12	5.8	0.7	1.29	0.151	15.7	56	72	54.5	0.918	607
МС321-7/6	5.5	1	0.8	0.281	0.135	5.7	216	49.5	0.945	420
МС323-14/8	6	0.8	0.8	0.196	1.32	14	45	45.5	0.945	500
МС321-6/6	4.6	0.8	0.65	0.28	0.12	5.2	135	43.5	0.88	420

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